Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.2-Cosine/86-4.2.1.2-g-sin-^p-a+b-cos-^m

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September 5, 2023 Compiled on September 5, 2023 at 4:10pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [88]. This is test number [86].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (88)	0.00 (0)
Mathematica	100.00 (88)	0.00 (0)
Maple	100.00 (88)	0.00 (0)
Fricas	64.77 (57)	35.23 (31)
Mupad	38.64 (34)	61.36 (54)
Giac	36.36 (32)	63.64 (56)
Maxima	30.68 (27)	69.32 (61)
Sympy	26.14 (23)	73.86 (65)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

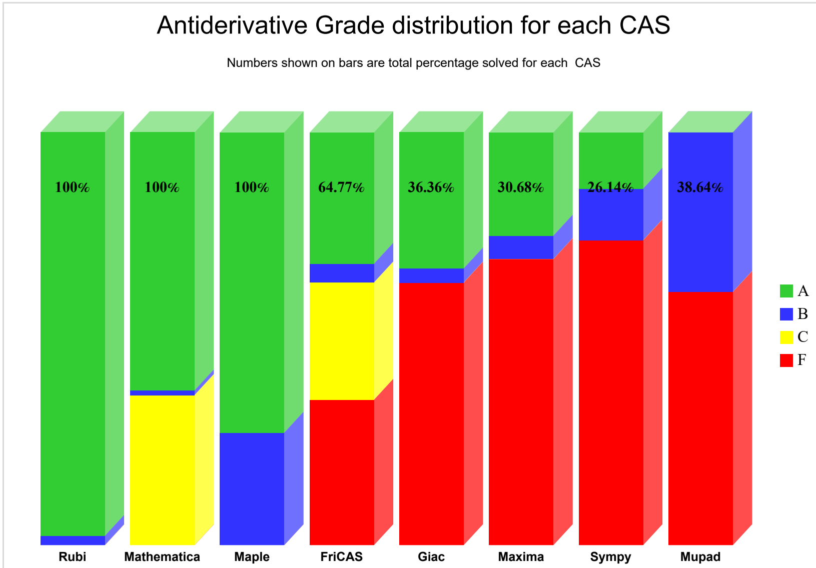
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

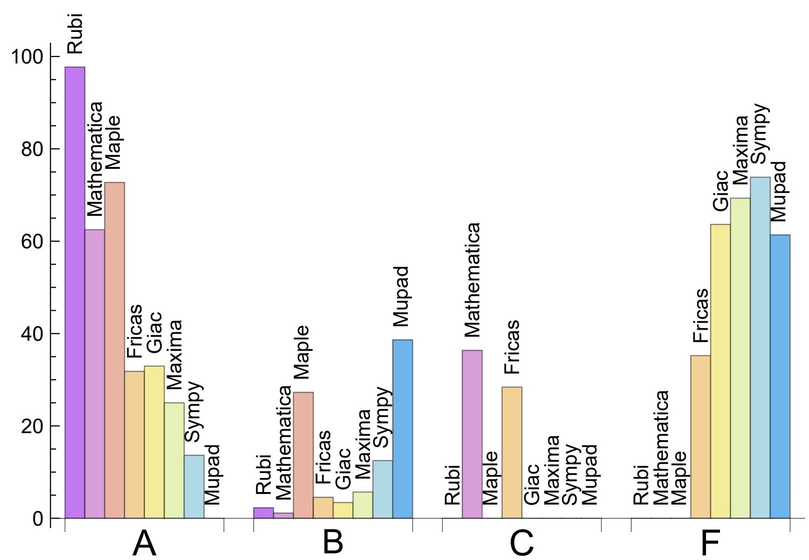
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.727	2.273	0.000	0.000
Maple	72.727	27.273	0.000	0.000
Mathematica	62.500	1.136	36.364	0.000
Giac	32.955	3.409	0.000	63.636
Fricas	31.818	4.545	28.409	35.227
Maxima	25.000	5.682	0.000	69.318
Sympy	13.636	12.500	0.000	73.864
Mupad	0.000	38.636	0.000	61.364

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Fricas	31	9.68	90.32	0.00
Mupad	54	0.00	100.00	0.00
Giac	56	100.00	0.00	0.00
Maxima	61	72.13	19.67	8.20
Sympy	65	47.69	52.31	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.19
Maxima	0.26
Giac	0.27
Rubi	0.63
Mathematica	4.84
Maple	9.89
Sympy	10.45
Mupad	10.79

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	27.85	1.32	14.00	1.00
Giac	40.88	1.22	14.00	1.01
Mupad	80.32	1.45	13.00	0.93
Fricas	107.60	1.41	105.00	1.19
Sympy	149.30	4.82	15.00	2.00
Rubi	226.90	1.05	142.50	1.00
Mathematica	403.90	1.34	101.50	1.07
Maple	672.16	2.01	227.50	1.59

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

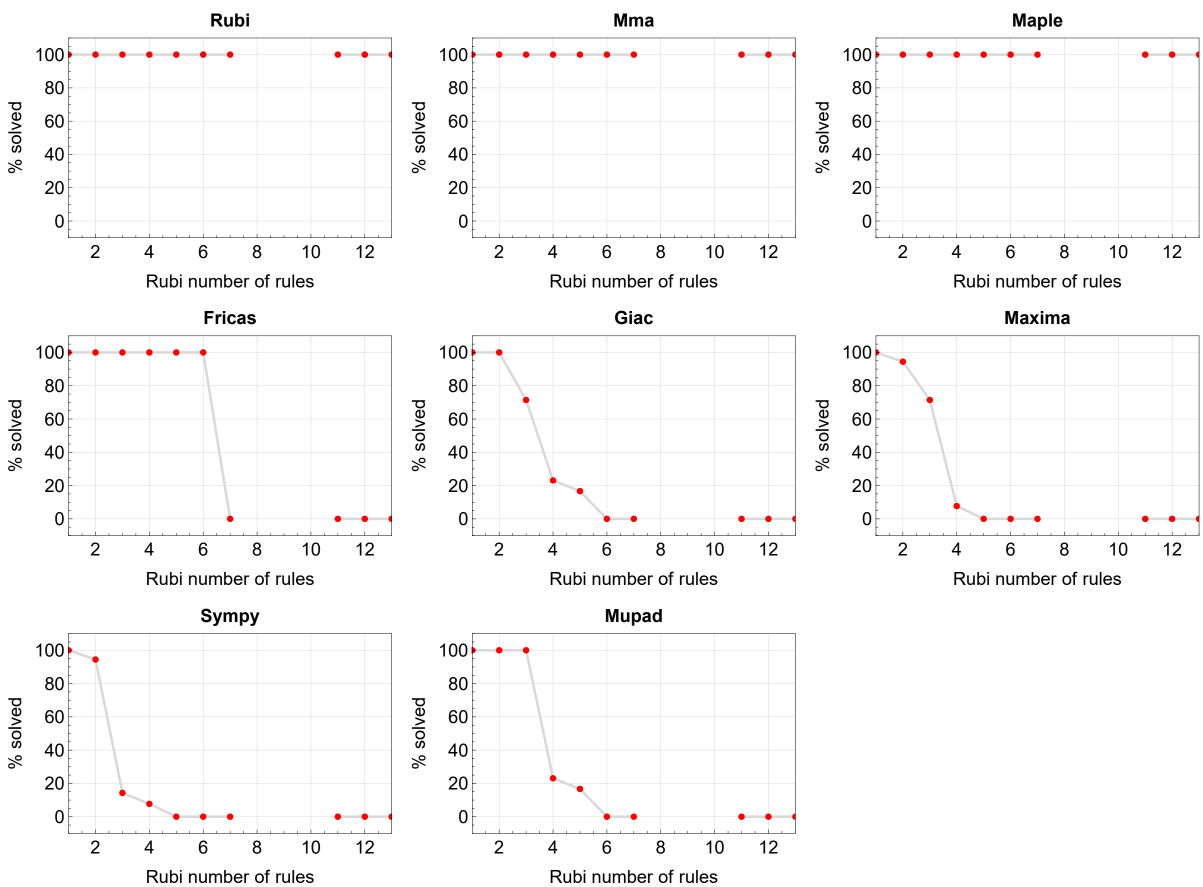


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

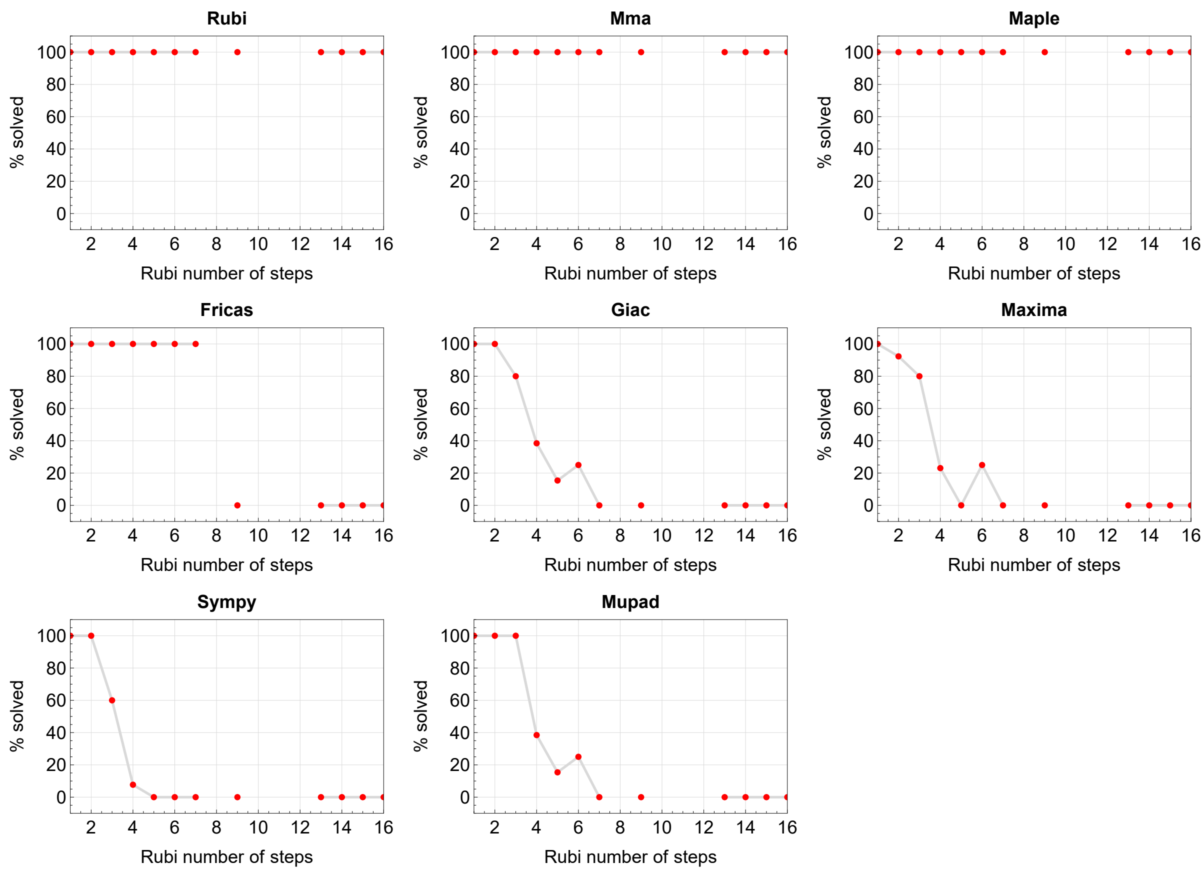


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

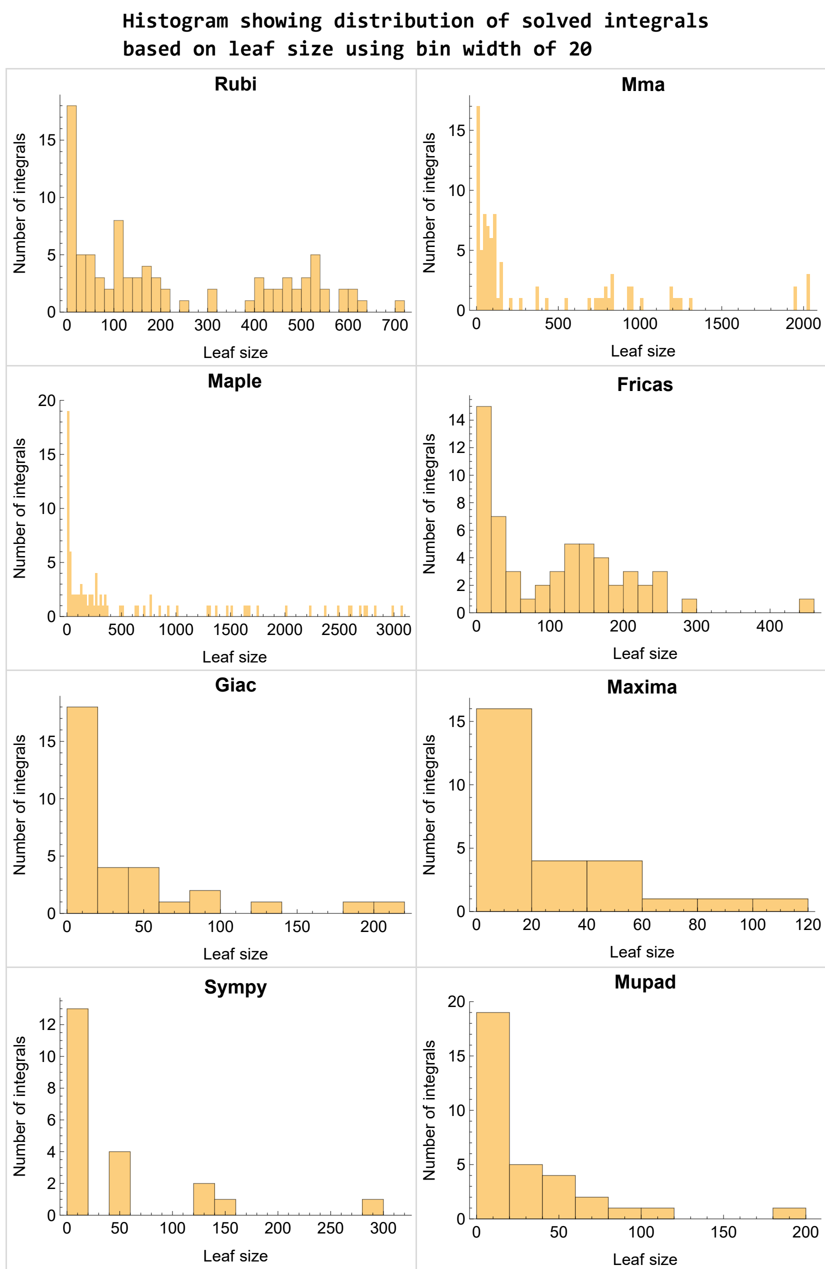


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

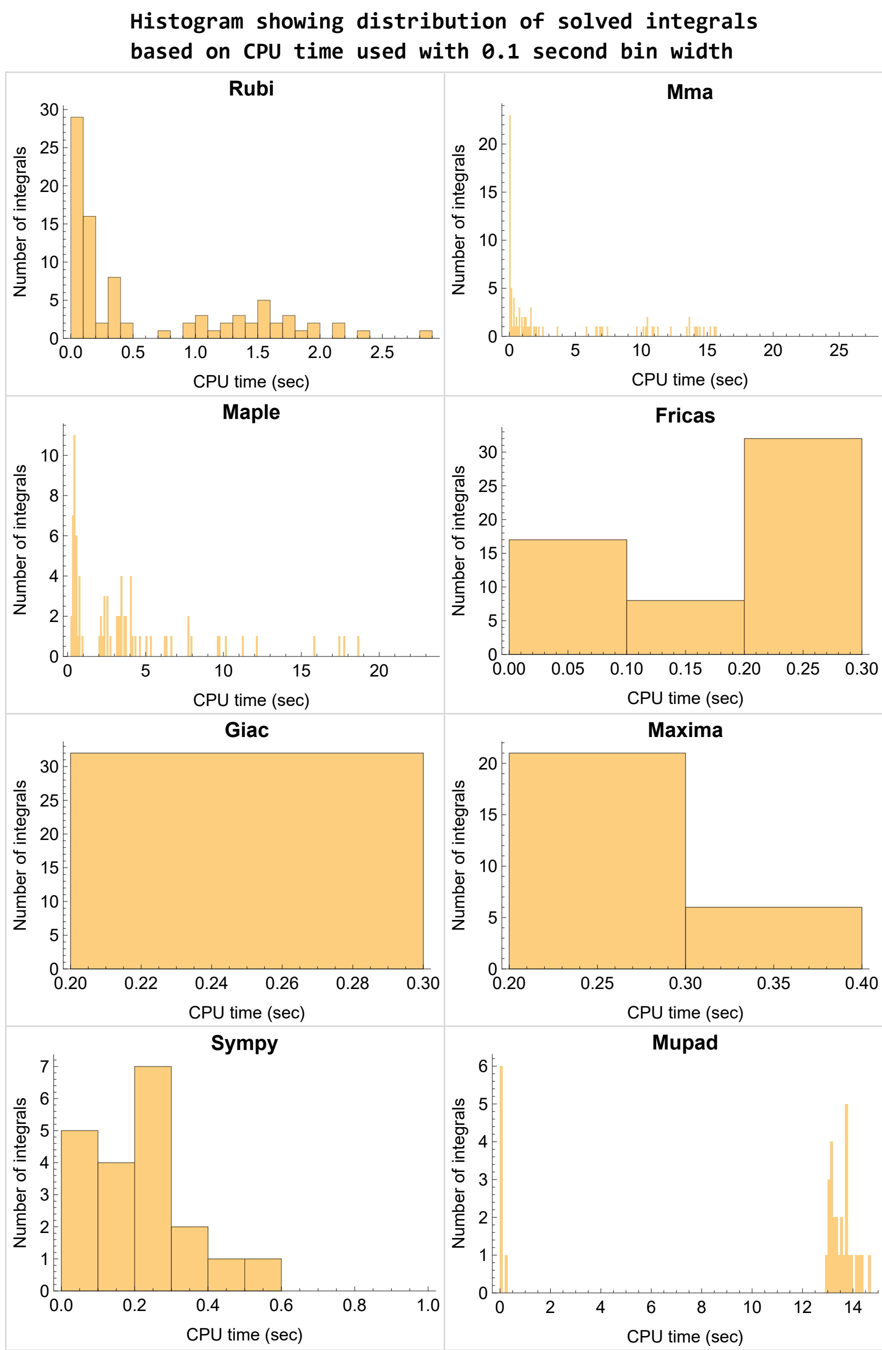


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

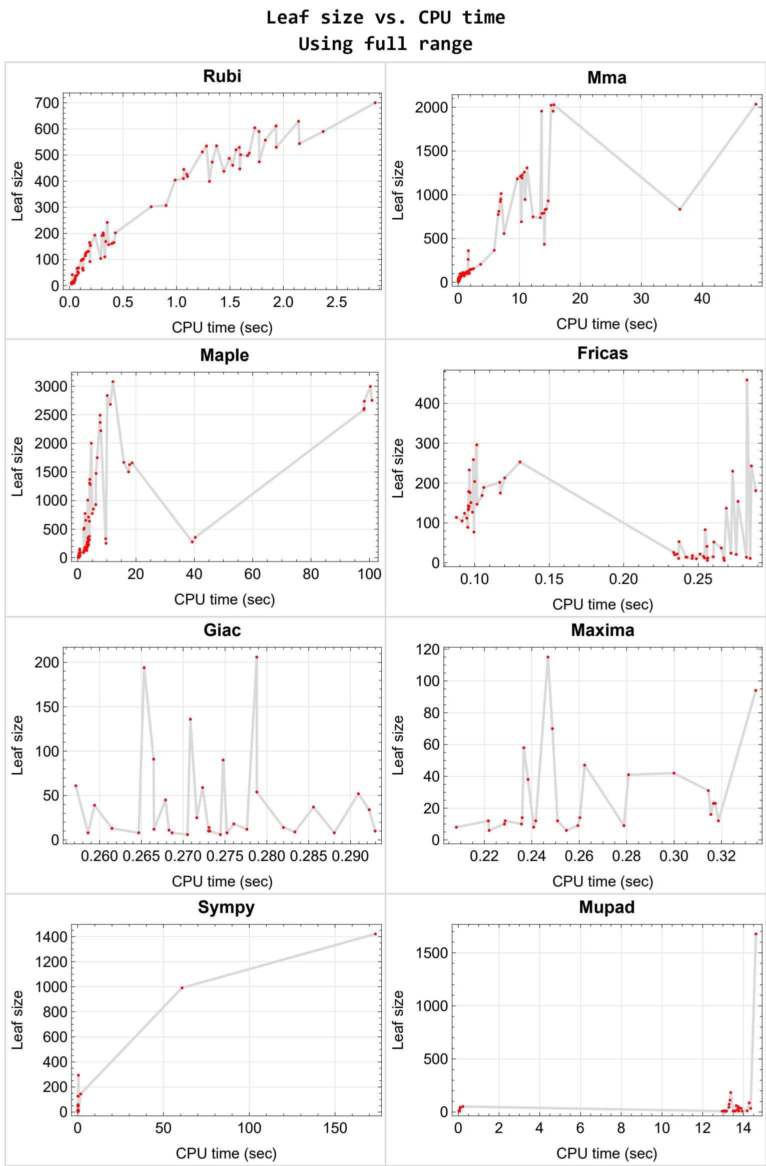


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88}

Maple {58, 68, 77, 78, 79, 81, 86, 87, 88}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command AbsoluteTiming[] was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command Usage was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's time.time() call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is ValueError. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

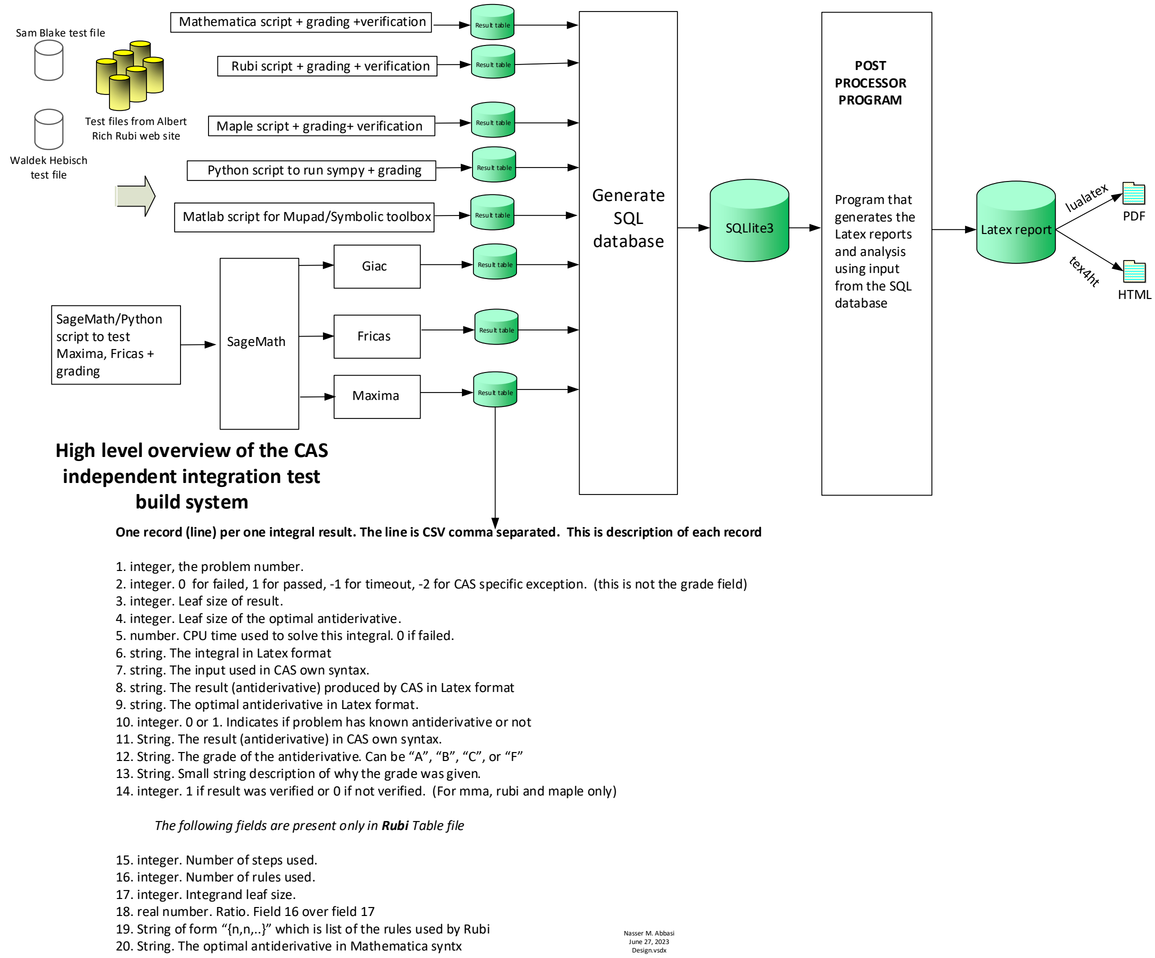
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

B grade { 10, 11 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57 }

B grade { 11 }

C grade { 15, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67 }

B grade { 11, 44, 46, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30 }

B grade { 10, 11, 31, 32 }

C grade { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57 }

F normal fail { 63, 66, 74 }

F(-1) timedout fail { 58, 59, 60, 61, 62, 64, 65, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

F(-2) exception fail { }

Maxima

A grade { 2, 4, 5, 6, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 27, 29, 31 }

B grade { 1, 3, 7, 9, 11 }

C grade { }

F normal fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 81, 82, 83, 84 }

F(-1) timedout fail { 67, 74, 75, 76, 77, 78, 79, 80, 85, 86, 87, 88 }

F(-2) exception fail { 24, 26, 28, 30, 32 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31 }

B grade { 11, 24, 32 }

C grade { }

F normal fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37 }

C grade { }

F normal fail { }

F(-1) timedout fail { 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

F(-2) exception fail { }

Sympy

A grade { 4, 5, 10, 12, 13, 14, 15, 18, 19, 20, 21, 27 }

B grade { 1, 2, 3, 11, 16, 17, 22, 23, 25, 26, 28 }

C grade { }

F normal fail { 6, 7, 8, 9, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 62, 63, 64, 65, 66, 73, 74 }

F(-1) timedout fail { 24, 33, 40, 41, 42, 48, 49, 50, 56, 57, 58, 59, 60, 61, 67, 68, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	25	24	94	24	294	45	34
N.S.	1	1.00	0.81	0.77	3.03	0.77	9.48	1.45	1.10
time (sec)	N/A	0.052	0.200	0.428	0.334	0.272	0.440	0.268	14.354

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	13	16	14	14	51	14	11
N.S.	1	1.00	0.68	0.84	0.74	0.74	2.68	0.74	0.58
time (sec)	N/A	0.043	0.015	0.445	0.236	0.282	0.248	0.282	14.184

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	17	11	42	10	46	25	10
N.S.	1	1.00	1.31	0.85	3.23	0.77	3.54	1.92	0.77
time (sec)	N/A	0.043	0.044	0.405	0.300	0.249	0.177	0.272	13.503

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	12	13	12	12	8	10	10
N.S.	1	1.00	1.20	1.30	1.20	1.20	0.80	1.00	1.00
time (sec)	N/A	0.027	0.008	0.348	0.242	0.267	0.063	0.293	13.597

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	9	12	11	5	8	8
N.S.	1	1.00	0.91	0.82	1.09	1.00	0.45	0.73	0.73
time (sec)	N/A	0.016	0.007	0.204	0.251	0.237	0.098	0.288	13.936

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	42	20	31	37	0	34	20
N.S.	1	1.00	1.83	0.87	1.35	1.61	0.00	1.48	0.87
time (sec)	N/A	0.054	0.032	0.562	0.314	0.266	0.000	0.292	13.705

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	30	25	41	26	0	37	35
N.S.	1	1.00	1.25	1.04	1.71	1.08	0.00	1.54	1.46
time (sec)	N/A	0.050	0.192	0.592	0.281	0.234	0.000	0.286	13.884

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	60	36	58	83	0	52	45
N.S.	1	1.00	1.22	0.73	1.18	1.69	0.00	1.06	0.92
time (sec)	N/A	0.084	0.126	0.630	0.237	0.255	0.000	0.291	0.088

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	38	45	70	53	0	59	45
N.S.	1	1.00	1.03	1.22	1.89	1.43	0.00	1.59	1.22
time (sec)	N/A	0.056	0.187	0.546	0.249	0.237	0.000	0.272	13.288

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	11	5	8	9	11	10	9	7
N.S.	1	2.20	1.00	1.60	1.80	2.20	2.00	1.80	1.40
time (sec)	N/A	0.021	0.007	0.420	0.279	0.246	0.054	0.283	0.062

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	13	7	12	9	11	8	11	9
N.S.	1	4.33	2.33	4.00	3.00	3.67	2.67	3.67	3.00
time (sec)	N/A	0.023	0.009	0.422	0.259	0.285	0.054	0.268	13.169

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	12	7	6	6	5	6	6
N.S.	1	1.00	2.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.020	0.011	0.309	0.222	0.256	0.164	0.271	13.072

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	6	6	5	6	6
N.S.	1	1.00	1.20	0.90	0.60	0.60	0.50	0.60	0.60
time (sec)	N/A	0.021	0.012	0.318	0.255	0.267	0.164	0.274	13.500

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	11	23	18	7	10	10
N.S.	1	1.00	1.29	0.79	1.64	1.29	0.50	0.71	0.71
time (sec)	N/A	0.033	0.008	0.358	0.317	0.246	0.215	0.273	13.112

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	26	17	23	16	8	12	10
N.S.	1	1.00	1.62	1.06	1.44	1.00	0.50	0.75	0.62
time (sec)	N/A	0.035	0.016	0.349	0.317	0.254	0.353	0.267	13.771

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	13	11	10	12	58	10	10
N.S.	1	1.00	1.30	1.10	1.00	1.20	5.80	1.00	1.00
time (sec)	N/A	0.039	0.023	0.468	0.228	0.256	0.195	0.273	13.760

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	13	11	10	12	58	12	10
N.S.	1	1.00	1.08	0.92	0.83	1.00	4.83	1.00	0.83
time (sec)	N/A	0.037	0.019	0.559	0.236	0.254	0.202	0.278	13.121

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	8	14	14	8	8
N.S.	1	1.00	1.20	0.90	0.80	1.40	1.40	0.80	0.80
time (sec)	N/A	0.021	0.010	0.423	0.241	0.242	0.244	0.265	0.037

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	8	14	15	8	8
N.S.	1	1.00	1.00	0.92	0.67	1.17	1.25	0.67	0.67
time (sec)	N/A	0.021	0.013	0.426	0.208	0.242	0.242	0.259	12.952

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	12	20	7	8	8
N.S.	1	1.00	0.86	0.64	0.86	1.43	0.50	0.57	0.57
time (sec)	N/A	0.032	0.083	0.311	0.222	0.234	0.341	0.275	13.047

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	9	12	22	10	8	8
N.S.	1	1.00	0.75	0.56	0.75	1.38	0.62	0.50	0.50
time (sec)	N/A	0.035	0.089	0.396	0.229	0.236	0.551	0.269	13.117

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	14	21	126	14	14
N.S.	1	1.00	1.29	1.07	1.00	1.50	9.00	1.00	1.00
time (sec)	N/A	0.045	0.011	0.574	0.260	0.276	0.264	0.273	0.047

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	29	17	16	22	126	18	16
N.S.	1	1.00	1.45	0.85	0.80	1.10	6.30	0.90	0.80
time (sec)	N/A	0.041	0.015	0.474	0.316	0.251	0.256	0.276	0.047

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	96	152	0	243	0	194	1677
N.S.	1	1.00	0.92	1.46	0.00	2.34	0.00	1.87	16.12
time (sec)	N/A	0.292	0.263	0.712	0.000	0.286	0.000	0.265	14.612

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	38	41	1421	39	38
N.S.	1	1.00	1.00	0.98	0.95	1.02	35.52	0.98	0.95
time (sec)	N/A	0.070	0.088	0.795	0.238	0.256	173.474	0.259	0.100

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	78	0	154	991	90	74
N.S.	1	1.00	0.92	1.32	0.00	2.61	16.80	1.53	1.25
time (sec)	N/A	0.126	0.109	0.464	0.000	0.277	60.780	0.275	13.298

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	15	17	13	12
N.S.	1	1.00	1.00	1.08	1.00	1.25	1.42	1.08	1.00
time (sec)	N/A	0.025	0.025	0.421	0.319	0.260	0.093	0.261	13.042

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	41	36	0	137	144	61	38
N.S.	1	1.00	0.98	0.86	0.00	3.26	3.43	1.45	0.90
time (sec)	N/A	0.026	0.031	0.286	0.000	0.269	1.710	0.257	13.776

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	47	47	52	0	54	52
N.S.	1	1.00	0.94	0.89	0.89	0.98	0.00	1.02	0.98
time (sec)	N/A	0.076	0.062	0.754	0.262	0.261	0.000	0.279	0.226

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	66	78	0	230	0	91	86
N.S.	1	1.00	0.99	1.16	0.00	3.43	0.00	1.36	1.28
time (sec)	N/A	0.122	0.392	0.592	0.000	0.273	0.000	0.266	14.283

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	99	91	115	181	0	136	112
N.S.	1	1.00	1.08	0.99	1.25	1.97	0.00	1.48	1.22
time (sec)	N/A	0.191	0.510	0.934	0.247	0.289	0.000	0.271	13.340

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	112	127	0	459	0	206	184
N.S.	1	1.00	1.02	1.15	0.00	4.17	0.00	1.87	1.67
time (sec)	N/A	0.329	0.763	0.729	0.000	0.283	0.000	0.279	13.374

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	108	127	0	143	0	0	0
N.S.	1	1.00	0.84	0.98	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.163	1.188	3.177	0.000	0.096	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	80	171	0	127	0	0	0
N.S.	1	1.00	0.80	1.71	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.125	0.776	3.214	0.000	0.098	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	80	116	0	105	0	0	0
N.S.	1	1.00	0.80	1.16	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.116	0.676	2.161	0.000	0.091	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	60	117	0	89	0	0	60
N.S.	1	1.00	0.88	1.72	0.00	1.31	0.00	0.00	0.88
time (sec)	N/A	0.085	0.316	2.332	0.000	0.095	0.000	0.000	13.648

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	54	92	0	77	0	0	50
N.S.	1	1.00	0.82	1.39	0.00	1.17	0.00	0.00	0.76
time (sec)	N/A	0.070	0.387	2.013	0.000	0.099	0.000	0.000	13.744

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	58	153	0	114	0	0	0
N.S.	1	1.00	0.60	1.59	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.109	0.327	2.359	0.000	0.088	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	59	124	0	135	0	0	0
N.S.	1	1.00	0.58	1.22	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	0.129	0.402	2.341	0.000	0.096	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	74	187	0	179	0	0	0
N.S.	1	1.00	0.56	1.43	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.176	0.568	2.568	0.000	0.096	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	157	252	0	202	0	0	0
N.S.	1	1.00	0.81	1.31	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.236	2.519	9.724	0.000	0.117	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	116	332	0	175	0	0	0
N.S.	1	1.00	0.75	2.16	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.194	1.320	9.602	0.000	0.117	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	117	229	0	151	0	0	0
N.S.	1	1.00	0.76	1.49	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.196	1.295	3.145	0.000	0.097	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	83	272	0	124	0	0	0
N.S.	1	1.00	0.73	2.39	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.146	0.703	3.290	0.000	0.093	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	79	170	0	112	0	0	0
N.S.	1	1.00	0.69	1.49	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.148	0.987	2.540	0.000	0.095	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	75	283	0	141	0	0	0
N.S.	1	1.00	0.64	2.40	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.149	0.961	3.485	0.000	0.096	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	76	202	0	176	0	0	0
N.S.	1	1.00	0.61	1.63	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.150	1.004	3.371	0.000	0.097	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	109	351	0	233	0	0	0
N.S.	1	1.00	0.66	2.13	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.188	1.413	3.495	0.000	0.096	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	205	276	0	253	0	0	0
N.S.	1	1.00	0.85	1.14	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.352	3.651	39.301	0.000	0.130	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	149	356	0	213	0	0	0
N.S.	1	1.00	0.74	1.76	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.315	2.243	40.314	0.000	0.120	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	147	272	0	189	0	0	0
N.S.	1	1.00	0.73	1.35	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.429	2.039	3.647	0.000	0.106	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	105	314	0	147	0	0	0
N.S.	1	1.00	0.65	1.95	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.395	1.226	4.064	0.000	0.101	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	98	210	0	133	0	0	0
N.S.	1	1.00	0.62	1.34	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.366	1.506	3.478	0.000	0.096	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	101	313	0	169	0	0	0
N.S.	1	1.00	0.61	1.90	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.415	1.100	3.637	0.000	0.105	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	102	226	0	204	0	0	0
N.S.	1	1.00	0.60	1.34	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.339	1.834	3.766	0.000	0.100	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	130	375	0	259	0	0	0
N.S.	1	1.00	0.68	1.95	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.304	1.680	4.020	0.000	0.099	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	144	265	0	296	0	0	0
N.S.	1	1.00	0.75	1.37	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.317	1.902	3.319	0.000	0.101	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	544	544	2035	930	0	0	0	0	0
N.S.	1	1.00	3.74	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.151	48.769	6.219	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	461	461	834	851	0	0	0	0	0
N.S.	1	1.00	1.81	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.525	36.314	5.372	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	1955	773	0	0	0	0	0
N.S.	1	1.00	4.12	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.774	15.539	5.013	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	692	639	0	0	0	0	0
N.S.	1	1.00	1.73	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.308	10.333	4.025	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	434	654	0	0	0	0	0
N.S.	1	1.00	1.06	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.065	14.106	2.770	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	361	496	0	0	0	0	0
N.S.	1	1.00	1.20	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.763	1.648	2.114	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	261	519	0	0	0	0	0
N.S.	1	1.00	0.85	1.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.902	1.616	2.232	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	791	774	0	0	0	0	0
N.S.	1	1.00	1.86	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.096	14.030	2.551	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	447	447	1192	711	0	0	0	0	0
N.S.	1	1.00	2.67	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.592	10.452	3.720	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	501	501	811	1007	0	0	0	0	0
N.S.	1	1.00	1.62	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.600	6.656	3.464	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	557	557	2029	1658	0	0	0	0	0
N.S.	1	1.00	3.64	2.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.831	15.671	18.677	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	739	1628	0	0	0	0	0
N.S.	1	1.00	1.56	3.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.334	13.435	17.785	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	1956	1501	0	0	0	0	0
N.S.	1	1.00	4.02	3.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.494	13.655	17.464	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	366	1668	0	0	0	0	0
N.S.	1	1.00	0.91	4.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.988	5.896	15.829	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	557	1370	0	0	0	0	0
N.S.	1	1.00	1.33	3.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.103	7.490	4.174	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	786	1306	0	0	0	0	0
N.S.	1	1.00	1.79	2.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.443	13.695	4.089	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	445	445	1182	1280	0	0	0	0	0
N.S.	1	1.00	2.66	2.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.068	9.667	4.308	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	507	507	774	2002	0	0	0	0	0
N.S.	1	1.00	1.53	3.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.681	6.541	4.672	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	530	530	1257	1474	0	0	0	0	0
N.S.	1	1.00	2.37	2.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.933	10.807	6.300	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	590	590	950	1749	0	0	0	0	0
N.S.	1	1.00	1.61	2.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.372	6.949	6.688	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	590	590	930	2995	0	0	0	0	0
N.S.	1	1.00	1.58	5.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.772	14.732	100.322	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	604	604	2024	2752	0	0	0	0	0
N.S.	1	1.00	3.35	4.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.732	15.205	100.882	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	498	498	837	2736	0	0	0	0	0
N.S.	1	1.00	1.68	5.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.665	14.435	98.270	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	512	512	946	2589	0	0	0	0	0
N.S.	1	1.00	1.85	5.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.241	10.942	98.093	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	520	520	831	2612	0	0	0	0	0
N.S.	1	1.00	1.60	5.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.557	14.238	98.230	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	1211	2493	0	0	0	0	0
N.S.	1	1.00	2.27	4.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.280	10.180	7.706	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	529	529	748	2365	0	0	0	0	0
N.S.	1	1.00	1.41	4.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.588	12.239	7.733	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	535	535	1226	2220	0	0	0	0	0
N.S.	1	1.00	2.29	4.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.375	10.454	7.953	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	611	611	922	2837	0	0	0	0	0
N.S.	1	1.00	1.51	4.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.931	6.894	10.136	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	629	629	1308	2681	0	0	0	0	0
N.S.	1	1.00	2.08	4.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.142	11.256	11.217	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	700	700	1014	3079	0	0	0	0	0
N.S.	1	1.00	1.45	4.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.858	7.019	12.100	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [78] had the largest ratio of [.52000000000000018]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	13	0.231
2	A	2	1	1.00	13	0.077
3	A	2	2	1.00	13	0.154
4	A	2	2	1.00	11	0.182
5	A	1	1	1.00	8	0.125
6	A	4	3	1.00	11	0.273
7	A	3	3	1.00	13	0.231
8	A	4	3	1.00	13	0.231
9	A	3	2	1.00	13	0.154
10	B	2	2	2.20	13	0.154
11	B	2	2	4.33	15	0.133
12	A	2	2	1.00	9	0.222
13	A	2	2	1.00	11	0.182
14	A	2	2	1.00	11	0.182
15	A	2	2	1.00	13	0.154
16	A	3	2	1.00	11	0.182
17	A	3	2	1.00	13	0.154
18	A	2	2	1.00	9	0.222
19	A	2	2	1.00	11	0.182
20	A	1	1	1.00	11	0.091
21	A	1	1	1.00	13	0.077
22	A	3	2	1.00	11	0.182
23	A	3	2	1.00	13	0.154
24	A	5	5	1.00	13	0.385

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	2	1.00	13	0.154
26	A	4	4	1.00	13	0.308
27	A	2	2	1.00	11	0.182
28	A	2	2	1.00	8	0.250
29	A	6	4	1.00	11	0.364
30	A	4	4	1.00	13	0.308
31	A	4	3	1.00	13	0.231
32	A	5	5	1.00	13	0.385
33	A	5	4	1.00	23	0.174
34	A	4	4	1.00	23	0.174
35	A	4	4	1.00	23	0.174
36	A	3	3	1.00	23	0.130
37	A	3	3	1.00	23	0.130
38	A	4	4	1.00	23	0.174
39	A	4	4	1.00	23	0.174
40	A	5	4	1.00	23	0.174
41	A	6	5	1.00	25	0.200
42	A	5	5	1.00	25	0.200
43	A	5	5	1.00	25	0.200
44	A	4	4	1.00	25	0.160
45	A	4	4	1.00	25	0.160
46	A	4	4	1.00	25	0.160
47	A	4	4	1.00	25	0.160
48	A	5	5	1.00	25	0.200
49	A	7	6	1.00	25	0.240
50	A	6	6	1.00	25	0.240
51	A	6	6	1.00	25	0.240
52	A	5	5	1.00	25	0.200
53	A	5	5	1.00	25	0.200
54	A	5	5	1.00	25	0.200
55	A	5	5	1.00	25	0.200
56	A	5	5	1.00	25	0.200
57	A	5	5	1.00	25	0.200
58	A	15	12	1.00	25	0.480
59	A	14	12	1.00	25	0.480

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	14	12	1.00	25	0.480
61	A	13	11	1.00	25	0.440
62	A	13	11	1.00	25	0.440
63	A	9	7	1.00	25	0.280
64	A	9	7	1.00	25	0.280
65	A	13	11	1.00	25	0.440
66	A	13	11	1.00	25	0.440
67	A	14	12	1.00	25	0.480
68	A	15	12	1.00	25	0.480
69	A	14	12	1.00	25	0.480
70	A	14	12	1.00	25	0.480
71	A	13	11	1.00	25	0.440
72	A	13	11	1.00	25	0.440
73	A	13	11	1.00	25	0.440
74	A	13	11	1.00	25	0.440
75	A	14	12	1.00	25	0.480
76	A	14	12	1.00	25	0.480
77	A	15	12	1.00	25	0.480
78	A	15	13	1.00	25	0.520
79	A	15	13	1.00	25	0.520
80	A	14	12	1.00	25	0.480
81	A	14	12	1.00	25	0.480
82	A	14	12	1.00	25	0.480
83	A	14	12	1.00	25	0.480
84	A	14	12	1.00	25	0.480
85	A	14	12	1.00	25	0.480
86	A	15	13	1.00	25	0.520
87	A	15	13	1.00	25	0.520
88	A	16	13	1.00	25	0.520

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx=\frac{x}{2a}-\frac{\cos(x)\sin(x)}{2a}-\frac{\sin^3(x)}{3a}$$

```
[Out] 1/2*x/a-1/2*cos(x)*sin(x)/a-1/3*sin(x)^3/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2761, 2715, 8}

$$\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx=\frac{x}{2a}-\frac{\sin^3(x)}{3a}-\frac{\sin(x)\cos(x)}{2a}$$

```
[In] Int[Sin[x]^4/(a + a*Cos[x]),x]
```

```
[Out] x/(2*a) - (Cos[x]*Sin[x])/(2*a) - Sin[x]^3/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sin^3(x)}{3a} + \frac{\int \sin^2(x)\,dx}{a}\\ &= -\frac{\cos(x)\sin(x)}{2a} - \frac{\sin^3(x)}{3a} + \frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a} - \frac{\cos(x)\sin(x)}{2a} - \frac{\sin^3(x)}{3a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sin^4(x)}{a + a\cos(x)}\,dx = \frac{6x - 3\sin(x) - 3\sin(2x) + \sin(3x)}{12a}$$

```
[In] Integrate[Sin[x]^4/(a + a*Cos[x]),x]
[Out] (6*x - 3*Sin[x] - 3*Sin[2*x] + Sin[3*x])/(12*a)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{6x-3\sin(x)+\sin(3x)-3\sin(2x)}{12a}$	24
risch	$\frac{x}{2a}-\frac{\sin(x)}{4a}+\frac{\sin(3x)}{12a}-\frac{\sin(2x)}{4a}$	33
default	$\frac{\frac{16\left(\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{16}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{6}-\frac{\tan\left(\frac{x}{2}\right)}{16}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}+\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$	48
norman	$\frac{\frac{\tan^7\left(\frac{x}{2}\right)}{a}-\frac{11\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a}-\frac{5\left(\tan^5\left(\frac{x}{2}\right)\right)}{3a}+\frac{x}{2a}-\frac{\tan\left(\frac{x}{2}\right)}{a}+\frac{2x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}+\frac{2x\left(\tan^6\left(\frac{x}{2}\right)\right)}{a}+\frac{x\left(\tan^8\left(\frac{x}{2}\right)\right)}{2a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	108

```
[In] int(sin(x)^4/(a+cos(x)*a),x,method=_RETURNVERBOSE)
[Out] 1/12*(6*x-3*sin(x)+sin(3*x)-3*sin(2*x))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\sin^4(x)}{a + a\cos(x)}\,dx = \frac{\left(2\,\cos\left(x\right)^2 - 3\,\cos\left(x\right) - 2\right)\sin\left(x\right) + 3\,x}{6\,a}$$

```
[In] integrate(sin(x)^4/(a+a*cos(x)),x, algorithm="fricas")
[Out] 1/6*((2*cos(x)^2 - 3*cos(x) - 2)*sin(x) + 3*x)/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(22) = 44.

Time = 0.44 (sec) , antiderivative size = 294, normalized size of antiderivative = 9.48

$$\begin{aligned}\int \frac{\sin^4(x)}{a + a\cos(x)}\,dx = {} & \frac{3x\tan^6\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a} \\ & + \frac{9x\tan^4\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a} \\ & + \frac{9x\tan^2\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a} \\ & + \frac{3x}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a} \\ & + \frac{6\tan^5\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a} \\ & - \frac{16\tan^3\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a} \\ & - \frac{6\tan\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a}\end{aligned}$$

```
[In] integrate(sin(x)**4/(a+a*cos(x)),x)
[Out] 3*x*tan(x/2)**6/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**2 + 6*
a) + 9*x*tan(x/2)**4/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**2
 + 6*a) + 9*x*tan(x/2)**2/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/
2)**2 + 6*a) + 3*x/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**2 +
 6*a) + 6*tan(x/2)**5/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**
2 + 6*a) - 16*tan(x/2)**3/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/
2)**2 + 6*a) - 6*tan(x/2)/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/
2)**2 + 6*a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(25) = 50.

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.03

$$\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx = -\frac{\frac{3\sin(x)}{\cos(x)+1}+\frac{8\sin(x)^3}{(\cos(x)+1)^3}-\frac{3\sin(x)^5}{(\cos(x)+1)^5}}{3\left(a+\frac{3a\sin(x)^2}{(\cos(x)+1)^2}+\frac{3a\sin(x)^4}{(\cos(x)+1)^4}+\frac{a\sin(x)^6}{(\cos(x)+1)^6}\right)}+\frac{\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

```
[In] integrate(sin(x)^4/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sin(x)/(cos(x) + 1) + 8*sin(x)^3/(cos(x) + 1)^3 - 3*sin(x)^5/(cos(x
) + 1)^5)/(a + 3*a*sin(x)^2/(cos(x) + 1)^2 + 3*a*sin(x)^4/(cos(x) + 1)^4 +
a*sin(x)^6/(cos(x) + 1)^6) + arctan(sin(x)/(cos(x) + 1))/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx = \frac{x}{2a}+\frac{3\tan\left(\frac{1}{2}x\right)^5-8\tan\left(\frac{1}{2}x\right)^3-3\tan\left(\frac{1}{2}x\right)}{3\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^3a}$$

```
[In] integrate(sin(x)^4/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] 1/2*x/a + 1/3*(3*tan(1/2*x)^5 - 8*tan(1/2*x)^3 - 3*tan(1/2*x))/((tan(1/2*x)
^2 + 1)^3*a)
```

Mupad [B] (verification not implemented)

Time = 14.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx = \frac{x}{2a}-\frac{\sin(x)}{3a}+\frac{\cos(x)^2\sin(x)}{3a}-\frac{\cos(x)\sin(x)}{2a}$$

```
[In] int(sin(x)^4/(a + a*cos(x)),x)
```

```
[Out] x/(2*a) - sin(x)/(3*a) + (cos(x)^2*sin(x))/(3*a) - (cos(x)*sin(x))/(2*a)
```

3.2 $\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx = -\frac{\cos(x)}{a}+\frac{\cos^2(x)}{2a}$$

```
[Out] -cos(x)/a+1/2*cos(x)^2/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2746}

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx = \frac{\cos^2(x)}{2a}-\frac{\cos(x)}{a}$$

```
[In] Int[Sin[x]^3/(a + a*Cos[x]),x]
```

```
[Out] -(Cos[x]/a) + Cos[x]^2/(2*a)
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int (a-x)\,dx, x, a\cos(x)\right)}{a^3} \\ &= -\frac{\cos(x)}{a}+\frac{\cos^2(x)}{2a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx=\frac{2\sin^4\left(\frac{x}{2}\right)}{a}$$

```
[In] Integrate[Sin[x]^3/(a + a*Cos[x]),x]
[Out] (2*Sin[x/2]^4)/a
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^2(x)\right)}{2}-\cos(x)}{a}$	16
default	$\frac{\frac{\left(\cos^2(x)\right)}{2}-\cos(x)}{a}$	16
parallelrisch	$\frac{\cos(2x)-5-4\cos(x)}{4a}$	16
risch	$-\frac{\cos(x)}{a}+\frac{\cos(2x)}{4a}$	18
norman	$\frac{-\frac{2}{a}-\frac{4\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}-\frac{6\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	40

```
[In] int(sin(x)^3/(a+cos(x)*a),x,method=_RETURNVERBOSE)
[Out] 1/a*(1/2*cos(x)^2-cos(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx=\frac{\cos\left(x\right)^2-2\,\cos\left(x\right)}{2\,a}$$

```
[In] integrate(sin(x)^3/(a+a*cos(x)),x, algorithm="fricas")
[Out] 1/2*(cos(x)^2 - 2*cos(x))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.68

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx = -\frac{4\tan^2\left(\frac{x}{2}\right)}{a\tan^4\left(\frac{x}{2}\right)+2a\tan^2\left(\frac{x}{2}\right)+a} - \frac{2}{a\tan^4\left(\frac{x}{2}\right)+2a\tan^2\left(\frac{x}{2}\right)+a}$$

```
[In] integrate(sin(x)**3/(a+a*cos(x)),x)
[Out] -4*tan(x/2)**2/(a*tan(x/2)**4 + 2*a*tan(x/2)**2 + a) - 2/(a*tan(x/2)**4 + 2
*a*tan(x/2)**2 + a)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx = \frac{\cos(x)^2-2\cos(x)}{2a}$$

```
[In] integrate(sin(x)^3/(a+a*cos(x)),x, algorithm="maxima")
[Out] 1/2*(cos(x)^2 - 2*cos(x))/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx = \frac{\cos(x)^2-2\cos(x)}{2a}$$

```
[In] integrate(sin(x)^3/(a+a*cos(x)),x, algorithm="giac")
[Out] 1/2*(cos(x)^2 - 2*cos(x))/a
```

Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\sin^3(x)}{a + a\cos(x)}\, dx = \frac{\cos(x)\ (\cos(x) - 2)}{2\, a}$$

```
[In] int(sin(x)^3/(a + a*cos(x)),x)
[Out] (cos(x)*(cos(x) - 2))/(2*a)
```

3.3 $\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx = \frac{x}{a} - \frac{\sin(x)}{a}$$

```
[Out] x/a-sin(x)/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2761, 8}

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx = \frac{x}{a} - \frac{\sin(x)}{a}$$

```
[In] Int[Sin[x]^2/(a + a*Cos[x]),x]
```

```
[Out] x/a - Sin[x]/a
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sin(x)}{a}+\frac{\int 1\,dx}{a}\\ &= \frac{x}{a}-\frac{\sin(x)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx=\frac{2\left(\frac{x}{2}-\frac{\sin(x)}{2}\right)}{a}$$

```
[In] Integrate[Sin[x]^2/(a + a*Cos[x]),x]
[Out] (2*(x/2 - Sin[x]/2))/a
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{x-\sin(x)}{a}$	11
risch	$\frac{x}{a}-\frac{\sin(x)}{a}$	14
default	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$	30
norman	$\frac{\frac{x}{a}+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}-\frac{2\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{2x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	61

```
[In] int(sin(x)^2/(a+cos(x)*a),x,method=_RETURNVERBOSE)
[Out] (x-sin(x))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\sin^2(x)}{a + a\cos(x)}\, dx = \frac{x - \sin(x)}{a}$$

```
[In] integrate(sin(x)^2/(a+a*cos(x)),x, algorithm="fricas")
[Out] (x - sin(x))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(7) = 14$.

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.54

$$\int \frac{\sin^2(x)}{a + a\cos(x)}\, dx = \frac{x\tan^2\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right) + a} + \frac{x}{a\tan^2\left(\frac{x}{2}\right) + a} - \frac{2\tan\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right) + a}$$

```
[In] integrate(sin(x)**2/(a+a*cos(x)),x)
[Out] x*tan(x/2)**2/(a*tan(x/2)**2 + a) + x/(a*tan(x/2)**2 + a) - 2*tan(x/2)/(a*t
an(x/2)**2 + a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(13) = 26$.

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.23

$$\int \frac{\sin^2(x)}{a + a\cos(x)}\, dx = \frac{2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a} - \frac{2\,\sin(x)}{\left(a + \frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)(\cos(x) + 1)}$$

```
[In] integrate(sin(x)^2/(a+a*cos(x)),x, algorithm="maxima")
[Out] 2*arctan(sin(x)/(cos(x) + 1))/a - 2*sin(x)/((a + a*sin(x)^2/(cos(x) + 1)^2)
*(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{\sin^2(x)}{a + a\cos(x)}\, dx = \frac{x}{a} - \frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\left(\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)a}$$

```
[In] integrate(sin(x)^2/(a+a*cos(x)),x, algorithm="giac")
[Out] x/a - 2*tan(1/2*x)/((tan(1/2*x)^2 + 1)*a)
```

Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\sin^2(x)}{a + a\cos(x)}\, dx = \frac{x - \sin(x)}{a}$$

```
[In] int(sin(x)^2/(a + a*cos(x)),x)
[Out] (x - sin(x))/a
```

3.4 $\int \frac{\sin(x)}{a+a\cos(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\sin(x)}{a+a\cos(x)}\,dx=-\frac{\log(1+\cos(x))}{a}$$

```
[Out] -ln(1+cos(x))/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2746, 31}

$$\int \frac{\sin(x)}{a+a\cos(x)}\,dx=-\frac{\log(\cos(x)+1)}{a}$$

```
[In] Int[Sin[x]/(a + a*Cos[x]),x]
```

```
[Out] -(Log[1 + Cos[x]]/a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\, dx, x, a\cos(x)\right)}{a}\\
&= -\frac{\log(1+\cos(x))}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{a+a\cos(x)}\, dx = -\frac{2\log\left(\cos\left(\frac{x}{2}\right)\right)}{a}$$

```
[In] Integrate[Sin[x]/(a + a*Cos[x]),x]
[Out] (-2*Log[Cos[x/2]])/a
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$-\frac{\ln(a+\cos(x)a)}{a}$	13
default	$-\frac{\ln(a+\cos(x)a)}{a}$	13
norman	$\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{a}$	14
parallelrisch	$\frac{\ln\left(\frac{2}{\cos(x)+1}\right)}{a}$	14
risch	$\frac{ix}{a}-\frac{2\ln\left(\mathrm{e}^{ix}+1\right)}{a}$	22

```
[In] int(sin(x)/(a+cos(x)*a),x,method=_RETURNVERBOSE)
[Out] -ln(a+cos(x)*a)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{a + a\cos(x)}\, dx = -\frac{\log\left(\frac{1}{2}\,\cos\left(x\right) + \frac{1}{2}\right)}{a}$$

```
[In] integrate(sin(x)/(a+a*cos(x)),x, algorithm="fricas")
[Out] -log(1/2*cos(x) + 1/2)/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin(x)}{a + a\cos(x)}\, dx = -\frac{\log\left(\cos\left(x\right) + 1\right)}{a}$$

```
[In] integrate(sin(x)/(a+a*cos(x)),x)
[Out] -log(cos(x) + 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{a + a\cos(x)}\, dx = -\frac{\log\left(a\cos\left(x\right) + a\right)}{a}$$

```
[In] integrate(sin(x)/(a+a*cos(x)),x, algorithm="maxima")
[Out] -log(a*cos(x) + a)/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a + a\cos(x)}\, dx = -\frac{\log\left(\cos\left(x\right) + 1\right)}{a}$$

```
[In] integrate(sin(x)/(a+a*cos(x)),x, algorithm="giac")
[Out] -log(cos(x) + 1)/a
```

Mupad [B] (verification not implemented)

Time = 13.60 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a + a\cos(x)}\,dx = -\frac{\ln(\cos(x) + 1)}{a}$$

```
[In] int(sin(x)/(a + a*cos(x)),x)
[Out] -log(cos(x) + 1)/a
```

3.5 $\int \frac{1}{a+a\cos(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 11

$$\int \frac{1}{a+a\cos(x)}\,dx=\frac{\sin(x)}{a+a\cos(x)}$$

```
[Out] sin(x)/(a+a*cos(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2727}

$$\int \frac{1}{a+a\cos(x)}\,dx=\frac{\sin(x)}{a\cos(x)+a}$$

```
[In] Int[(a + a*Cos[x])^(-1),x]
```

```
[Out] Sin[x]/(a + a*Cos[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral}=\frac{\sin(x)}{a+a\cos(x)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{a + a\cos(x)} \, dx = \frac{\tan\left(\frac{x}{2}\right)}{a}$$

```
[In] Integrate[(a + a*Cos[x])^(-1),x]
[Out] Tan[x/2]/a
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\tan\left(\frac{x}{2}\right)}{a}$	9
norman	$\frac{\tan\left(\frac{x}{2}\right)}{a}$	9
parallelrisch	$\frac{\tan\left(\frac{x}{2}\right)}{a}$	9
risch	$\frac{2i}{(e^{ix}+1)a}$	16

```
[In] int(1/(a+cos(x)*a),x,method=_RETURNVERBOSE)
[Out] tan(1/2*x)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{a + a\cos(x)} \, dx = \frac{\sin(x)}{a\cos(x) + a}$$

```
[In] integrate(1/(a+a*cos(x)),x, algorithm="fricas")
[Out] sin(x)/(a*cos(x) + a)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int \frac{1}{a + a\cos(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{a}$$

```
[In] integrate(1/(a+a*cos(x)),x)
[Out] tan(x/2)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{a + a\cos(x)}\,dx = \frac{\sin\left(x\right)}{a(\cos\left(x\right) + 1)}$$

```
[In] integrate(1/(a+a*cos(x)),x, algorithm="maxima")
[Out] sin(x)/(a*(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{a + a\cos(x)}\,dx = \frac{\tan\left(\frac{1}{2}\,x\right)}{a}$$

```
[In] integrate(1/(a+a*cos(x)),x, algorithm="giac")
[Out] tan(1/2*x)/a
```

Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{a + a\cos(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{a}$$

```
[In] int(1/(a + a*cos(x)),x)
[Out] tan(x/2)/a
```

3.6 $\int \frac{\csc(x)}{a+a\cos(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\csc(x)}{a+a\cos(x)}\,dx = -\frac{\operatorname{arctanh}(\cos(x))}{2a}+\frac{1}{2(a+a\cos(x))}$$

```
[Out] -1/2*arctanh(cos(x))/a+1/2/(a+a*cos(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2746, 46, 212}

$$\int \frac{\csc(x)}{a+a\cos(x)}\,dx = \frac{1}{2(a\cos(x)+a)}-\frac{\operatorname{arctanh}(\cos(x))}{2a}$$

```
[In] Int[Csc[x]/(a + a*Cos[x]),x]
```

```
[Out] -1/2*ArcTanh[Cos[x]]/a + 1/(2*(a + a*Cos[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(a\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^2}\,dx, x, a\cos(x)\right)\right)\\
&= -\left(a\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^2}+\frac{1}{2a\,(a^2-x^2)}\right)\,dx, x, a\cos(x)\right)\right)\\
&= \frac{1}{2(a+a\cos(x))}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\cos(x)\right)\\
&= -\frac{\text{arctanh}(\cos(x))}{2a}+\frac{1}{2(a+a\cos(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{\csc(x)}{a+a\cos(x)}\,dx = \frac{1-2\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{2a(1+\cos(x))}$$

```
[In] Integrate[Csc[x]/(a + a*Cos[x]),x]
[Out] (1 - 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(2*a*(1 + Cos[x]))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{\tan^2\left(\frac{x}{2}\right)+2\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4a}$	20
norman	$\frac{\tan^2\left(\frac{x}{2}\right)}{4a}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}$	23
default	$\frac{\frac{1}{2\cos(x)+2}-\frac{\ln(\cos(x)+1)}{4}+\frac{\ln(\cos(x)-1)}{4}}{a}$	28
risch	$\frac{e^{ix}}{\left(e^{ix}+1\right)^2 a}-\frac{\ln\left(e^{ix}+1\right)}{2a}+\frac{\ln\left(e^{ix}-1\right)}{2a}$	46

[In] int(csc(x)/(a+cos(x)*a),x,method=_RETURNVERBOSE)

[Out] 1/4*(tan(1/2*x)^2+2*ln(tan(1/2*x)))/a

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{\csc(x)}{a+a\cos(x)}\,dx = -\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{4\,(a\cos(x)+a)}$$

[In] integrate(csc(x)/(a+a*cos(x)),x, algorithm="fricas")

[Out] -1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) - 2)/(a*cos(x) + a)

Sympy [F]

$$\int \frac{\csc(x)}{a+a\cos(x)}\,dx = \frac{\int \frac{\csc(x)}{\cos(x)+1}\,dx}{a}$$

[In] integrate(csc(x)/(a+a*cos(x)),x)

[Out] Integral(csc(x)/(cos(x) + 1), x)/a

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{\csc(x)}{a + a\cos(x)}\,dx = -\frac{\log\left(\cos\left(x\right)+1\right)}{4\,a} + \frac{\log\left(\cos\left(x\right)-1\right)}{4\,a} + \frac{1}{2\left(a\cos\left(x\right)+a\right)}$$

```
[In] integrate(csc(x)/(a+a*cos(x)),x, algorithm="maxima")
[Out] -1/4*log(cos(x) + 1)/a + 1/4*log(cos(x) - 1)/a + 1/2/(a*cos(x) + a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{\csc(x)}{a + a\cos(x)}\,dx = -\frac{\log\left(\cos\left(x\right)+1\right)}{4\,a} + \frac{\log\left(-\cos\left(x\right)+1\right)}{4\,a} + \frac{1}{2\,a(\cos\left(x\right)+1)}$$

```
[In] integrate(csc(x)/(a+a*cos(x)),x, algorithm="giac")
[Out] -1/4*log(cos(x) + 1)/a + 1/4*log(-cos(x) + 1)/a + 1/2/(a*(cos(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\csc(x)}{a + a\cos(x)}\,dx = \frac{1}{2\,a\,\left(\cos\left(x\right)+1\right)} - \frac{\mathrm{atanh}(\cos\left(x\right))}{2\,a}$$

```
[In] int(1/(sin(x)*(a + a*cos(x))),x)
[Out] 1/(2*a*(cos(x) + 1)) - atanh(cos(x))/(2*a)
```

3.7 $\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx=-\frac{2\cot(x)}{3a}+\frac{\csc(x)}{3(a+a\cos(x))}$$

```
[Out] -2/3*cot(x)/a+1/3*csc(x)/(a+a*cos(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2751, 3852, 8}

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx=\frac{\csc(x)}{3(a\cos(x)+a)}-\frac{2\cot(x)}{3a}$$

```
[In] Int[Csc[x]^2/(a + a*Cos[x]),x]
```

```
[Out] (-2*Cot[x])/(3*a) + Csc[x]/(3*(a + a*Cos[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
```

```
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\csc(x)}{3(a+a\cos(x))}+\frac{2\int \csc^2(x)\,dx}{3a}\\ &= \frac{\csc(x)}{3(a+a\cos(x))}-\frac{2\text{Subst}\left(\int 1\,dx,x,\cot(x)\right)}{3a}\\ &= -\frac{2\cot(x)}{3a}+\frac{\csc(x)}{3(a+a\cos(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx = -\frac{(2\cos(x)+\cos(2x))\csc\left(\frac{x}{2}\right)\sec^3\left(\frac{x}{2}\right)}{12a}$$

```
[In] Integrate[Csc[x]^2/(a + a*Cos[x]),x]
[Out] -1/12*((2*Cos[x] + Cos[2*x])*Csc[x/2]*Sec[x/2]^3)/a
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$\frac{\tan^3\left(\frac{x}{2}\right)+6\tan\left(\frac{x}{2}\right)-3\cot\left(\frac{x}{2}\right)}{12a}$	25
default	$\frac{\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}+2\tan\left(\frac{x}{2}\right)-\frac{1}{\tan\left(\frac{x}{2}\right)}}{4a}$	29
risch	$-\frac{4i\left(1+2\,e^{ix}\right)}{3\left(e^{ix}+1\right)^3 a\left(e^{ix}-1\right)}$	34
norman	$\frac{-\frac{1}{4a}+\frac{\tan^2\left(\frac{x}{2}\right)}{2a}+\frac{\tan^4\left(\frac{x}{2}\right)}{12a}}{\tan\left(\frac{x}{2}\right)}$	36

```
[In] int(csc(x)^2/(a+cos(x)*a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(tan(1/2*x)^3+6*tan(1/2*x)-3*cot(1/2*x))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(x)}{a + a\cos(x)}\,dx = -\frac{2\,\cos{(x)}^2 + 2\,\cos{(x)} - 1}{3\,(a\cos{(x)} + a)\sin{(x)}}$$

```
[In] integrate(csc(x)^2/(a+a*cos(x)),x, algorithm="fricas")
[Out] -1/3*(2*cos(x)^2 + 2*cos(x) - 1)/((a*cos(x) + a)*sin(x))
```

Sympy [F]

$$\int \frac{\csc^2(x)}{a + a\cos(x)}\,dx = \frac{\int \frac{\csc^2{(x)}}{\cos{(x)}+1}\,dx}{a}$$

```
[In] integrate(csc(x)**2/(a+a*cos(x)),x)
[Out] Integral(csc(x)**2/(cos(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{\csc^2(x)}{a + a\cos(x)}\,dx = \frac{\frac{6\,\sin(x)}{\cos(x)+1} + \frac{\sin(x)^3}{(\cos(x)+1)^3}}{12\,a} - \frac{\cos{(x)} + 1}{4\,a\sin{(x)}}$$

```
[In] integrate(csc(x)^2/(a+a*cos(x)),x, algorithm="maxima")
[Out] 1/12*(6*sin(x)/(cos(x) + 1) + sin(x)^3/(cos(x) + 1)^3)/a - 1/4*(cos(x) + 1)
/(a*sin(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^2(x)}{a + a\cos(x)}\,dx = \frac{a^2 \tan\left(\frac{1}{2}\,x\right)^3 + 6\,a^2 \tan\left(\frac{1}{2}\,x\right)}{12\,a^3} - \frac{1}{4\,a\tan\left(\frac{1}{2}\,x\right)}$$

```
[In] integrate(csc(x)^2/(a+a*cos(x)),x, algorithm="giac")
[Out] 1/12*(a^2*tan(1/2*x)^3 + 6*a^2*tan(1/2*x))/a^3 - 1/4/(a*tan(1/2*x))
```

Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\csc^2(x)}{a + a\cos(x)}\,dx = \frac{-8\cos\left(\frac{x}{2}\right)^4 + 4\cos\left(\frac{x}{2}\right)^2 + 1}{12\,a\cos\left(\frac{x}{2}\right)^3\,\sin\left(\frac{x}{2}\right)}$$

```
[In] int(1/(sin(x)^2*(a + a*cos(x))),x)
[Out] (4*cos(x/2)^2 - 8*cos(x/2)^4 + 1)/(12*a*cos(x/2)^3*sin(x/2))
```

3.8 $\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx = -\frac{3\text{arctanh}(\cos(x))}{8a} - \frac{1}{8(a-a\cos(x))} + \frac{a}{8(a+a\cos(x))^2} + \frac{1}{4(a+a\cos(x))}$$

```
[Out] -3/8*arctanh(cos(x))/a-1/8/(a-a*cos(x))+1/8*a/(a+a*cos(x))^2+1/4/(a+a*cos(x
))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2746, 46, 212}

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx = -\frac{3\text{arctanh}(\cos(x))}{8a} + \frac{a}{8(a\cos(x)+a)^2} - \frac{1}{8(a-a\cos(x))} + \frac{1}{4(a\cos(x)+a)}$$

```
[In] Int[Csc[x]^3/(a + a*Cos[x]),x]
```

```
[Out] (-3*ArcTanh[Cos[x]])/(8*a) - 1/(8*(a - a*Cos[x])) + a/(8*(a + a*Cos[x])^2)
+ 1/(4*(a + a*Cos[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(a^3\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, a\cos(x)\right)\right) \\
&= -\left(a^3\text{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2}+\frac{1}{4a^2(a+x)^3}+\frac{1}{4a^3(a+x)^2}+\frac{3}{8a^3\left(a^2-x^2\right)}\right)\,dx, x, a\cos(x)\right)\right) \\
&= -\frac{1}{8(a-a\cos(x))}+\frac{a}{8(a+a\cos(x))^2}+\frac{1}{4(a+a\cos(x))}-\frac{3}{8}\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\cos(x)\right) \\
&= -\frac{3\operatorname{arctanh}(\cos(x))}{8a}-\frac{1}{8(a-a\cos(x))}+\frac{a}{8(a+a\cos(x))^2}+\frac{1}{4(a+a\cos(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx = \frac{4-2\cot^2\left(\frac{x}{2}\right)-12\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+\sec^2\left(\frac{x}{2}\right)}{16a(1+\cos(x))}$$

```
[In] Integrate[Csc[x]^3/(a + a*Cos[x]),x]
[Out] (4 - 2*Cot[x/2]^2 - 12*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]) + Sec[x/2
]^2)/(16*a*(1 + Cos[x]))
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
parallelrisch	$\frac{\tan^{4}\left(\frac{x}{2}\right)+6\left(\tan^{2}\left(\frac{x}{2}\right)\right)-2\left(\cot^{2}\left(\frac{x}{2}\right)\right)+12\ln\left(\tan\left(\frac{x}{2}\right)\right)}{32a}$	36
default	$\frac{\frac{1}{8\cos(x)-8}+\frac{3\ln(\cos(x)-1)}{16}+\frac{1}{8(\cos(x)+1)^{2}}+\frac{1}{4\cos(x)+4}-\frac{3\ln(\cos(x)+1)}{16}}{a}$	44
norman	$\frac{-\frac{1}{16a}+\frac{3\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{16a}+\frac{\tan^{6}\left(\frac{x}{2}\right)}{32a}}{\tan\left(\frac{x}{2}\right)^{2}}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8a}$	47
risch	$\frac{3\,\mathrm{e}^{5ix}+6\,\mathrm{e}^{4ix}-2\,\mathrm{e}^{3ix}+6\,\mathrm{e}^{2ix}+3\,\mathrm{e}^{ix}}{4\left(\mathrm{e}^{ix}+1\right)^{4}a\left(\mathrm{e}^{ix}-1\right)^{2}}-\frac{3\ln\left(\mathrm{e}^{ix}+1\right)}{8a}+\frac{3\ln\left(\mathrm{e}^{ix}-1\right)}{8a}$	87

```
[In] int(csc(x)^3/(a+cos(x)*a),x,method=_RETURNVERBOSE)
[Out] 1/32*(tan(1/2*x)^4+6*tan(1/2*x)^2-2*cot(1/2*x)^2+12*ln(tan(1/2*x)))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{\csc^{3}(x)}{a+a\cos(x)}\,dx = \frac{6\cos\left(x\right)^{2}-3\left(\cos\left(x\right)^{3}+\cos\left(x\right)^{2}-\cos\left(x\right)-1\right)\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+3\left(\cos\left(x\right)^{3}+\cos\left(x\right)^{2}-\cos\left(x\right)-1}{16\left(a\cos\left(x\right)^{3}+a\cos\left(x\right)^{2}-a\cos\left(x\right)-a\right)}$$

```
[In] integrate(csc(x)^3/(a+a*cos(x)),x, algorithm="fricas")
[Out] 1/16*(6*cos(x)^2 - 3*(cos(x)^3 + cos(x)^2 - cos(x) - 1)*log(1/2*cos(x) + 1/
2) + 3*(cos(x)^3 + cos(x)^2 - cos(x) - 1)*log(-1/2*cos(x) + 1/2) + 6*cos(x)
 - 4)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) - a)
```

Sympy [F]

$$\int \frac{\csc^{3}(x)}{a+a\cos(x)}\,dx = \frac{\int \frac{\csc^{3}(x)}{\cos(x)+1}\,dx}{a}$$

```
[In] integrate(csc(x)**3/(a+a*cos(x)),x)
[Out] Integral(csc(x)**3/(cos(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{\csc^3(x)}{a + a\cos(x)}\, dx = \frac{3\, \cos(x)^2 + 3\, \cos(x) - 2}{8\, \left(a\cos(x)^3 + a\cos(x)^2 - a\cos(x) - a\right)} - \frac{3\, \log(\cos(x) + 1)}{16\, a} + \frac{3\, \log(\cos(x) - 1)}{16\, a}$$

```
[In] integrate(csc(x)^3/(a+a*cos(x)),x, algorithm="maxima")
[Out] 1/8*(3*cos(x)^2 + 3*cos(x) - 2)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) - a) -
3/16*log(cos(x) + 1)/a + 3/16*log(cos(x) - 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{\csc^3(x)}{a + a\cos(x)}\, dx = -\frac{3\, \log(\cos(x) + 1)}{16\, a} + \frac{3\, \log(-\cos(x) + 1)}{16\, a} + \frac{3\, \cos(x)^2 + 3\, \cos(x) - 2}{8\, a(\cos(x) + 1)^2(\cos(x) - 1)}$$

```
[In] integrate(csc(x)^3/(a+a*cos(x)),x, algorithm="giac")
[Out] -3/16*log(cos(x) + 1)/a + 3/16*log(-cos(x) + 1)/a + 1/8*(3*cos(x)^2 + 3*cos
(x) - 2)/(a*(cos(x) + 1)^2*(cos(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{\csc^3(x)}{a + a\cos(x)}\, dx = -\frac{\frac{3\cos(x)^2}{8} + \frac{3\, \cos(x)}{8} - \frac{1}{4}}{-a\cos(x)^3 - a\cos(x)^2 + a\, \cos(x) + a} - \frac{3\, \mathrm{atanh}(\cos(x))}{8\, a}$$

```
[In] int(1/(sin(x)^3*(a + a*cos(x))),x)
[Out] - ((3*cos(x))/8 + (3*cos(x)^2)/8 - 1/4)/(a + a*cos(x) - a*cos(x)^2 - a*cos(
x)^3) - (3*atanh(cos(x)))/(8*a)
```

3.9 $\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx=-\frac{4\cot(x)}{5a}-\frac{4\cot^3(x)}{15a}+\frac{\csc^3(x)}{5(a+a\cos(x))}$$

```
[Out] -4/5*cot(x)/a-4/15*cot(x)^3/a+1/5*csc(x)^3/(a+a*cos(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2751, 3852}

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx=-\frac{4\cot^3(x)}{15a}-\frac{4\cot(x)}{5a}+\frac{\csc^3(x)}{5(a\cos(x)+a)}$$

```
[In] Int[Csc[x]^4/(a + a*Cos[x]),x]
```

```
[Out] (-4*Cot[x])/(5*a) - (4*Cot[x]^3)/(15*a) + Csc[x]^3/(5*(a + a*Cos[x]))
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\csc^3(x)}{5(a+a\cos(x))}+\frac{4\int\csc^4(x)\,dx}{5a}\\ &= \frac{\csc^3(x)}{5(a+a\cos(x))}-\frac{4\text{Subst}\left(\int(1+x^2)\,dx,x,\cot(x)\right)}{5a}\\ &= -\frac{4\cot(x)}{5a}-\frac{4\cot^3(x)}{15a}+\frac{\csc^3(x)}{5(a+a\cos(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int\frac{\csc^4(x)}{a+a\cos(x)}\,dx=\frac{(-6\cos(x)-2\cos(2x)+2\cos(3x)+\cos(4x))\csc^3(x)}{15a(1+\cos(x))}$$

```
[In] Integrate[Csc[x]^4/(a + a*Cos[x]),x]
[Out] ((-6*Cos[x] - 2*Cos[2*x] + 2*Cos[3*x] + Cos[4*x])*Csc[x]^3)/(15*a*(1 + Cos[
x]))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{5}+\frac{4\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}+6\tan\left(\frac{x}{2}\right)-\frac{4}{\tan\left(\frac{x}{2}\right)}-\frac{1}{3\tan\left(\frac{x}{2}\right)^3}}{16a}$	45
risch	$\frac{16i(6\,e^{3ix}+2\,e^{2ix}-2\,e^{ix}-1)}{15(e^{ix}-1)^3a(e^{ix}+1)^5}$	48
parallelrisch	$-\frac{4\csc(x)(-\cos(4x)+2\cos(2x)-2\cos(3x)+6\cos(x))}{15a(\cos(x)-\cos(3x)-2\cos(2x)+2)}$	49
norman	$\frac{-\frac{1}{48a}-\frac{\tan^2\left(\frac{x}{2}\right)}{4a}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{8a}+\frac{\tan^6\left(\frac{x}{2}\right)}{12a}+\frac{\tan^8\left(\frac{x}{2}\right)}{80a}}{\tan\left(\frac{x}{2}\right)^3}$	58

```
[In] int(csc(x)^4/(a+cos(x)*a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a*(1/5*tan(1/2*x)^5+4/3*tan(1/2*x)^3+6*tan(1/2*x)-4/tan(1/2*x)-1/3/tan
(1/2*x)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = -\frac{8\cos(x)^4+8\cos(x)^3-12\cos(x)^2-12\cos(x)+3}{15\left(a\cos(x)^3+a\cos(x)^2-a\cos(x)-a\right)\sin(x)}$$

```
[In] integrate(csc(x)^4/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(x)^4 + 8*cos(x)^3 - 12*cos(x)^2 - 12*cos(x) + 3)/((a*cos(x)^3
+ a*cos(x)^2 - a*cos(x) - a)*sin(x))
```

Sympy [F]

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = \frac{\int \frac{\csc^4(x)}{\cos(x)+1}\,dx}{a}$$

```
[In] integrate(csc(x)**4/(a+a*cos(x)),x)
```

```
[Out] Integral(csc(x)**4/(cos(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(31) = 62.

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.89

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = \frac{\frac{90\sin(x)}{\cos(x)+1}+\frac{20\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^5}{(\cos(x)+1)^5}}{240\,a} - \frac{\left(\frac{12\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)^3}{48\,a\sin(x)^3}$$

```
[In] integrate(csc(x)^4/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] 1/240*(90*sin(x)/(cos(x) + 1) + 20*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^5/(co
s(x) + 1)^5)/a - 1/48*(12*sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)^3/(a*si
n(x)^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = -\frac{12\,\tan\left(\frac{1}{2}\,x\right)^2+1}{48\,a\tan\left(\frac{1}{2}\,x\right)^3}+\frac{3\,a^4\tan\left(\frac{1}{2}\,x\right)^5+20\,a^4\tan\left(\frac{1}{2}\,x\right)^3+90\,a^4\tan\left(\frac{1}{2}\,x\right)}{240\,a^5}$$

```
[In] integrate(csc(x)^4/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] -1/48*(12*tan(1/2*x)^2 + 1)/(a*tan(1/2*x)^3) + 1/240*(3*a^4*tan(1/2*x)^5 +
20*a^4*tan(1/2*x)^3 + 90*a^4*tan(1/2*x))/a^5
```

Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = \frac{3\tan\left(\frac{x}{2}\right)^8+20\tan\left(\frac{x}{2}\right)^6+90\tan\left(\frac{x}{2}\right)^4-60\tan\left(\frac{x}{2}\right)^2-5}{240\,a\tan\left(\frac{x}{2}\right)^3}$$

```
[In] int(1/(sin(x)^4*(a + a*cos(x))),x)
```

```
[Out] (90*tan(x/2)^4 - 60*tan(x/2)^2 + 20*tan(x/2)^6 + 3*tan(x/2)^8 - 5)/(240*a*t
an(x/2)^3)
```

3.10 $\int \frac{\sin(2x)}{1+\cos(2x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 5

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\log(\cos(x))$$

```
[Out] -ln(cos(x))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2746, 31}

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{1}{2}\log(\cos(2x)+1)$$

```
[In] Int[Sin[2*x]/(1 + Cos[2*x]),x]
[Out] -1/2*Log[1 + Cos[2*x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(2x)\right)\right)\\ &= -\frac{1}{2}\log(1+\cos(2x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\log(\cos(x))$$

```
[In] Integrate[Sin[2*x]/(1 + Cos[2*x]),x]
[Out] -Log[Cos[x]]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

method	result	size
parallelrisch	$\ln\left(\sqrt{\sec^2(x)}\right)$	8
derivativedivides	$-\frac{\ln(1+\cos(2x))}{2}$	10
default	$-\frac{\ln(1+\cos(2x))}{2}$	10
norman	$\frac{\ln(1+\tan^2(x))}{2}$	10
risch	$ix - \ln\left(e^{2ix}+1\right)$	16

```
[In] int(sin(2*x)/(1+cos(2*x)),x,method=_RETURNVERBOSE)
[Out] ln((sec(x)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{1}{2}\,\log\left(\frac{1}{2}\,\cos\left(2\,x\right) + \frac{1}{2}\right)$$

```
[In] integrate(sin(2*x)/(1+cos(2*x)),x, algorithm="fricas")
[Out] -1/2*log(1/2*cos(2*x) + 1/2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.00

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{\log\left(\cos\left(2x\right) + 1\right)}{2}$$

```
[In] integrate(sin(2*x)/(1+cos(2*x)),x)
[Out] -log(cos(2*x) + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{1}{2}\,\log\left(\cos\left(2\,x\right) + 1\right)$$

```
[In] integrate(sin(2*x)/(1+cos(2*x)),x, algorithm="maxima")
[Out] -1/2*log(cos(2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{1}{2}\,\log\left(\cos\left(2\,x\right) + 1\right)$$

```
[In] integrate(sin(2*x)/(1+cos(2*x)),x, algorithm="giac")
[Out] -1/2*log(cos(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{\ln\left(\cos\left(x\right)^2\right)}{2}$$

```
[In] int(sin(2*x)/(cos(2*x) + 1),x)
[Out] -log(cos(x)^2)/2
```

3.11 $\int \frac{\sin(2x)}{1-\cos(2x)} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 3

$$\int \frac{\sin(2x)}{1-\cos(2x)} \, dx = \log(\sin(x))$$

```
[Out] ln(sin(x))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 13 vs. 2(3) = 6.

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 4.33, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2746, 31}

$$\int \frac{\sin(2x)}{1-\cos(2x)} \, dx = \frac{1}{2}\log(1-\cos(2x))$$

```
[In] Int[Sin[2*x]/(1 - Cos[2*x]),x]
```

```
[Out] Log[1 - Cos[2*x]]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, -\cos(2x)\right)\\ &= \frac{1}{2}\log(1-\cos(2x))\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \log(\cos(x)) + \log(\tan(x))$$

```
[In] Integrate[Sin[2*x]/(1 - Cos[2*x]),x]
[Out] Log[Cos[x]] + Log[Tan[x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.42 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

method	result	size
derivativedivides	$\frac{\ln(1-\cos(2x))}{2}$	12
default	$\frac{\ln(1-\cos(2x))}{2}$	12
parallelrisch	$\ln\left(\frac{1}{\sqrt{\sec^2(x)}}\right) + \ln(\tan(x))$	12
norman	$-\frac{\ln(1+\tan^2(x))}{2} + \ln(\tan(x))$	14
risch	$-ix + \ln(e^{2ix} - 1)$	14

```
[In] int(sin(2*x)/(1-cos(2*x)),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(1-cos(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{1}{2}\log\left(-\frac{1}{2}\cos\left(2\,x\right)+\frac{1}{2}\right)$$

```
[In] integrate(sin(2*x)/(1-cos(2*x)),x, algorithm="fricas")
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. $2(3) = 6$.

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.67

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{\log\left(\cos\left(2x\right)-1\right)}{2}$$

```
[In] integrate(sin(2*x)/(1-cos(2*x)),x)
[Out] log(cos(2*x) - 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9 vs. $2(3) = 6$.

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 3.00

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{1}{2}\log\left(\cos\left(2\,x\right)-1\right)$$

```
[In] integrate(sin(2*x)/(1-cos(2*x)),x, algorithm="maxima")
[Out] 1/2*log(cos(2*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(3) = 6.

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{1}{2}\,\log\left(-\cos\left(2\,x\right)+1\right)$$

```
[In] integrate(sin(2*x)/(1-cos(2*x)),x, algorithm="giac")
[Out] 1/2*log(-cos(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 13.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 3.00

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{\ln\left(-{\sin\left(x\right)}^2\right)}{2}$$

```
[In] int(-sin(2*x)/(cos(2*x) - 1),x)
[Out] log(-sin(x)^2)/2
```

3.12 $\int \frac{\sin(x)}{(1+\cos(x))^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 6

$$\int \frac{\sin(x)}{(1+\cos(x))^2} \, dx = \frac{1}{1+\cos(x)}$$

```
[Out] 1/(1+cos(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2746, 32}

$$\int \frac{\sin(x)}{(1+\cos(x))^2} \, dx = \frac{1}{\cos(x)+1}$$

```
[In] Int[Sin[x]/(1 + Cos[x])^2,x]
```

```
[Out] (1 + Cos[x])^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{(1+x)^2}\,dx, x, \cos(x)\right)$$
$$= \frac{1}{1+\cos(x)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$$

```
[In] Integrate[Sin[x]/(1 + Cos[x])^2,x]
[Out] Sec[x/2]^2/2
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{1}{\cos(x)+1}$	7
default	$\frac{1}{\cos(x)+1}$	7
norman	$\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}$	9
parallelrisch	$\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}$	9
risch	$\frac{2\,e^{ix}}{\left(e^{ix}+1\right)^2}$	17

```
[In] int(sin(x)/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/(cos(x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos(x)+1}$$

```
[In] integrate(sin(x)/(1+cos(x))^2,x, algorithm="fricas")
[Out] 1/(cos(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos(x)+1}$$

```
[In] integrate(sin(x)/(1+cos(x))**2,x)
[Out] 1/(cos(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos(x)+1}$$

```
[In] integrate(sin(x)/(1+cos(x))^2,x, algorithm="maxima")
[Out] 1/(cos(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos(x)+1}$$

```
[In] integrate(sin(x)/(1+cos(x))^2,x, algorithm="giac")
[Out] 1/(cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos(x)+1}$$

```
[In] int(sin(x)/(cos(x) + 1)^2,x)
[Out] 1/(cos(x) + 1)
```

3.13 $\int \frac{\sin(x)}{(1-\cos(x))^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\sin(x)}{(1-\cos(x))^2} \, dx = -\frac{1}{1-\cos(x)}$$

```
[Out] -1/(1-cos(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2746, 32}

$$\int \frac{\sin(x)}{(1-\cos(x))^2} \, dx = -\frac{1}{1-\cos(x)}$$

```
[In] Int[Sin[x]/(1 - Cos[x])^2,x]
```

```
[Out] -(1 - Cos[x])^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(1+x)^2}\,dx, x, -\cos(x)\right)$$
$$= -\frac{1}{1-\cos(x)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = -\frac{1}{2}\csc^2\left(\frac{x}{2}\right)$$

```
[In] Integrate[Sin[x]/(1 - Cos[x])^2,x]
[Out] -1/2*Csc[x/2]^2
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$-\frac{1}{2\tan\left(\frac{x}{2}\right)^2}$	9
derivativedivides	$-\frac{1}{1-\cos(x)}$	11
default	$-\frac{1}{1-\cos(x)}$	11
risch	$\frac{2\,\mathrm{e}^{ix}}{(\mathrm{e}^{ix}-1)^2}$	17
norman	$\frac{-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}-\frac{\tan\left(\frac{x}{2}\right)}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)^3}$	33

```
[In] int(sin(x)/(1-cos(x))^2,x,method=_RETURNVERBOSE)
[Out] -1/2/tan(1/2*x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos(x)-1}$$

```
[In] integrate(sin(x)/(1-cos(x))^2,x, algorithm="fricas")
[Out] 1/(cos(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.50

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos(x)-1}$$

```
[In] integrate(sin(x)/(1-cos(x))**2,x)
[Out] 1/(cos(x) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos(x)-1}$$

```
[In] integrate(sin(x)/(1-cos(x))^2,x, algorithm="maxima")
[Out] 1/(cos(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos(x)-1}$$

```
[In] integrate(sin(x)/(1-cos(x))^2,x, algorithm="giac")
[Out] 1/(cos(x) - 1)
```

Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\, dx = \frac{1}{\cos(x)-1}$$

```
[In] int(sin(x)/(cos(x) - 1)^2,x)
[Out] 1/(cos(x) - 1)
```

3.14 $\int \frac{\sin^2(x)}{(1+\cos(x))^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2} \, dx = -x + \frac{2\sin(x)}{1+\cos(x)}$$

```
[Out] -x+2*sin(x)/(1+cos(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2759, 8}

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2} \, dx = \frac{2\sin(x)}{\cos(x)+1} - x$$

```
[In] Int[Sin[x]^2/(1 + Cos[x])^2,x]
```

```
[Out] -x + (2*Sin[x])/(1 + Cos[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sin(x)}{1+\cos(x)} - \int 1\,dx\\ &= -x + \frac{2\sin(x)}{1+\cos(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx = -2\arctan\left(\tan\left(\frac{x}{2}\right)\right) + 2\tan\left(\frac{x}{2}\right)$$

```
[In] Integrate[Sin[x]^2/(1 + Cos[x])^2,x]
[Out] -2*ArcTan[Tan[x/2]] + 2*Tan[x/2]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$-x+2\tan\left(\frac{x}{2}\right)$	11
default	$2\tan\left(\frac{x}{2}\right)-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	15
risch	$-x+\frac{4i}{{\mathrm e}^{ix}+1}$	17
norman	$\frac{-x+4\left(\tan^{3}\left(\frac{x}{2}\right)\right)+2\left(\tan^{5}\left(\frac{x}{2}\right)\right)-2\left(\tan^{2}\left(\frac{x}{2}\right)\right)x-\left(\tan^{4}\left(\frac{x}{2}\right)\right)x+2\tan\left(\frac{x}{2}\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	56

```
[In] int(sin(x)^2/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
[Out] -x+2*tan(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2} \, dx = -\frac{x \cos(x) + x - 2 \sin(x)}{\cos(x) + 1}$$

```
[In] integrate(sin(x)^2/(1+cos(x))^2,x, algorithm="fricas")
[Out] -(x*cos(x) + x - 2*sin(x))/(cos(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2} \, dx = -x + 2 \tan\left(\frac{x}{2}\right)$$

```
[In] integrate(sin(x)**2/(1+cos(x))**2,x)
[Out] -x + 2*tan(x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2} \, dx = \frac{2 \sin(x)}{\cos(x) + 1} - 2 \arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

```
[In] integrate(sin(x)^2/(1+cos(x))^2,x, algorithm="maxima")
[Out] 2*sin(x)/(cos(x) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2} \, dx = -x + 2 \tan\left(\frac{1}{2} x\right)$$

```
[In] integrate(sin(x)^2/(1+cos(x))^2,x, algorithm="giac")
[Out] -x + 2*tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 13.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx = 2\tan\left(\frac{x}{2}\right) - x$$

```
[In] int(sin(x)^2/(cos(x) + 1)^2,x)
[Out] 2*tan(x/2) - x
```

3.15 $\int \frac{\sin^2(x)}{(1-\cos(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2} dx = -x - \frac{2\sin(x)}{1-\cos(x)}$$

```
[Out] -x-2*sin(x)/(1-cos(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2759, 8}

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2} dx = -x - \frac{2\sin(x)}{1-\cos(x)}$$

```
[In] Int[Sin[x]^2/(1 - Cos[x])^2,x]
```

```
[Out] -x - (2*Sin[x])/(1 - Cos[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\sin(x)}{1-\cos(x)} - \int 1\,dx \\ &= -x - \frac{2\sin(x)}{1-\cos(x)}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx = -2\cot\left(\frac{x}{2}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Sin[x]^2/(1 - Cos[x])^2,x]
[Out] -2*Cot[x/2]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x/2]^2]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{2}{\tan\left(\frac{x}{2}\right)}-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	17
risch	$-x-\frac{4i}{{\rm e}^{ix}-1}$	17
parallelrisch	$\frac{-\tan\left(\frac{x}{2}\right)x-2}{\tan\left(\frac{x}{2}\right)}$	17
norman	$\frac{-2\left(\tan^2\left(\frac{x}{2}\right)\right)-4\left(\tan^4\left(\frac{x}{2}\right)\right)-2\left(\tan^6\left(\frac{x}{2}\right)\right)-x\left(\tan^7\left(\frac{x}{2}\right)\right)-\left(\tan^3\left(\frac{x}{2}\right)\right)x-2\left(\tan^5\left(\frac{x}{2}\right)\right)x}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\tan\left(\frac{x}{2}\right)^3}$	70

```
[In] int(sin(x)^2/(1-cos(x))^2,x,method=_RETURNVERBOSE)
[Out] -2/tan(1/2*x)-2*arctan(tan(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx = -\frac{x\sin(x) + 2\cos(x) + 2}{\sin(x)}$$

```
[In] integrate(sin(x)^2/(1-cos(x))^2,x, algorithm="fricas")
[Out] -(x*sin(x) + 2*cos(x) + 2)/sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx = -x - \frac{2}{\tan\left(\frac{x}{2}\right)}$$

```
[In] integrate(sin(x)**2/(1-cos(x))**2,x)
[Out] -x - 2/tan(x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx = -\frac{2(\cos(x)+1)}{\sin(x)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

```
[In] integrate(sin(x)^2/(1-cos(x))^2,x, algorithm="maxima")
[Out] -2*(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx = -x - \frac{2}{\tan\left(\frac{1}{2}x\right)}$$

```
[In] integrate(sin(x)^2/(1-cos(x))^2,x, algorithm="giac")
[Out] -x - 2/tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx = -x - 2\cot\left(\frac{x}{2}\right)$$

```
[In] int(sin(x)^2/(cos(x) - 1)^2,x)
[Out] - x - 2*cot(x/2)
```

3.16 $\int \frac{\sin^3(x)}{(1+\cos(x))^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2} \, dx = \cos(x) - 2\log(1+\cos(x))$$

```
[Out] cos(x)-2*ln(1+cos(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2746, 45}

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2} \, dx = \cos(x) - 2\log(\cos(x)+1)$$

```
[In] Int[Sin[x]^3/(1 + Cos[x])^2,x]
```

```
[Out] Cos[x] - 2*Log[1 + Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1-x}{1+x}\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \left(-1+\frac{2}{1+x}\right)\,dx, x, \cos(x)\right)\\ &= \cos(x) - 2\log(1+\cos(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = -1+\cos(x) - 4\log\left(\cos\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Sin[x]^3/(1 + Cos[x])^2,x]
[Out] -1 + Cos[x] - 4*Log[Cos[x/2]]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\cos(x) - 2\ln(\cos(x)+1)$	11
default	$\cos(x) - 2\ln(\cos(x)+1)$	11
parallelrisch	$2\ln\left(\frac{1}{\cos(x)+1}\right)+\ln(4)+\cos(x)+1$	16
risch	$2ix+\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}-4\ln(e^{ix}+1)$	30
norman	$\frac{2(\tan^4(\frac{x}{2}))+4(\tan^2(\frac{x}{2}))+2}{(1+\tan^2(\frac{x}{2}))^3}+2\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	42

```
[In] int(sin(x)^3/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
[Out] cos(x)-2*ln(cos(x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = \cos(x) - 2\,\log\left(\frac{1}{2}\,\cos(x) + \frac{1}{2}\right)$$

```
[In] integrate(sin(x)^3/(1+cos(x))^2,x, algorithm="fricas")
[Out] cos(x) - 2*log(1/2*cos(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 5.80

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = -\frac{2\log(\cos(x)+1)\cos(x)}{\cos(x)+1} - \frac{2\log(\cos(x)+1)}{\cos(x)+1} + \frac{\sin^2(x)}{\cos(x)+1} + \frac{2\cos^2(x)}{\cos(x)+1} - \frac{2}{\cos(x)+1}$$

```
[In] integrate(sin(x)**3/(1+cos(x))**2,x)
[Out] -2*log(cos(x) + 1)*cos(x)/(cos(x) + 1) - 2*log(cos(x) + 1)/(cos(x) + 1) + s
in(x)**2/(cos(x) + 1) + 2*cos(x)**2/(cos(x) + 1) - 2/(cos(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = \cos(x) - 2\,\log(\cos(x)+1)$$

```
[In] integrate(sin(x)^3/(1+cos(x))^2,x, algorithm="maxima")
[Out] cos(x) - 2*log(cos(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = \cos(x) - 2\,\log(\cos(x)+1)$$

```
[In] integrate(sin(x)^3/(1+cos(x))^2,x, algorithm="giac")
[Out] cos(x) - 2*log(cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = \cos(x) - 2\,\ln(\cos(x)+1)$$

```
[In] int(sin(x)^3/(cos(x) + 1)^2,x)
[Out] cos(x) - 2*log(cos(x) + 1)
```

3.17 $\int \frac{\sin^3(x)}{(1-\cos(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2} \, dx = \cos(x)+2\log(1-\cos(x))$$

```
[Out] cos(x)+2*ln(1-cos(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2746, 45}

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2} \, dx = \cos(x)+2\log(1-\cos(x))$$

```
[In] Int[Sin[x]^3/(1 - Cos[x])^2,x]
```

```
[Out] Cos[x] + 2*Log[1 - Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1-x}{1+x}\,dx, x, -\cos(x)\right)\\ &= \text{Subst}\left(\int \left(-1+\frac{2}{1+x}\right)\,dx, x, -\cos(x)\right)\\ &= \cos(x)+2\log(1-\cos(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = -1+\cos(x)+4\log\left(\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Sin[x]^3/(1 - Cos[x])^2,x]
[Out] -1 + Cos[x] + 4*Log[Sin[x/2]]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\cos(x)+2\ln(\cos(x)-1)$	11
default	$\cos(x)+2\ln(\cos(x)-1)$	11
parallelrisch	$4\ln(\csc(x)-\cot(x))-2\ln\left(\frac{1}{\cos(x)+1}\right)+\ln\left(\frac{1}{4}\right)+\cos(x)+1$	26
risch	$-2ix+\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+4\ln(e^{ix}-1)$	30
norman	$\frac{2(\tan^3(\frac{x}{2}))+2(\tan^7(\frac{x}{2}))+4(\tan^5(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^3\tan(\frac{x}{2})^3}+4\ln\left(\tan\left(\frac{x}{2}\right)\right)-2\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	62

```
[In] int(sin(x)^3/(1-cos(x))^2,x,method=_RETURNVERBOSE)
[Out] cos(x)+2*ln(cos(x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = \cos(x) + 2\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

```
[In] integrate(sin(x)^3/(1-cos(x))^2,x, algorithm="fricas")
[Out] cos(x) + 2*log(-1/2*cos(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(10) = 20.

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.83

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = \frac{2\log(\cos(x)-1)\cos(x)}{\cos(x)-1} - \frac{2\log(\cos(x)-1)}{\cos(x)-1} + \frac{\sin^2(x)}{\cos(x)-1} + \frac{2\cos^2(x)}{\cos(x)-1} - \frac{2}{\cos(x)-1}$$

```
[In] integrate(sin(x)**3/(1-cos(x))**2,x)
[Out] 2*log(cos(x) - 1)*cos(x)/(cos(x) - 1) - 2*log(cos(x) - 1)/(cos(x) - 1) + si
n(x)**2/(cos(x) - 1) + 2*cos(x)**2/(cos(x) - 1) - 2/(cos(x) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = \cos(x) + 2\log(\cos(x) - 1)$$

```
[In] integrate(sin(x)^3/(1-cos(x))^2,x, algorithm="maxima")
[Out] cos(x) + 2*log(cos(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = \cos(x) + 2\log(-\cos(x) + 1)$$

```
[In] integrate(sin(x)^3/(1-cos(x))^2,x, algorithm="giac")
[Out] cos(x) + 2*log(-cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 13.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = 2\ln(\cos(x) - 1) + \cos(x)$$

```
[In] int(sin(x)^3/(cos(x) - 1)^2,x)
[Out] 2*log(cos(x) - 1) + cos(x)
```

3.18 $\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2(1+\cos(x))^2}$$

```
[Out] 1/2/(1+cos(x))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2746, 32}

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2(\cos(x)+1)^2}$$

```
[In] Int[Sin[x]/(1 + Cos[x])^3,x]
```

```
[Out] 1/(2*(1 + Cos[x])^2)
```

<u>Rule 32</u>

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

<u>Rule 2746</u>

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, \cos(x)\right)\\ &= \frac{1}{2(1+\cos(x))^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{8}\sec^4\left(\frac{x}{2}\right)$$

```
[In] Integrate[Sin[x]/(1 + Cos[x])^3,x]
[Out] Sec[x/2]^4/8
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{1}{2(\cos(x)+1)^2}$	9
default	$\frac{1}{2(\cos(x)+1)^2}$	9
risch	$\frac{2\,\mathrm{e}^{2ix}}{(\mathrm{e}^{ix}+1)^4}$	17
parallelrisch	$\frac{(\tan^2(\frac{x}{2}))(\tan^2(\frac{x}{2})+2)}{8}$	17
norman	$\frac{(\tan^2(\frac{x}{2}))}{4}+\frac{(\tan^4(\frac{x}{2}))}{8}$	18

```
[In] int(sin(x)/(cos(x)+1)^3,x,method=_RETURNVERBOSE)
[Out] 1/2/(cos(x)+1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\left(\cos\left(x\right)^2 + 2\,\cos\left(x\right) + 1\right)}$$

```
[In] integrate(sin(x)/(1+cos(x))^3,x, algorithm="fricas")
[Out] 1/2/(cos(x)^2 + 2*cos(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\cos^2\left(x\right) + 4\cos\left(x\right) + 2}$$

```
[In] integrate(sin(x)/(1+cos(x))**3,x)
[Out] 1/(2*cos(x)**2 + 4*cos(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\left(\cos\left(x\right) + 1\right)^2}$$

```
[In] integrate(sin(x)/(1+cos(x))^3,x, algorithm="maxima")
[Out] 1/2/(cos(x) + 1)^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\left(\cos\left(x\right) + 1\right)^2}$$

```
[In] integrate(sin(x)/(1+cos(x))^3,x, algorithm="giac")
[Out] 1/2/(cos(x) + 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin(x)}{(1+\cos(x))^3} \, dx = \frac{1}{2\left(\cos\left(x\right)+1\right)^2}$$

```
[In] int(sin(x)/(cos(x) + 1)^3,x)
[Out] 1/(2*(cos(x) + 1)^2)
```

3.19 $\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2(1-\cos(x))^2}$$

```
[Out] -1/2/(1-cos(x))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2746, 32}

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2(1-\cos(x))^2}$$

```
[In] Int[Sin[x]/(1 - Cos[x])^3,x]
```

```
[Out] -1/2*1/(1 - Cos[x])^2
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, -\cos(x)\right)\\ &= -\frac{1}{2(1-\cos(x))^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{8}\csc^4\left(\frac{x}{2}\right)$$

```
[In] Integrate[Sin[x]/(1 - Cos[x])^3,x]
[Out] -1/8*Csc[x/2]^4
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{2(1-\cos(x))^2}$	11
default	$-\frac{1}{2(1-\cos(x))^2}$	11
risch	$-\frac{2\,e^{2ix}}{(e^{ix}-1)^4}$	17
parallelrisch	$-\frac{(\cot^2(\frac{x}{2}))(\cot^2(\frac{x}{2})+2)}{8}$	17
norman	$\frac{-\frac{(\tan^5(\frac{x}{2}))}{4}-\frac{3(\tan^3(\frac{x}{2}))}{8}-\frac{\tan(\frac{x}{2})}{8}}{(1+\tan^2(\frac{x}{2}))\tan(\frac{x}{2})^5}$	41

```
[In] int(sin(x)/(1-cos(x))^3,x,method=_RETURNVERBOSE)
[Out] -1/2/(1-cos(x))^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2\left(\cos\left(x\right)^2 - 2\cos\left(x\right) + 1\right)}$$

```
[In] integrate(sin(x)/(1-cos(x))^3,x, algorithm="fricas")
[Out] -1/2/(cos(x)^2 - 2*cos(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2\cos^2\left(x\right) - 4\cos\left(x\right) + 2}$$

```
[In] integrate(sin(x)/(1-cos(x))**3,x)
[Out] -1/(2*cos(x)**2 - 4*cos(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2\left(\cos\left(x\right) - 1\right)^2}$$

```
[In] integrate(sin(x)/(1-cos(x))^3,x, algorithm="maxima")
[Out] -1/2/(cos(x) - 1)^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2\left(\cos\left(x\right) - 1\right)^2}$$

```
[In] integrate(sin(x)/(1-cos(x))^3,x, algorithm="giac")
[Out] -1/2/(cos(x) - 1)^2
```

Mupad [B] (verification not implemented)

Time = 12.95 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sin(x)}{(1-\cos(x))^3} \, dx = -\frac{1}{2\left(\cos\left(x\right)-1\right)^2}$$

```
[In] int(-sin(x)/(cos(x) - 1)^3,x)
[Out] -1/(2*(cos(x) - 1)^2)
```

3.20 $\int \frac{\sin^2(x)}{(1+\cos(x))^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3} \, dx = \frac{\sin^3(x)}{3(1+\cos(x))^3}$$

```
[Out] 1/3*sin(x)^3/(1+cos(x))^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2750}

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3} \, dx = \frac{\sin^3(x)}{3(\cos(x)+1)^3}$$

```
[In] Int[Sin[x]^2/(1 + Cos[x])^3,x]
```

```
[Out] Sin[x]^3/(3*(1 + Cos[x])^3)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\sin^3(x)}{3(1+\cos(x))^3}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{1}{3}\tan^3\left(\frac{x}{2}\right)$$

```
[In] Integrate[Sin[x]^2/(1 + Cos[x])^3,x]
[Out] Tan[x/2]^3/3
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}$	9
parallelrisch	$\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}$	9
risch	$-\frac{2i\left(3\,e^{2ix}+1\right)}{3\left(e^{ix}+1\right)^3}$	22
norman	$\frac{\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}+\frac{2\left(\tan^5\left(\frac{x}{2}\right)\right)}{3}+\frac{\left(\tan^7\left(\frac{x}{2}\right)\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	37

```
[In] int(sin(x)^2/(cos(x)+1)^3,x,method=_RETURNVERBOSE)
[Out] 1/3*tan(1/2*x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = -\frac{(\cos(x)-1)\sin(x)}{3\left(\cos(x)^2+2\cos(x)+1\right)}$$

```
[In] integrate(sin(x)^2/(1+cos(x))^3,x, algorithm="fricas")
[Out] -1/3*(cos(x) - 1)*sin(x)/(cos(x)^2 + 2*cos(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3} \, dx = \frac{\tan^3\left(\frac{x}{2}\right)}{3}$$

```
[In] integrate(sin(x)**2/(1+cos(x))**3,x)
[Out] tan(x/2)**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3} \, dx = \frac{\sin\left(x\right)^3}{3\left(\cos\left(x\right)+1\right)^3}$$

```
[In] integrate(sin(x)^2/(1+cos(x))^3,x, algorithm="maxima")
[Out] 1/3*sin(x)^3/(cos(x) + 1)^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3} \, dx = \frac{1}{3} \tan\left(\frac{1}{2}x\right)^3$$

```
[In] integrate(sin(x)^2/(1+cos(x))^3,x, algorithm="giac")
[Out] 1/3*tan(1/2*x)^3
```

Mupad [B] (verification not implemented)

Time = 13.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3} \, dx = \frac{\tan\left(\frac{x}{2}\right)^3}{3}$$

```
[In] int(sin(x)^2/(cos(x) + 1)^3,x)
[Out] tan(x/2)^3/3
```

3.21 $\int \frac{\sin^2(x)}{(1-\cos(x))^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3} \, dx = -\frac{\sin^3(x)}{3(1-\cos(x))^3}$$

```
[Out] -1/3*sin(x)^3/(1-cos(x))^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2750}

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3} \, dx = -\frac{\sin^3(x)}{3(1-\cos(x))^3}$$

```
[In] Int[Sin[x]^2/(1 - Cos[x])^3,x]
```

```
[Out] -1/3*Sin[x]^3/(1 - Cos[x])^3
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sin^3(x)}{3(1-\cos(x))^3}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{3}\cot^3\left(\frac{x}{2}\right)$$

```
[In] Integrate[Sin[x]^2/(1 - Cos[x])^3,x]
[Out] -1/3*Cot[x/2]^3
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{1}{3\tan\left(\frac{x}{2}\right)^{3}}$	9
parallelrisch	$-\frac{1}{3\tan\left(\frac{x}{2}\right)^{3}}$	9
risch	$\frac{2i\left(3\,\mathrm{e}^{2ix}+1\right)}{3\left(\mathrm{e}^{ix}-1\right)^{3}}$	22
norman	$\frac{-\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3}-\frac{2\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{3}-\frac{\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{3}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}\tan\left(\frac{x}{2}\right)^{5}}$	43

```
[In] int(sin(x)^2/(1-cos(x))^3,x,method=_RETURNVERBOSE)
[Out] -1/3/tan(1/2*x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = \frac{\cos\left(x\right)^2+2\,\cos\left(x\right)+1}{3\left(\cos\left(x\right)-1\right)\sin\left(x\right)}$$

```
[In] integrate(sin(x)^2/(1-cos(x))^3,x, algorithm="fricas")
[Out] 1/3*(cos(x)^2 + 2*cos(x) + 1)/((cos(x) - 1)*sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{3\tan^3\left(\frac{x}{2}\right)}$$

```
[In] integrate(sin(x)**2/(1-cos(x))**3,x)
[Out] -1/(3*tan(x/2)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{(\cos(x)+1)^3}{3\sin(x)^3}$$

```
[In] integrate(sin(x)^2/(1-cos(x))^3,x, algorithm="maxima")
[Out] -1/3*(cos(x) + 1)^3/sin(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{3\tan\left(\frac{1}{2}x\right)^3}$$

```
[In] integrate(sin(x)^2/(1-cos(x))^3,x, algorithm="giac")
[Out] -1/3/tan(1/2*x)^3
```

Mupad [B] (verification not implemented)

Time = 13.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{\cot\left(\frac{x}{2}\right)^3}{3}$$

```
[In] int(-sin(x)^2/(cos(x) - 1)^3,x)
[Out] -cot(x/2)^3/3
```

3.22 $\int \frac{\sin^3(x)}{(1+\cos(x))^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3} \, dx = \frac{2}{1+\cos(x)} + \log(1+\cos(x))$$

```
[Out] 2/(1+cos(x))+ln(1+cos(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2746, 45}

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3} \, dx = \frac{2}{\cos(x)+1} + \log(\cos(x)+1)$$

```
[In] Int[Sin[x]^3/(1 + Cos[x])^3,x]
```

```
[Out] 2/(1 + Cos[x]) + Log[1 + Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)\,dx, x, \cos(x)\right)\\ &= \frac{2}{1+\cos(x)}+\log(1+\cos(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = 2\log\left(\cos\left(\frac{x}{2}\right)\right)+\tan^2\left(\frac{x}{2}\right)$$

```
[In] Integrate[Sin[x]^3/(1 + Cos[x])^3,x]
[Out] 2*Log[Cos[x/2]] + Tan[x/2]^2
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{2}{\cos(x)+1}+\ln\left(\cos\left(x\right)+1\right)$	15
default	$\frac{2}{\cos(x)+1}+\ln\left(\cos\left(x\right)+1\right)$	15
parallelrisch	$\tan^2\left(\frac{x}{2}\right)-\ln\left(\frac{2}{\cos(x)+1}\right)$	19
risch	$-ix+\frac{4\,e^{ix}}{\left(e^{ix}+1\right)^2}+2\ln\left(e^{ix}+1\right)$	32
norman	$\frac{\tan^8\left(\frac{x}{2}\right)-8\left(\tan^2\left(\frac{x}{2}\right)\right)-6\left(\tan^4\left(\frac{x}{2}\right)\right)-3}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}-\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	48

```
[In] int(sin(x)^3/(cos(x)+1)^3,x,method=_RETURNVERBOSE)
[Out] 2/(cos(x)+1)+ln(cos(x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{\cos(x)+1}$$

```
[In] integrate(sin(x)^3/(1+cos(x))^3,x, algorithm="fricas")
[Out] ((cos(x) + 1)*log(1/2*cos(x) + 1/2) + 2)/(cos(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(12) = 24.

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 9.00

$$\begin{aligned}\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx &= \frac{2\log(\cos(x)+1)\cos^2(x)}{2\cos^2(x)+4\cos(x)+2}+\frac{4\log(\cos(x)+1)\cos(x)}{2\cos^2(x)+4\cos(x)+2}\\ &\quad+\frac{2\log(\cos(x)+1)}{2\cos^2(x)+4\cos(x)+2}+\frac{\sin^2(x)}{2\cos^2(x)+4\cos(x)+2}\\ &\quad+\frac{2\cos(x)}{2\cos^2(x)+4\cos(x)+2}+\frac{2}{2\cos^2(x)+4\cos(x)+2}\end{aligned}$$

```
[In] integrate(sin(x)**3/(1+cos(x))**3,x)
[Out] 2*log(cos(x) + 1)*cos(x)**2/(2*cos(x)**2 + 4*cos(x) + 2) + 4*log(cos(x) + 1
)*cos(x)/(2*cos(x)**2 + 4*cos(x) + 2) + 2*log(cos(x) + 1)/(2*cos(x)**2 + 4*
cos(x) + 2) + sin(x)**2/(2*cos(x)**2 + 4*cos(x) + 2) + 2*cos(x)/(2*cos(x)**
2 + 4*cos(x) + 2) + 2/(2*cos(x)**2 + 4*cos(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \frac{2}{\cos(x)+1}+\log(\cos(x)+1)$$

```
[In] integrate(sin(x)^3/(1+cos(x))^3,x, algorithm="maxima")
[Out] 2/(cos(x) + 1) + log(cos(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \frac{2}{\cos(x)+1} + \log(\cos(x)+1)$$

```
[In] integrate(sin(x)^3/(1+cos(x))^3,x, algorithm="giac")
[Out] 2/(cos(x) + 1) + log(cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \ln(\cos(x)+1) + \frac{2}{\cos(x)+1}$$

```
[In] int(sin(x)^3/(cos(x) + 1)^3,x)
[Out] log(cos(x) + 1) + 2/(cos(x) + 1)
```

3.23 $\int \frac{\sin^3(x)}{(1-\cos(x))^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3} dx = -\frac{2}{1-\cos(x)} - \log(1-\cos(x))$$

```
[Out] -2/(1-cos(x))-ln(1-cos(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2746, 45}

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3} dx = -\frac{2}{1-\cos(x)} - \log(1-\cos(x))$$

```
[In] Int[Sin[x]^3/(1 - Cos[x])^3,x]
```

```
[Out] -2/(1 - Cos[x]) - Log[1 - Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, -\cos(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)\,dx, x, -\cos(x)\right)\\
&= -\frac{2}{1-\cos(x)}-\log(1-\cos(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = -\cot^2\left(\frac{x}{2}\right)-2\log\left(\cos\left(\frac{x}{2}\right)\right)-2\log\left(\tan\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Sin[x]^3/(1 - Cos[x])^3,x]
[Out] -Cot[x/2]^2 - 2*Log[Cos[x/2]] - 2*Log[Tan[x/2]]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2}{\cos(x)-1}-\ln\left(\cos\left(x\right)-1\right)$	17
default	$\frac{2}{\cos(x)-1}-\ln\left(\cos\left(x\right)-1\right)$	17
parallelrisch	$-2\ln\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(\frac{2}{\cos(x)+1}\right)-\left(\cot^2\left(\frac{x}{2}\right)\right)$	26
risch	$ix+\frac{4\,e^{ix}}{\left(e^{ix}-1\right)^2}-2\ln\left(e^{ix}-1\right)$	32
norman	$\frac{-3\left(\tan^5\left(\frac{x}{2}\right)\right)-3\left(\tan^7\left(\frac{x}{2}\right)\right)-\left(\tan^9\left(\frac{x}{2}\right)\right)-\left(\tan^3\left(\frac{x}{2}\right)\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3\tan\left(\frac{x}{2}\right)^5}-2\ln\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	68

```
[In] int(sin(x)^3/(1-cos(x))^3,x,method=_RETURNVERBOSE)
[Out] 2/(cos(x)-1)-ln(cos(x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = -\frac{(\cos(x)-1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{\cos(x)-1}$$

```
[In] integrate(sin(x)^3/(1-cos(x))^3,x, algorithm="fricas")
[Out] -((cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 2)/(cos(x) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(14) = 28.

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 6.30

$$\begin{aligned}\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = &-\frac{2\log(\cos(x)-1)\cos^2(x)}{2\cos^2(x)-4\cos(x)+2}+\frac{4\log(\cos(x)-1)\cos(x)}{2\cos^2(x)-4\cos(x)+2}\\&-\frac{2\log(\cos(x)-1)}{2\cos^2(x)-4\cos(x)+2}-\frac{\sin^2(x)}{2\cos^2(x)-4\cos(x)+2}\\&+\frac{2\cos(x)}{2\cos^2(x)-4\cos(x)+2}-\frac{2}{2\cos^2(x)-4\cos(x)+2}\end{aligned}$$

```
[In] integrate(sin(x)**3/(1-cos(x))**3,x)
[Out] -2*log(cos(x) - 1)*cos(x)**2/(2*cos(x)**2 - 4*cos(x) + 2) + 4*log(cos(x) -
1)*cos(x)/(2*cos(x)**2 - 4*cos(x) + 2) - 2*log(cos(x) - 1)/(2*cos(x)**2 - 4
*cos(x) + 2) - sin(x)**2/(2*cos(x)**2 - 4*cos(x) + 2) + 2*cos(x)/(2*cos(x)*
*2 - 4*cos(x) + 2) - 2/(2*cos(x)**2 - 4*cos(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = \frac{2}{\cos(x)-1}-\log(\cos(x)-1)$$

```
[In] integrate(sin(x)^3/(1-cos(x))^3,x, algorithm="maxima")
[Out] 2/(cos(x) - 1) - log(cos(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = \frac{2}{\cos(x)-1} - \log(-\cos(x)+1)$$

```
[In] integrate(sin(x)^3/(1-cos(x))^3,x, algorithm="giac")
[Out] 2/(cos(x) - 1) - log(-cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = \frac{2}{\cos(x)-1} - \ln(\cos(x)-1)$$

```
[In] int(-sin(x)^3/(cos(x) - 1)^3,x)
[Out] 2/(cos(x) - 1) - log(cos(x) - 1)
```

3.24 $\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 104

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = -\frac{a(2a^2-3b^2)\,x}{2b^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4}+\frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2b^3}-\frac{\sin^3(x)}{3b}$$

```
[Out] -1/2*a*(2*a^2-3*b^2)*x/b^4+2*(a-b)^(3/2)*(a+b)^(3/2)*arctan((a-b)^(1/2)*tan
(1/2*x)/(a+b)^(1/2))/b^4+1/2*(2*a^2-2*b^2-a*b*cos(x))*sin(x)/b^3-1/3*sin(x)
^3/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2774, 2944, 2814, 2738, 211}

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = -\frac{ax(2a^2-3b^2)}{2b^4}+\frac{\sin(x)\,(2(a^2-b^2)-ab\cos(x))}{2b^3}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4}-\frac{\sin^3(x)}{3b}$$

```
[In] Int[Sin[x]^4/(a + b*Cos[x]),x]
```

```
[Out] -1/2*(a*(2*a^2 - 3*b^2)*x)/b^4 + (2*(a - b)^(3/2)*(a + b)^(3/2)*ArcTan[(Sqr
t[a - b]*Tan[x/2])/Sqrt[a + b]])/b^4 + ((2*(a^2 - b^2) - a*b*Cos[x])*Sin[x]
)/(2*b^3) - Sin[x]^3/(3*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sin^3(x)}{3b} - \frac{\int \frac{(-b-a\cos(x))\sin^2(x)}{a+b\cos(x)}\,dx}{b}\\ &= \frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2b^3} - \frac{\sin^3(x)}{3b} - \frac{\int \frac{b(a^2-2b^2)+a(2a^2-3b^2)\cos(x)}{a+b\cos(x)}\,dx}{2b^3}\end{aligned}$$

$$= -\frac{a(2a^2-3b^2)x}{2b^4} + \frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2b^3} - \frac{\sin^3(x)}{3b} + \frac{(a^2-b^2)^2\int \frac{1}{a+b\cos(x)}\,dx}{b^4}$$

$$= -\frac{a(2a^2-3b^2)x}{2b^4} + \frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2b^3} - \frac{\sin^3(x)}{3b} + \frac{\left(2(a^2-b^2)^2\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b^4}$$

$$= -\frac{a(2a^2-3b^2)x}{2b^4} + \frac{2(a-b)^{3/2}(a+b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4} + \frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2b^3} - \frac{\sin^3(x)}{3b}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = \frac{-12a^3x+18ab^2x-24(-a^2+b^2)^{3/2}\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+3b(4a^2-5b^2)\sin(x)-3ab^2\sin(2x)+b^3\sin(3x)}{12b^4}$$

```
[In] Integrate[Sin[x]^4/(a + b*Cos[x]),x]
```

```
[Out] (-12*a^3*x + 18*a*b^2*x - 24*(-a^2 + b^2)^(3/2)*ArcTanh[((a - b)*Tan[x/2])/
Sqrt[-a^2 + b^2]] + 3*b*(4*a^2 - 5*b^2)*Sin[x] - 3*a*b^2*Sin[2*x] + b^3*Sin
[3*x])/(12*b^4)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.46

method	result
default	$-\frac{2\left(\frac{\left(-a^2b-\frac{1}{2}ab^2+b^3\right)\left(\tan^5\left(\frac{x}{2}\right)\right)+\left(-2a^2b+\frac{10}{3}b^3\right)\left(\tan^3\left(\frac{x}{2}\right)\right)+\left(-a^2b+b^3+\frac{1}{2}ab^2\right)\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}+\frac{a\left(2a^2-3b^2\right)\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{2}\right)}{b^4} + \frac{2(a+b)^2(\ldots}{\ldots}$
risch	$-\frac{a^3x}{b^4}+\frac{3ax}{2b^2}-\frac{ie^{ix}a^2}{2b^3}+\frac{5ie^{ix}}{8b}+\frac{ie^{-ix}a^2}{2b^3}-\frac{5ie^{-ix}}{8b}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{ix}-\frac{i\sqrt{-a^2+b^2}-a}{b}\right)a^2}{b^4}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{ix}-\frac{i\sqrt{-a^2\ldots}}{\ldots}\right)}{b^2}$

```
[In] int(sin(x)^4/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^4*(((-a^2*b-1/2*a*b^2+b^3)*tan(1/2*x)^5+(-2*a^2*b+10/3*b^3)*tan(1/2*x)
^3+(-a^2*b+b^3+1/2*a*b^2)*tan(1/2*x))/(1+tan(1/2*x)^2)^3+1/2*a*(2*a^2-3*b^2
```

```
)*arctan(tan(1/2*x)))+2*(a+b)^2*(a-b)^2/b^4/((a-b)*(a+b))^(1/2)*arctan((a-b
)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.34

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx$$
$$= \left[-\frac{3\,(a^2-b^2)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+(2\,a^2-b^2)\cos(x)^2+2\,\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)+3\,(2\,a^3-3\,ab^2)x-}{6\,b^4}\right.$$

```
[In] integrate(sin(x)^4/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2 - b^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos
(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)
^2 + 2*a*b*cos(x) + a^2)) + 3*(2*a^3 - 3*a*b^2)*x - (2*b^3*cos(x)^2 - 3*a*b
^2*cos(x) + 6*a^2*b - 8*b^3)*sin(x))/b^4, 1/6*(6*(a^2 - b^2)^(3/2)*arctan(-
(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))) - 3*(2*a^3 - 3*a*b^2)*x + (2*b^3*c
os(x)^2 - 3*a*b^2*cos(x) + 6*a^2*b - 8*b^3)*sin(x))/b^4]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(x)**4/(a+b*cos(x)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sin(x)^4/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(86) = 172.

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.87

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = -\frac{(2\,a^3-3\,ab^2)x}{2\,b^4} - \frac{2\,(a^4-2\,a^2b^2+b^4)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan(\frac{1}{2}\,x)-b\tan(\frac{1}{2}\,x)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}b^4} + \frac{6\,a^2\tan\left(\frac{1}{2}\,x\right)^5+3\,ab\tan\left(\frac{1}{2}\,x\right)^5-6\,b^2\tan\left(\frac{1}{2}\,x\right)^5+12\,a^2\tan\left(\frac{1}{2}\,x\right)^3-20\,b^2\tan\left(\frac{1}{2}\,x\right)^3+6\,a^2\tan\left(\frac{1}{2}\,x\right) -}{3\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^3b^3}$$

```
[In] integrate(sin(x)^4/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3 - 3*a*b^2)*x/b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*x/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x))/sqrt(a^2 -
b^2)))/(sqrt(a^2 - b^2)*b^4) + 1/3*(6*a^2*tan(1/2*x)^5 + 3*a*b*tan(1/2*x)^5
 - 6*b^2*tan(1/2*x)^5 + 12*a^2*tan(1/2*x)^3 - 20*b^2*tan(1/2*x)^3 + 6*a^2*t
an(1/2*x) - 3*a*b*tan(1/2*x) - 6*b^2*tan(1/2*x))/((tan(1/2*x)^2 + 1)^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 1677, normalized size of antiderivative = 16.12

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = \text{Too large to display}$$

```
[In] int(sin(x)^4/(a + b*cos(x)),x)
```

```
[Out] ((4*tan(x/2)^3*(3*a^2 - 5*b^2))/(3*b^3) - (tan(x/2)*(a*b - 2*a^2 + 2*b^2))/
b^3 + (tan(x/2)^5*(a*b + 2*a^2 - 2*b^2))/b^3)/(3*tan(x/2)^2 + 3*tan(x/2)^4
+ tan(x/2)^6 + 1) - (2*atanh((64*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^
2)^(1/2))/(128*a*b^2 + 112*a^2*b - 352*a^3 - 64*b^3 + (16*a^4)/b + (320*a^5
)/b^2 - (112*a^6)/b^3 - (96*a^7)/b^4 + (48*a^8)/b^5) + (144*a^2*tan(x/2)*(b
^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b^4 + 16*a^4*b + 320*a^5 -
64*b^5 + 112*a^2*b^3 - 352*a^3*b^2 - (112*a^6)/b - (96*a^7)/b^2 + (48*a^8)/
b^3) + (80*a^3*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b
^5 + 320*a^5*b - 112*a^6 - 64*b^6 + 112*a^2*b^4 - 352*a^3*b^3 + 16*a^4*b^2
- (96*a^7)/b + (48*a^8)/b^2) - (144*a^4*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3
*a^4*b^2)^(1/2))/(128*a*b^6 - 112*a^6*b - 96*a^7 - 64*b^7 + 112*a^2*b^5 - 3
52*a^3*b^4 + 16*a^4*b^3 + 320*a^5*b^2 + (48*a^8)/b) + (48*a^5*tan(x/2)*(b^6
 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b^7 - 96*a^7*b + 48*a^8 - 64*
b^8 + 112*a^2*b^6 - 352*a^3*b^5 + 16*a^4*b^4 + 320*a^5*b^3 - 112*a^6*b^2) -
```

```
 (192*a*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b^3 - 35
2*a^3*b + 16*a^4 - 64*b^4 + 112*a^2*b^2 + (320*a^5)/b - (112*a^6)/b^2 - (96
*a^7)/b^3 + (48*a^8)/b^4))*(-(a + b)^3*(a - b)^3)^(1/2))/b^4 + (a*atan(((a*
(2*a^2 - 3*b^2)*((8*tan(x/2)*(4*a*b^8 - 16*a^8*b + 8*a^9 - 4*b^9 + 7*a^2*b^
7 + 11*a^3*b^6 - 39*a^4*b^5 - 3*a^5*b^4 + 48*a^6*b^3 - 16*a^7*b^2))/b^6 - (
a*((8*(2*a*b^12 - 4*b^13 + 10*a^2*b^11 - 6*a^3*b^10 - 6*a^4*b^9 + 4*a^5*b^8
))/b^9 - (a*tan(x/2)*(2*a^2 - 3*b^2)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*4i
)/b^10)*(2*a^2 - 3*b^2)*1i)/(2*b^4)))/(2*b^4) + (a*(2*a^2 - 3*b^2)*((8*tan(
x/2)*(4*a*b^8 - 16*a^8*b + 8*a^9 - 4*b^9 + 7*a^2*b^7 + 11*a^3*b^6 - 39*a^4*
b^5 - 3*a^5*b^4 + 48*a^6*b^3 - 16*a^7*b^2))/b^6 + (a*((8*(2*a*b^12 - 4*b^13
 + 10*a^2*b^11 - 6*a^3*b^10 - 6*a^4*b^9 + 4*a^5*b^8))/b^9 + (a*tan(x/2)*(2*
a^2 - 3*b^2)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*4i)/b^10)*(2*a^2 - 3*b^2)*
1i)/(2*b^4)))/(2*b^4))/((16*(6*a^10*b - 6*a*b^10 - 4*a^11 + 15*a^2*b^9 + 10
*a^3*b^8 - 49*a^4*b^7 + 8*a^5*b^6 + 59*a^6*b^5 - 26*a^7*b^4 - 31*a^8*b^3 +
18*a^9*b^2))/b^9 + (a*(2*a^2 - 3*b^2)*((8*tan(x/2)*(4*a*b^8 - 16*a^8*b + 8*
a^9 - 4*b^9 + 7*a^2*b^7 + 11*a^3*b^6 - 39*a^4*b^5 - 3*a^5*b^4 + 48*a^6*b^3
- 16*a^7*b^2))/b^6 - (a*((8*(2*a*b^12 - 4*b^13 + 10*a^2*b^11 - 6*a^3*b^10 -
 6*a^4*b^9 + 4*a^5*b^8))/b^9 - (a*tan(x/2)*(2*a^2 - 3*b^2)*(8*a*b^10 - 16*a
^2*b^9 + 8*a^3*b^8)*4i)/b^10)*(2*a^2 - 3*b^2)*1i)/(2*b^4))*1i)/(2*b^4) - (a
*(2*a^2 - 3*b^2)*((8*tan(x/2)*(4*a*b^8 - 16*a^8*b + 8*a^9 - 4*b^9 + 7*a^2*b
^7 + 11*a^3*b^6 - 39*a^4*b^5 - 3*a^5*b^4 + 48*a^6*b^3 - 16*a^7*b^2))/b^6 +
(a*((8*(2*a*b^12 - 4*b^13 + 10*a^2*b^11 - 6*a^3*b^10 - 6*a^4*b^9 + 4*a^5*b^
8))/b^9 + (a*tan(x/2)*(2*a^2 - 3*b^2)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*4
i)/b^10)*(2*a^2 - 3*b^2)*1i)/(2*b^4))*1i)/(2*b^4)))*(2*a^2 - 3*b^2))/b^4
```

3.25 $\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = -\frac{a\cos(x)}{b^2}+\frac{\cos^2(x)}{2b}+\frac{(a^2-b^2)\log(a+b\cos(x))}{b^3}$$

```
[Out] -a*cos(x)/b^2+1/2*cos(x)^2/b+(a^2-b^2)*ln(a+b*cos(x))/b^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2747, 711}

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \frac{(a^2-b^2)\log(a+b\cos(x))}{b^3}-\frac{a\cos(x)}{b^2}+\frac{\cos^2(x)}{2b}$$

```
[In] Int[Sin[x]^3/(a + b*Cos[x]),x]
```

```
[Out] -((a*Cos[x])/b^2) + Cos[x]^2/(2*b) + ((a^2 - b^2)*Log[a + b*Cos[x]])/b^3
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
```

```
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{a+x}\,dx, x, b\cos(x)\right)}{b^3} \\
&= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2+b^2}{a+x}\right)\,dx, x, b\cos(x)\right)}{b^3} \\
&= -\frac{a\cos(x)}{b^2}+\frac{\cos^2(x)}{2b}+\frac{(a^2-b^2)\log(a+b\cos(x))}{b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = -\frac{a\cos(x)}{b^2}+\frac{\cos(2x)}{4b}+\frac{(a^2-b^2)\log(a+b\cos(x))}{b^3}$$

```
[In] Integrate[Sin[x]^3/(a + b*Cos[x]),x]
[Out] -((a*Cos[x])/b^2) + Cos[2*x]/(4*b) + ((a^2 - b^2)*Log[a + b*Cos[x]])/b^3
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{-\frac{b\left(\cos^2(x)\right)}{2}+\cos(x)a}{b^2}+\frac{(a^2-b^2)\ln(a+\cos(x)b)}{b^3}$
default	$-\frac{-\frac{b\left(\cos^2(x)\right)}{2}+\cos(x)a}{b^2}+\frac{(a^2-b^2)\ln(a+\cos(x)b)}{b^3}$
parallelrisch	$\frac{(a^2-b^2)\ln\left(\frac{a+\cos(x)b}{\cos(x)+1}\right)+(-a^2+b^2)\ln\left(\frac{1}{\cos(x)+1}\right)-b\left(\cos(x)a-\frac{b\cos(2x)}{4}+a+\frac{b}{4}\right)}{b^3}$
norman	$\frac{\frac{2a\left(\tan^4\left(\frac{x}{2}\right)\right)}{b^2}-\frac{2a-2b}{3b^2}+\frac{(4a+2b)\left(\tan^6\left(\frac{x}{2}\right)\right)}{3b^2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}+\frac{(a-b)(a+b)\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+a+b\right)}{b^3}-\frac{(a-b)(a+b)\ln(1+\tan^2}{b^3}$
risch	$-\frac{ia^2x}{b^3}+\frac{ix}{b}+\frac{e^{2ix}}{8b}-\frac{a\,e^{ix}}{2b^2}-\frac{a\,e^{-ix}}{2b^2}+\frac{e^{-2ix}}{8b}+\frac{\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}+1\right)a^2}{b^3}-\frac{\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}+1\right)}{b}$

```
[In] int(sin(x)^3/(a+cos(x)*b),x,method=_RETURNVERBOSE)
[Out] -1/b^2*(-1/2*b*cos(x)^2+cos(x)*a)+(a^2-b^2)*ln(a+cos(x)*b)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \frac{b^2\cos(x)^2 - 2\,ab\cos(x) + 2\,(a^2-b^2)\log(-b\cos(x)-a)}{2\,b^3}$$

```
[In] integrate(sin(x)^3/(a+b*cos(x)),x, algorithm="fricas")
[Out] 1/2*(b^2*cos(x)^2 - 2*a*b*cos(x) + 2*(a^2 - b^2)*log(-b*cos(x) - a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. 2(34) = 68.

Time = 173.47 (sec) , antiderivative size = 1421, normalized size of antiderivative = 35.52

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(x)**3/(a+b*cos(x)),x)
[Out] Piecewise((zoo*(-log(tan(x/2) - 1)*tan(x/2)**4/(tan(x/2)**4 + 2*tan(x/2)**2
 + 1) - 2*log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) -
 log(tan(x/2) - 1)/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - log(tan(x/2) + 1)*ta
n(x/2)**4/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - 2*log(tan(x/2) + 1)*tan(x/2)*
*2/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - log(tan(x/2) + 1)/(tan(x/2)**4 + 2*t
an(x/2)**2 + 1) + log(tan(x/2)**2 + 1)*tan(x/2)**4/(tan(x/2)**4 + 2*tan(x/2
)**2 + 1) + 2*log(tan(x/2)**2 + 1)*tan(x/2)**2/(tan(x/2)**4 + 2*tan(x/2)**2
 + 1) + log(tan(x/2)**2 + 1)/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - 2*tan(x/2)
**2/(tan(x/2)**4 + 2*tan(x/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (-4*tan(x/2)*
*2/(b*tan(x/2)**4 + 2*b*tan(x/2)**2 + b) - 2/(b*tan(x/2)**4 + 2*b*tan(x/2)*
*2 + b), Eq(a, b)), ((-sin(x)**2*cos(x) - 2*cos(x)**3/3)/a, Eq(b, 0)), (a**
2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**4/(b**3*tan(x/2)*
*4 + 2*b**3*tan(x/2)**2 + b**3) + 2*a**2*log(-sqrt(-a/(a - b) - b/(a - b))
+ tan(x/2))*tan(x/2)**2/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) + a*
*2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**3*tan(x/2)**4 + 2*b**3
*tan(x/2)**2 + b**3) + a**2*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*ta
n(x/2)**4/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) + 2*a**2*log(sqrt(
-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**2/(b**3*tan(x/2)**4 + 2*b**3*
tan(x/2)**2 + b**3) + a**2*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b*
*3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - a**2*log(tan(x/2)**2 + 1)*tan
(x/2)**4/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - 2*a**2*log(tan(x/
2)**2 + 1)*tan(x/2)**2/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - a**
```

```
2*log(tan(x/2)**2 + 1)/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - 2*a
*b*tan(x/2)**2/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - 2*a*b/(b**3
*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - b**2*log(-sqrt(-a/(a - b) - b/(
a - b)) + tan(x/2))*tan(x/2)**4/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b*
*3) - 2*b**2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**2/(b**
3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - b**2*log(-sqrt(-a/(a - b) - b/
(a - b)) + tan(x/2))/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - b**2*
log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**4/(b**3*tan(x/2)**4
+ 2*b**3*tan(x/2)**2 + b**3) - 2*b**2*log(sqrt(-a/(a - b) - b/(a - b)) + ta
n(x/2))*tan(x/2)**2/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - b**2*l
og(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**3*tan(x/2)**4 + 2*b**3*tan(
x/2)**2 + b**3) + b**2*log(tan(x/2)**2 + 1)*tan(x/2)**4/(b**3*tan(x/2)**4 +
 2*b**3*tan(x/2)**2 + b**3) + 2*b**2*log(tan(x/2)**2 + 1)*tan(x/2)**2/(b**3
*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) + b**2*log(tan(x/2)**2 + 1)/(b**3
*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - 2*b**2*tan(x/2)**2/(b**3*tan(x/
2)**4 + 2*b**3*tan(x/2)**2 + b**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \frac{b\cos{(x)}^2-2\,a\cos{(x)}}{2\,b^2}+\frac{(a^2-b^2)\log{(b\cos{(x)}+a)}}{b^3}$$

```
[In] integrate(sin(x)^3/(a+b*cos(x)),x, algorithm="maxima")
[Out] 1/2*(b*cos(x)^2 - 2*a*cos(x))/b^2 + (a^2 - b^2)*log(b*cos(x) + a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \frac{b\cos{(x)}^2-2\,a\cos{(x)}}{2\,b^2}+\frac{(a^2-b^2)\log{(|b\cos{(x)}+a|)}}{b^3}$$

```
[In] integrate(sin(x)^3/(a+b*cos(x)),x, algorithm="giac")
[Out] 1/2*(b*cos(x)^2 - 2*a*cos(x))/b^2 + (a^2 - b^2)*log(abs(b*cos(x) + a))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \frac{\cos{(x)}^2}{2\,b}+\frac{\ln{(a+b\,\cos{(x)})}\,\left(a^2-b^2\right)}{b^3}-\frac{a\,\cos{(x)}}{b^2}$$

```
[In] int(sin(x)^3/(a + b*cos(x)),x)
[Out] cos(x)^2/(2*b) + (log(a + b*cos(x))*(a^2 - b^2))/b^3 - (a*cos(x))/b^2
```

3.26 $\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \frac{ax}{b^2} - \frac{2\sqrt{a-b}\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2} - \frac{\sin(x)}{b}$$

```
[Out] a*x/b^2-sin(x)/b-2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))*(a-b)^(1/2)*(
a+b)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2774, 2814, 2738, 211}

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = -\frac{2\sqrt{a-b}\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2} + \frac{ax}{b^2} - \frac{\sin(x)}{b}$$

```
[In] Int[Sin[x]^2/(a + b*Cos[x]),x]
```

```
[Out] (a*x)/b^2 - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a
 + b]])/b^2 - Sin[x]/b
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(x)}{b} - \frac{\int \frac{-b-a\cos(x)}{a+b\cos(x)}\,dx}{b} \\
&= \frac{ax}{b^2} - \frac{\sin(x)}{b} + \left(1-\frac{a^2}{b^2}\right)\int \frac{1}{a+b\cos(x)}\,dx \\
&= \frac{ax}{b^2} - \frac{\sin(x)}{b} + \left(2\left(1-\frac{a^2}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) \\
&= \frac{ax}{b^2} - \frac{2\sqrt{a-b}\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2} - \frac{\sin(x)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \frac{ax - 2\sqrt{-a^2+b^2}\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right) - b\sin(x)}{b^2}$$

```
[In] Integrate[Sin[x]^2/(a + b*Cos[x]),x]
[Out] (a*x - 2*Sqrt[-a^2 + b^2]*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]] - b*
Sin[x])/b^2
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{-\frac{2b\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}+2a\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^2}-\frac{2(a+b)(a-b)\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}$	78
risch	$\frac{ax}{b^2}+\frac{i e^{ix}}{2b}-\frac{i e^{-ix}}{2b}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{ix}-\frac{i\sqrt{-a^2+b^2}-a}{b}\right)}{b^2}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{ix}+\frac{i\sqrt{-a^2+b^2}+a}{b}\right)}{b^2}$	118

```
[In] int(sin(x)^2/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(-b*tan(1/2*x)/(1+tan(1/2*x)^2)+a*arctan(tan(1/2*x)))-2*(a+b)*(a-b)/b
^2/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.61

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \left[\frac{2\,ax-2\,b\sin\left(x\right)+\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+\left(2\,a^2-b^2\right)\cos(x)^2+2\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)}{2\,b^2},\ \frac{ax-b\sin\left(x\right)\right.$$

```
[In] integrate(sin(x)^2/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x - 2*b*sin(x) + sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b^
2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*
cos(x)^2 + 2*a*b*cos(x) + a^2)))/b^2, (a*x - b*sin(x) - sqrt(a^2 - b^2)*arc
tan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. 2(49) = 98.

Time = 60.78 (sec) , antiderivative size = 991, normalized size of antiderivative = 16.80

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(x)**2/(a+b*cos(x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**2 + 1) - log(tan(
x/2) - 1)/(tan(x/2)**2 + 1) + log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)**2 +
1) + log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - 2*tan(x/2)/(tan(x/2)**2 + 1)), E
q(a, 0) & Eq(b, 0)), (x*tan(x/2)**2/(b*tan(x/2)**2 + b) + x/(b*tan(x/2)**2
+ b) - 2*tan(x/2)/(b*tan(x/2)**2 + b), Eq(a, b)), (-x*tan(x/2)**2/(b*tan(x/
2)**2 + b) - x/(b*tan(x/2)**2 + b) - 2*tan(x/2)/(b*tan(x/2)**2 + b), Eq(a,
-b)), ((x*sin(x)**2/2 + x*cos(x)**2/2 - sin(x)*cos(x)/2)/a, Eq(b, 0)), (a*x
*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2/(b**2*sqrt(-a/(a - b) - b/(a - b)
)*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))) + a*x*sqrt(-a/(a - b) -
b/(a - b))/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a
 - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x
/2)**2/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b
) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**2*sqr
t(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b)))
+ a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**2/(b**2*sqrt(-a/
(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))) + a*l
og(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**2*sqrt(-a/(a - b) - b/(a -
b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*b*sqrt(-a/(a - b)
- b/(a - b))*tan(x/2)/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2
*sqrt(-a/(a - b) - b/(a - b))) - b*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(
x/2))*tan(x/2)**2/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqr
t(-a/(a - b) - b/(a - b))) - b*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2)
)/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b
/(a - b))) + b*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**2/(b*
*2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a -
 b))) + b*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**2*sqrt(-a/(a - b
) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sin(x)^2/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \frac{ax}{b^2} + \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,x\right)-b\tan\left(\frac{1}{2}\,x\right)}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2} - \frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)b}$$

```
[In] integrate(sin(x)^2/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] a*x/b^2 + 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
x) - b*tan(1/2*x))/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2 - 2*tan(1/2*x)/((t
an(1/2*x)^2 + 1)*b)
```

Mupad [B] (verification not implemented)

Time = 13.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.25

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)\sqrt{b^2-a^2}}{a\,\cos\left(\frac{x}{2}\right)+b\,\cos\left(\frac{x}{2}\right)}\right)\,\sqrt{b^2-a^2}}{b^2} - \frac{\sin\left(x\right)}{b} + \frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{b^2}$$

```
[In] int(sin(x)^2/(a + b*cos(x)),x)
```

```
[Out] (2*atanh((sin(x/2)*(b^2 - a^2)^(1/2))/(a*cos(x/2) + b*cos(x/2)))*(b^2 - a^2
)^(1/2))/b^2 - sin(x)/b + (2*a*atan(sin(x/2)/cos(x/2)))/b^2
```

3.27 $\int \frac{\sin(x)}{a+b\cos(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log(a+b\cos(x))}{b}$$

```
[Out] -ln(a+b*cos(x))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2747, 31}

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log(a+b\cos(x))}{b}$$

```
[In] Int[Sin[x]/(a + b*Cos[x]),x]
```

```
[Out] -(Log[a + b*Cos[x]]/b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cos(x)\right)}{b}$$
$$= -\frac{\log(a+b\cos(x))}{b}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log(a+b\cos(x))}{b}$$

```
[In] Integrate[Sin[x]/(a + b*Cos[x]),x]
[Out] -(Log[a + b*Cos[x]]/b)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+\cos(x)b)}{b}$	13
default	$-\frac{\ln(a+\cos(x)b)}{b}$	13
parallelrisch	$\frac{\ln\left(\frac{1}{\cos(x)+1}\right)-\ln\left(\frac{a+\cos(x)b}{\cos(x)+1}\right)}{b}$	29
risch	$\frac{ix}{b}-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2a\,\mathrm{e}^{ix}}{b}+1\right)}{b}$	33
norman	$\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{b}-\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+a+b\right)}{b}$	41

```
[In] int(sin(x)/(a+cos(x)*b),x,method=_RETURNVERBOSE)
[Out] -ln(a+cos(x)*b)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log\left(-b\cos\left(x\right)-a\right)}{b}$$

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] -log(-b*cos(x) - a)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = \begin{cases} -\frac{\log\left(\frac{a}{b}+\cos\left(x\right)\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cos\left(x\right)}{a} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(x)/(a+b*cos(x)),x)
```

```
[Out] Piecewise((-log(a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log\left(b\cos\left(x\right)+a\right)}{b}$$

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] -log(b*cos(x) + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log\left(|b\cos\left(x\right)+a|\right)}{b}$$

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="giac")
[Out] -log(abs(b*cos(x) + a))/b
```

Mupad [B] (verification not implemented)

Time = 13.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\ln\left(a+b\,\cos\left(x\right)\right)}{b}$$

```
[In] int(sin(x)/(a + b*cos(x)),x)
[Out] -log(a + b*cos(x))/b
```

3.28 $\int \frac{1}{a+b\cos(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int \frac{1}{a+b\cos(x)}\,dx=\frac{2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] 2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2738, 211}

$$\int \frac{1}{a+b\cos(x)}\,dx=\frac{2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

```
[In] Int[(a + b*Cos[x])^(-1),x]
```

```
[Out] (2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \frac{2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\cos(x)}\,dx = -\frac{2\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

```
[In] Integrate[(a + b*Cos[x])^(-1),x]
[Out] (-2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}$	36
risch	$-\frac{\ln\left(\mathrm{e}^{ix}+\frac{i a^2-i b^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{\ln\left(\mathrm{e}^{ix}-\frac{i a^2-i b^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	125

```
[In] int(1/(a+cos(x)*b),x,method=_RETURNVERBOSE)
[Out] 2/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.26

$$\int \frac{1}{a+b\cos(x)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+(2\,a^2-b^2)\cos(x)^2+2\,\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)}{2\,(a^2-b^2)}, \frac{\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\sin(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

```
[In] integrate(1/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos(x)^2 + 2*sqrt(
-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*cos(
x) + a^2))/(a^2 - b^2), arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x)))/sq
rt(a^2 - b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(34) = 68$.

Time = 1.71 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.43

$$\int \frac{1}{a+b\cos(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\log\left(\tan\left(\frac{x}{2}\right)-1\right)+\log\left(\tan\left(\frac{x}{2}\right)+1\right)\right) & \text{for } a=0\wedge b=0 \\ \frac{\tan\left(\frac{x}{2}\right)}{b} & \text{for } a=b \\ \frac{1}{b\tan\left(\frac{x}{2}\right)} & \text{for } a=-b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*cos(x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x/2) - 1) + log(tan(x/2) + 1)), Eq(a, 0) & Eq(b, 0
)), (tan(x/2)/b, Eq(a, b)), (1/(b*tan(x/2)), Eq(a, -b)), (log(-sqrt(-a/(a -
 b) - b/(a - b)) + tan(x/2))/(a*sqrt(-a/(a - b) - b/(a - b)) - b*sqrt(-a/(a
 - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(a*sqrt(
-a/(a - b) - b/(a - b)) - b*sqrt(-a/(a - b) - b/(a - b))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \frac{1}{a + b\cos(x)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor \operatorname{sgn}(-2a + 2b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right) - b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2 - b^2}}$$

```
[In] integrate(1/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x))/sqrt(a^2 - b^2)))/sqrt(a^2 - b^2)
```

Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1}{a + b\cos(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)(2a-2b)}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2 - b^2}}$$

```
[In] int(1/(a + b*cos(x)),x)
```

```
[Out] (2*atan((tan(x/2)*(2*a - 2*b))/(2*(a^2 - b^2)^(1/2))))/(a^2 - b^2)^(1/2)
```

3.29 $\int \frac{\csc(x)}{a+b\cos(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 53

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx=\frac{\log(1-\cos(x))}{2(a+b)}-\frac{\log(1+\cos(x))}{2(a-b)}+\frac{b\log(a+b\cos(x))}{a^2-b^2}$$

```
[Out] 1/2*ln(1-cos(x))/(a+b)-1/2*ln(1+cos(x))/(a-b)+b*ln(a+b*cos(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2747, 720, 31, 647}

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx=\frac{b\log(a+b\cos(x))}{a^2-b^2}+\frac{\log(1-\cos(x))}{2(a+b)}-\frac{\log(\cos(x)+1)}{2(a-b)}$$

```
[In] Int[Csc[x]/(a + b*Cos[x]),x]
```

```
[Out] Log[1 - Cos[x]]/(2*(a + b)) - Log[1 + Cos[x]]/(2*(a - b)) + (b*Log[a + b*Co
s[x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
```

```
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(b\text{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cos(x)\right)\right)\\
&= \frac{b\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cos(x)\right)}{a^2-b^2} + \frac{b\text{Subst}\left(\int \frac{-a+x}{b^2-x^2}\,dx, x, b\cos(x)\right)}{a^2-b^2}\\
&= \frac{b\log(a+b\cos(x))}{a^2-b^2} + \frac{\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\cos(x)\right)}{2(a-b)} - \frac{\text{Subst}\left(\int \frac{1}{b-x}\,dx, x, b\cos(x)\right)}{2(a+b)}\\
&= \frac{\log(1-\cos(x))}{2(a+b)} - \frac{\log(1+\cos(x))}{2(a-b)} + \frac{b\log(a+b\cos(x))}{a^2-b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{(a-b)\log(1-\cos(x)) - (a+b)\log(1+\cos(x)) + 2b\log(a+b\cos(x))}{2(a-b)(a+b)}$$

```
[In] Integrate[Csc[x]/(a + b*Cos[x]),x]
[Out] ((a - b)*Log[1 - Cos[x]] - (a + b)*Log[1 + Cos[x]] + 2*b*Log[a + b*Cos[x]])
/(2*(a - b)*(a + b))
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

method	result	size
norman	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a+b}+\frac{b\ln\left(a\left(\tan^{2}\left(\frac{x}{2}\right)\right)-b\left(\tan^{2}\left(\frac{x}{2}\right)\right)+a+b\right)}{a^{2}-b^{2}}$	47
parallelrisch	$\frac{b\ln\left(\frac{2\cos(x)b+2a}{\cos(x)+1}\right)+\ln(\csc(x)-\cot(x))(a-b)}{a^{2}-b^{2}}$	47
default	$-\frac{\ln(\cos(x)+1)}{2a-2b}+\frac{\ln(\cos(x)-1)}{2a+2b}+\frac{b\ln(a+\cos(x)b)}{(a-b)(a+b)}$	54
risch	$-\frac{ix}{a+b}+\frac{ix}{a-b}-\frac{2ixb}{a^{2}-b^{2}}+\frac{\ln(e^{ix}-1)}{a+b}-\frac{\ln(e^{ix}+1)}{a-b}+\frac{b\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}+1\right)}{a^{2}-b^{2}}$	101

```
[In] int(csc(x)/(a+cos(x)*b),x,method=_RETURNVERBOSE)
[Out] 1/(a+b)*ln(tan(1/2*x))+1/(a^2-b^2)*b*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+a+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{2\,b\log\left(-b\cos\left(x\right)-a\right)-\left(a+b\right)\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+\left(a-b\right)\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)}{2\,\left(a^{2}-b^{2}\right)}$$

```
[In] integrate(csc(x)/(a+b*cos(x)),x, algorithm="fricas")
[Out] 1/2*(2*b*log(-b*cos(x) - a) - (a + b)*log(1/2*cos(x) + 1/2) + (a - b)*log(-
1/2*cos(x) + 1/2))/(a^2 - b^2)
```

Sympy [F]

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \int \frac{\csc\left(x\right)}{a+b\cos\left(x\right)}\,dx$$

```
[In] integrate(csc(x)/(a+b*cos(x)),x)
[Out] Integral(csc(x)/(a + b*cos(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{b\log\left(b\cos\left(x\right)+a\right)}{a^2-b^2} - \frac{\log\left(\cos\left(x\right)+1\right)}{2\left(a-b\right)} + \frac{\log\left(\cos\left(x\right)-1\right)}{2\left(a+b\right)}$$

```
[In] integrate(csc(x)/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] b*log(b*cos(x) + a)/(a^2 - b^2) - 1/2*log(cos(x) + 1)/(a - b) + 1/2*log(cos
(x) - 1)/(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{b^2\log\left(\left|b\cos\left(x\right)+a\right|\right)}{a^2b-b^3} - \frac{\log\left(\cos\left(x\right)+1\right)}{2\left(a-b\right)} + \frac{\log\left(-\cos\left(x\right)+1\right)}{2\left(a+b\right)}$$

```
[In] integrate(csc(x)/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(b*cos(x) + a))/(a^2*b - b^3) - 1/2*log(cos(x) + 1)/(a - b) + 1/
2*log(-cos(x) + 1)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{\ln\left(\cos\left(x\right)-1\right)}{2\,\left(a+b\right)} - \frac{\ln\left(\cos\left(x\right)+1\right)}{2\,\left(a-b\right)} + \frac{b\,\ln\left(a+b\,\cos\left(x\right)\right)}{a^2-b^2}$$

```
[In] int(1/(sin(x)*(a + b*cos(x))),x)
```

```
[Out] log(cos(x) - 1)/(2*(a + b)) - log(cos(x) + 1)/(2*(a - b)) + (b*log(a + b*co
s(x)))/(a^2 - b^2)
```

3.30 $\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx=-\frac{2b^2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cos(x))\csc(x)}{a^2-b^2}$$

```
[Out] -2*b^2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)+(
b-a*cos(x))*csc(x)/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2775, 12, 2738, 211}

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx=\frac{\csc(x)(b-a\cos(x))}{a^2-b^2}-\frac{2b^2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

```
[In] Int[Csc[x]^2/(a + b*Cos[x]),x]
```

```
[Out] (-2*b^2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)^
(3/2)) + ((b - a*Cos[x])*Csc[x])/(a^2 - b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(b-a\cos(x))\csc(x)}{a^2-b^2}+\frac{\int \frac{b^2}{a+b\cos(x)}\,dx}{-a^2+b^2}\\
&= \frac{(b-a\cos(x))\csc(x)}{a^2-b^2}-\frac{b^2\int \frac{1}{a+b\cos(x)}\,dx}{a^2-b^2}\\
&= \frac{(b-a\cos(x))\csc(x)}{a^2-b^2}-\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= -\frac{2b^2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cos(x))\csc(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx=-\frac{2b^2\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}+\frac{(b-a\cos(x))\csc(x)}{a^2-b^2}$$

```
[In] Integrate[Csc[x]^2/(a + b*Cos[x]),x]
[Out] (-2*b^2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) +
((b - a*Cos[x])*Csc[x])/(a^2 - b^2)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{\tan\left(\frac{x}{2}\right)}{2a-2b}-\frac{1}{2(a+b)\tan\left(\frac{x}{2}\right)}-\frac{2b^{2}\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}$	78
risch	$-\frac{2i\left(-\mathrm{e}^{ix}b+a\right)}{\left(\mathrm{e}^{2ix}-1\right)\left(a^{2}-b^{2}\right)}+\frac{b^{2}\ln\left(\mathrm{e}^{ix}+\frac{i a^{2}-i b^{2}+a\sqrt{-a^{2}+b^{2}}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}-\frac{b^{2}\ln\left(\mathrm{e}^{ix}+\frac{-i a^{2}+i b^{2}+a\sqrt{-a^{2}+b^{2}}}{\sqrt{-a^{2}+b^{2}}\,b}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}$	186

```
[In] int(csc(x)^2/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tan(1/2*x)/(a-b)-1/2/(a+b)/tan(1/2*x)-2/(a-b)/(a+b)*b^2/((a-b)*(a+b))^(
1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.43

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx$$
$$=\left[\frac{\sqrt{-a^2+b^2}b^2\log\left(\frac{2\,ab\cos(x)+(2\,a^2-b^2)\cos(x)^2+2\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)\sin(x)+2\,a^2b-2\,b^3-2\,(a^3-}{2\,(a^4-2\,a^2b^2+b^4)\sin(x)}\right.$$
$$\left.-\frac{\sqrt{a^2-b^2}b^2\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\sin(x)}\right)\sin(x)-a^2b+b^3+(a^3-ab^2)\cos(x)}{(a^4-2\,a^2b^2+b^4)\sin(x)}\right]$$

```
[In] integrate(csc(x)^2/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*b^2*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos(x)^2 + 2*s
qrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*
cos(x) + a^2))*sin(x) + 2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(x))/((a^4 - 2
*a^2*b^2 + b^4)*sin(x)), -(sqrt(a^2 - b^2)*b^2*arctan(-(a*cos(x) + b)/(sqrt
(a^2 - b^2)*sin(x)))*sin(x) - a^2*b + b^3 + (a^3 - a*b^2)*cos(x))/((a^4 - 2
*a^2*b^2 + b^4)*sin(x))]
```

Sympy [F]

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = \int \frac{\csc^2(x)}{a+b\cos(x)}\,dx$$

```
[In] integrate(csc(x)**2/(a+b*cos(x)),x)
```

```
[Out] Integral(csc(x)**2/(a + b*cos(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csc(x)^2/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)b^2}{\left(a^2-b^2\right)^{\frac{3}{2}}} + \frac{\tan\left(\frac{1}{2}x\right)}{2(a-b)} - \frac{1}{2(a+b)\tan\left(\frac{1}{2}x\right)}$$

```
[In] integrate(csc(x)^2/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan
(1/2*x))/sqrt(a^2 - b^2)))*b^2/(a^2 - b^2)^(3/2) + 1/2*tan(1/2*x)/(a - b) -
 1/2/((a + b)*tan(1/2*x))
```

Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{2a-2b} - \frac{2b^2\,\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)\,(a^2-b^2)}{(a+b)^{3/2}\,\sqrt{a-b}}\right)}{(a+b)^{3/2}\,(a-b)^{3/2}} - \frac{a-b}{\tan\left(\frac{x}{2}\right)\,(a+b)\,(2a-2b)}$$

```
[In] int(1/(sin(x)^2*(a + b*cos(x))),x)
```

```
[Out] tan(x/2)/(2*a - 2*b) - (2*b^2*atan((tan(x/2)*(a^2 - b^2))/((a + b)^(3/2)*(a
 - b)^(1/2))))/((a + b)^(3/2)*(a - b)^(3/2)) - (a - b)/(tan(x/2)*(a + b)*(2
*a - 2*b))
```

3.31 $\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = \frac{(b-a\cos(x))\csc^2(x)}{2(a^2-b^2)} + \frac{(a+2b)\log(1-\cos(x))}{4(a+b)^2} - \frac{(a-2b)\log(1+\cos(x))}{4(a-b)^2} - \frac{b^3\log(a+b\cos(x))}{(a^2-b^2)^2}$$

```
[Out] 1/2*(b-a*cos(x))*csc(x)^2/(a^2-b^2)+1/4*(a+2*b)*ln(1-cos(x))/(a+b)^2-1/4*(a
-2*b)*ln(1+cos(x))/(a-b)^2-b^3*ln(a+b*cos(x))/(a^2-b^2)^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2747, 755, 815}

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = \frac{\csc^2(x)(b-a\cos(x))}{2(a^2-b^2)} - \frac{b^3\log(a+b\cos(x))}{(a^2-b^2)^2} + \frac{(a+2b)\log(1-\cos(x))}{4(a+b)^2} - \frac{(a-2b)\log(\cos(x)+1)}{4(a-b)^2}$$

```
[In] Int[Csc[x]^3/(a + b*Cos[x]),x]
```

```
[Out] ((b - a*Cos[x])*Csc[x]^2)/(2*(a^2 - b^2)) + ((a + 2*b)*Log[1 - Cos[x]])/(4*
(a + b)^2) - ((a - 2*b)*Log[1 + Cos[x]])/(4*(a - b)^2) - (b^3*Log[a + b*Cos
[x]])/(a^2 - b^2)^2
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(b^3\text{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\cos(x)\right)\right)\\
&= \frac{(b-a\cos(x))\csc^2(x)}{2\left(a^2-b^2\right)} - \frac{b\text{Subst}\left(\int \frac{a^2-2b^2+ax}{(a+x)(b^2-x^2)}\,dx, x, b\cos(x)\right)}{2\left(a^2-b^2\right)}\\
&= \frac{(b-a\cos(x))\csc^2(x)}{2\left(a^2-b^2\right)}\\
&\quad - \frac{b\text{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right)dx, x, b\cos(x)\right)}{2\left(a^2-b^2\right)}\\
&= \frac{(b-a\cos(x))\csc^2(x)}{2\left(a^2-b^2\right)} + \frac{(a+2b)\log(1-\cos(x))}{4(a+b)^2}\\
&\quad - \frac{(a-2b)\log(1+\cos(x))}{4(a-b)^2} - \frac{b^3\log(a+b\cos(x))}{\left(a^2-b^2\right)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = \frac{1}{8}\left(-\frac{\csc^2\left(\frac{x}{2}\right)}{a+b}-\frac{4(a-2b)\log\left(\cos\left(\frac{x}{2}\right)\right)}{(a-b)^2}-\frac{8b^3\log(a+b\cos(x))}{(a^2-b^2)^2}+\frac{4(a+2b)\log\left(\sin\left(\frac{x}{2}\right)\right)}{(a+b)^2}+\frac{\sec^2\left(\frac{x}{2}\right)}{a-b}\right)$$

```
[In] Integrate[Csc[x]^3/(a + b*Cos[x]),x]
```

```
[Out] (-(Csc[x/2]^2/(a + b)) - (4*(a - 2*b)*Log[Cos[x/2]])/(a - b)^2 - (8*b^3*Log
[a + b*Cos[x]])/(a^2 - b^2)^2 + (4*(a + 2*b)*Log[Sin[x/2]])/(a + b)^2 + Sec
[x/2]^2/(a - b))/8
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{-4b^3\ln\left(\frac{2\cos(x)b+2a}{\cos(x)+1}\right)-2(a-b)\left(-(a+2b)(a-b)\ln(\csc(x)-\cot(x))+\left(\cot(x)\csc(x)a-\frac{b\left(\cot^2(x)\right)}{2}-\frac{b\left(\csc^2(x)\right)}{2}\right)(a+b)\right)}{4(a-b)^2(a+b)^2}$
default	$\frac{1}{(4a+4b)(\cos(x)-1)}+\frac{(a+2b)\ln(\cos(x)-1)}{4(a+b)^2}-\frac{b^3\ln(a+\cos(x)b)}{(a+b)^2(a-b)^2}+\frac{1}{(4a-4b)(\cos(x)+1)}+\frac{(-a+2b)\ln(\cos(x)+1)}{4(a-b)^2}$
norman	$\frac{-\frac{1}{8(a+b)}+\frac{\tan^4\left(\frac{x}{2}\right)}{8a-8b}}{\tan\left(\frac{x}{2}\right)^2}-\frac{b^3\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+a+b\right)}{a^4-2a^2b^2+b^4}+\frac{(a+2b)\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^2+4ab+2b^2}$
risch	$\frac{ixa}{2a^2-4ab+2b^2}-\frac{ixb}{a^2-2ab+b^2}-\frac{ixa}{2(a^2+2ab+b^2)}-\frac{ixb}{a^2+2ab+b^2}+\frac{2ix\,b^3}{a^4-2a^2b^2+b^4}-\frac{a\,e^{3ix}-2\,e^{2ix}b+a\,e^{ix}}{(e^{2ix}-1)^2(-a^2+b^2)}-\frac{\ln\left(e^{ix}+1\right)a}{2(a^2-2ab+b}$

```
[In] int(csc(x)^3/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-4*b^3*ln((2*cos(x)*b+2*a)/(cos(x)+1))-2*(a-b)*(-(a+2*b)*(a-b)*ln(csc(
x)-cot(x))+(cot(x)*csc(x)*a-1/2*b*cot(x)^2-1/2*b*csc(x)^2)*(a+b)))/(a-b)^2/
(a+b)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(87) = 174.

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.97

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = \frac{2\,a^2b-2\,b^3-2\,(a^3-ab^2)\cos(x)+4\,(b^3\cos(x)^2-b^3)\log(-b\cos(x)-a)-(a^3-3\,ab^2-2\,b^3-(a^3-3}{4\,(a^4-2\,a^2b^2+b^4-}$$

```
[In] integrate(csc(x)^3/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(x) + 4*(b^3*cos(x)^2 - b^3)*log(
-b*cos(x) - a) - (a^3 - 3*a*b^2 - 2*b^3 - (a^3 - 3*a*b^2 - 2*b^3)*cos(x)^2)
*log(1/2*cos(x) + 1/2) + (a^3 - 3*a*b^2 + 2*b^3 - (a^3 - 3*a*b^2 + 2*b^3)*c
os(x)^2)*log(-1/2*cos(x) + 1/2))/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2
+ b^4)*cos(x)^2)
```

Sympy [F]

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = \int \frac{\csc^3(x)}{a+b\cos(x)}\,dx$$

```
[In] integrate(csc(x)**3/(a+b*cos(x)),x)
```

```
[Out] Integral(csc(x)**3/(a + b*cos(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.25

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = -\frac{b^3\log(b\cos(x)+a)}{a^4-2\,a^2b^2+b^4} - \frac{(a-2\,b)\log(\cos(x)+1)}{4\,(a^2-2\,ab+b^2)} + \frac{(a+2\,b)\log(\cos(x)-1)}{4\,(a^2+2\,ab+b^2)} + \frac{a\cos(x)-b}{2\,((a^2-b^2)\cos(x)^2-a^2+b^2)}$$

```
[In] integrate(csc(x)^3/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log(b*cos(x) + a)/(a^4 - 2*a^2*b^2 + b^4) - 1/4*(a - 2*b)*log(cos(x) +
 1)/(a^2 - 2*a*b + b^2) + 1/4*(a + 2*b)*log(cos(x) - 1)/(a^2 + 2*a*b + b^2)
 + 1/2*(a*cos(x) - b)/((a^2 - b^2)*cos(x)^2 - a^2 + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.48

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = -\frac{b^4\log(|b\cos(x)+a|)}{a^4b-2\,a^2b^3+b^5} - \frac{(a-2\,b)\log(\cos(x)+1)}{4\,(a^2-2\,ab+b^2)} + \frac{(a+2\,b)\log(-\cos(x)+1)}{4\,(a^2+2\,ab+b^2)} - \frac{a^2b-b^3-(a^3-ab^2)\cos(x)}{2\,(a+b)^2(a-b)^2(\cos(x)+1)(\cos(x)-1)}$$

```
[In] integrate(csc(x)^3/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -b^4*log(abs(b*cos(x) + a))/(a^4*b - 2*a^2*b^3 + b^5) - 1/4*(a - 2*b)*log(c
os(x) + 1)/(a^2 - 2*a*b + b^2) + 1/4*(a + 2*b)*log(-cos(x) + 1)/(a^2 + 2*a*
b + b^2) - 1/2*(a^2*b - b^3 - (a^3 - a*b^2)*cos(x))/((a + b)^2*(a - b)^2*(c
os(x) + 1)*(cos(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.22

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = \ln(\cos(x)-1)\,\left(\frac{b}{4\,(a+b)^2}+\frac{1}{4\,(a+b)}\right)+\frac{\frac{b}{2\,(a^2-b^2)}-\frac{a\,\cos(x)}{2\,(a^2-b^2)}}{\sin(x)^2} - \frac{b^3\,\ln(a+b\,\cos(x))}{a^4-2\,a^2\,b^2+b^4} - \frac{\ln(\cos(x)+1)\,(a-2\,b)}{4\,(a-b)^2}$$

```
[In] int(1/(sin(x)^3*(a + b*cos(x))),x)
```

```
[Out] log(cos(x) - 1)*(b/(4*(a + b)^2) + 1/(4*(a + b))) + (b/(2*(a^2 - b^2)) - (a
*cos(x))/(2*(a^2 - b^2)))/sin(x)^2 - (b^3*log(a + b*cos(x)))/(a^4 + b^4 - 2
*a^2*b^2) - (log(cos(x) + 1)*(a - 2*b))/(4*(a - b)^2)
```

3.32 $\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 110

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \frac{2b^4\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{(3b^3+a(2a^2-5b^2)\cos(x))\csc(x)}{3(a^2-b^2)^2} + \frac{(b-a\cos(x))\csc^3(x)}{3(a^2-b^2)}$$

```
[Out] 2*b^4*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)-1/
3*(3*b^3+a*(2*a^2-5*b^2)*cos(x))*csc(x)/(a^2-b^2)^2+1/3*(b-a*cos(x))*csc(x)
^3/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2775, 2945, 12, 2738, 211}

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \frac{\csc^3(x)(b-a\cos(x))}{3(a^2-b^2)} - \frac{\csc(x)\left(a(2a^2-5b^2)\cos(x)+3b^3\right)}{3(a^2-b^2)^2} + \frac{2b^4\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

```
[In] Int[Csc[x]^4/(a + b*Cos[x]),x]
```

```
[Out] (2*b^4*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(
5/2)) - ((3*b^3 + a*(2*a^2 - 5*b^2)*Cos[x])*Csc[x])/(3*(a^2 - b^2)^2) + ((b
 - a*Cos[x])*Csc[x]^3)/(3*(a^2 - b^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = \frac{(b - a\cos(x))\csc^3(x)}{3(a^2 - b^2)} - \frac{\int \frac{(-2a^2+3b^2-2ab\cos(x))\csc^2(x)}{a+b\cos(x)}\,dx}{3(a^2 - b^2)}$$

$$= -\frac{(3b^3 + a(2a^2 - 5b^2)\cos(x))\csc(x)}{3(a^2 - b^2)^2} + \frac{(b - a\cos(x))\csc^3(x)}{3(a^2 - b^2)} + \frac{\int \frac{3b^4}{a+b\cos(x)}\,dx}{3(a^2 - b^2)^2}$$

$$= -\frac{(3b^3 + a(2a^2 - 5b^2)\cos(x))\csc(x)}{3(a^2-b^2)^2} + \frac{(b - a\cos(x))\csc^3(x)}{3(a^2-b^2)} + \frac{b^4 \int \frac{1}{a+b\cos(x)}\,dx}{(a^2-b^2)^2}$$

$$= -\frac{(3b^3 + a(2a^2 - 5b^2)\cos(x))\csc(x)}{3(a^2-b^2)^2} + \frac{(b - a\cos(x))\csc^3(x)}{3(a^2-b^2)} + \frac{(2b^4)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{(a^2-b^2)^2}$$

$$= \frac{2b^4 \arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{(3b^3 + a(2a^2 - 5b^2)\cos(x))\csc(x)}{3(a^2-b^2)^2} + \frac{(b - a\cos(x))\csc^3(x)}{3(a^2-b^2)}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = -\frac{2b^4 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{((-6a^3+9ab^2)\cos(x) + 6b^3\cos(2x) + (2a^2-5b^2)(2b + a\cos(3x)))\csc^3(x)}{12(a-b)^2(a+b)^2}$$

```
[In] Integrate[Csc[x]^4/(a + b*Cos[x]),x]
```

```
[Out] (-2*b^4*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) +
(((-6*a^3 + 9*a*b^2)*Cos[x] + 6*b^3*Cos[2*x] + (2*a^2 - 5*b^2)*(2*b + a*Cos
[3*x]))*Csc[x]^3)/(12*(a - b)^2*(a + b)^2)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

method	result
default	$\frac{\frac{a\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}-\frac{b\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}+3a\tan\left(\frac{x}{2}\right)-5b\tan\left(\frac{x}{2}\right)}{8(a-b)^2} - \frac{1}{24(a+b)\tan\left(\frac{x}{2}\right)^3} - \frac{3a+5b}{8(a+b)^2\tan\left(\frac{x}{2}\right)} + \frac{2b^4\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2\sqrt{(a-b)(a+b)}}$
risch	$-\frac{2i\left(3b^3e^{5ix}-3ab^2e^{4ix}+4a^2be^{3ix}-10b^3e^{3ix}-6a^3e^{2ix}+12ab^2e^{2ix}+3b^3e^{ix}+2a^3-5ab^2\right)}{3(a^4-2a^2b^2+b^4)(e^{2ix}-1)^3} - \frac{b^4\ln\left(e^{ix}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}(a+b)^2(a-b)^2} +$

```
[In] int(csc(x)^4/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/(a-b)^2*(1/3*a*tan(1/2*x)^3-1/3*b*tan(1/2*x)^3+3*a*tan(1/2*x)-5*b*tan(1
/2*x))-1/24/(a+b)/tan(1/2*x)^3-1/8*(3*a+5*b)/(a+b)^2/tan(1/2*x)+2/(a-b)^2/(
a+b)^2*b^4/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(97) = 194$.

Time = 0.28 (sec) , antiderivative size = 459, normalized size of antiderivative = 4.17

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx$$
$$= \left[\frac{2\,a^4b - 10\,a^2b^3 + 8\,b^5 + 2\,(2\,a^5 - 7\,a^3b^2 + 5\,ab^4)\cos(x)^3 + 3\,(b^4\cos(x)^2 - b^4)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+(2\ldots}{}\right.}{6\,(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6 - (a^6 - \ldots}\right.$$

```
[In] integrate(csc(x)^4/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^4*b - 10*a^2*b^3 + 8*b^5 + 2*(2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos(x)
^3 + 3*(b^4*cos(x)^2 - b^4)*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b
^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2
*cos(x)^2 + 2*a*b*cos(x) + a^2))*sin(x) + 6*(a^2*b^3 - b^5)*cos(x)^2 - 6*(a
^5 - 3*a^3*b^2 + 2*a*b^4)*cos(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - (a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(x)^2)*sin(x)), 1/3*(a^4*b - 5*a^2*b^3
+ 4*b^5 + (2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos(x)^3 - 3*(b^4*cos(x)^2 - b^4)*s
qrt(a^2 - b^2)*arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x)))*sin(x) + 3*
(a^2*b^3 - b^5)*cos(x)^2 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cos(x))/((a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(x)^2)*s
in(x))]
```

Sympy [F]

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \int \frac{\csc^4(x)}{a+b\cos(x)}\,dx$$

```
[In] integrate(csc(x)**4/(a+b*cos(x)),x)
```

```
[Out] Integral(csc(x)**4/(a + b*cos(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csc(x)^4/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(97) = 194.

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.87

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)b^4}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{a^2\tan\left(\frac{1}{2}x\right)^3-2ab\tan\left(\frac{1}{2}x\right)^3+b^2\tan\left(\frac{1}{2}x\right)^3+9a^2\tan\left(\frac{1}{2}x\right)-24ab\tan\left(\frac{1}{2}x\right)+15b^2\tan\left(\frac{1}{2}x\right)}{24(a^3-3a^2b+3ab^2-b^3)} - \frac{9a\tan\left(\frac{1}{2}x\right)^2+15b\tan\left(\frac{1}{2}x\right)^2+a+b}{24(a^2+2ab+b^2)\tan\left(\frac{1}{2}x\right)^3}$$

```
[In] integrate(csc(x)^4/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x))/sqrt(a^2 - b^2)))*b^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) +
 1/24*(a^2*tan(1/2*x)^3 - 2*a*b*tan(1/2*x)^3 + b^2*tan(1/2*x)^3 + 9*a^2*tan
(1/2*x) - 24*a*b*tan(1/2*x) + 15*b^2*tan(1/2*x))/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) - 1/24*(9*a*tan(1/2*x)^2 + 15*b*tan(1/2*x)^2 + a + b)/((a^2 + 2*a*b +
 b^2)*tan(1/2*x)^3)
```

Mupad [B] (verification not implemented)

Time = 13.37 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)\left(\frac{4}{8a-8b}-\frac{8a+8b}{(8a-8b)^2}\right)+\frac{\tan\left(\frac{x}{2}\right)^3}{3\,(8a-8b)} - \frac{\frac{a^2-2ab+b^2}{3(a+b)}-\frac{\tan\left(\frac{x}{2}\right)^2(-3a^3+a^2b+7ab^2-5b^3)}{(a+b)^2}}{\tan\left(\frac{x}{2}\right)^3(8a^2-16ab+8b^2)} + \frac{2b^4\,\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{3/2}}\right)}{(a+b)^{5/2}(a-b)^{5/2}}$$

```
[In] int(1/(sin(x)^4*(a + b*cos(x))),x)
```

```
[Out] tan(x/2)*(4/(8*a - 8*b) - (8*a + 8*b)/(8*a - 8*b)^2) + tan(x/2)^3/(3*(8*a -
 8*b)) - ((a^2 - 2*a*b + b^2)/(3*(a + b)) - (tan(x/2)^2*(7*a*b^2 + a^2*b -
3*a^3 - 5*b^3))/(a + b)^2)/(tan(x/2)^3*(8*a^2 - 16*a*b + 8*b^2)) + (2*b^4*a
tan((tan(x/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/((a
+ b)^(5/2)*(a - b)^(5/2))
```

3.33 $\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx=\frac{10ae^4\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}}-\frac{10ae^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d}-\frac{2ae\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

```
[Out] -2/7*a*e*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/d+2/9*b*(e*sin(d*x+c))^(9/2)/d/e-1
0/21*a*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*sin(d*x+c)
)^(1/2)-10/21*a*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used

$= \{2748, 2715, 2721, 2720\}$

$$\int (a + b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \frac{10ae^4\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21d\sqrt{e\sin(c+dx)}} - \frac{10ae^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d} + \frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

```
[In] Int[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (10*a*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*d*Sqrt[e
*Sin[c + d*x]]) - (10*a*e^3*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(21*d) - (2*
a*e*Cos[c + d*x]*(e*Sin[c + d*x])^(5/2))/(7*d) + (2*b*(e*Sin[c + d*x])^(9/2
))/(9*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(e\sin(c+dx))^{9/2}}{9de} + a\int (e\sin(c+dx))^{7/2}\,dx \\
&= -\frac{2ae\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d} + \frac{2b(e\sin(c+dx))^{9/2}}{9de} + \frac{1}{7}\left(5ae^2\right)\int (e\sin(c+dx))^{3/2}\,dx \\
&= -\frac{10ae^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d} \\
&\quad + \frac{2b(e\sin(c+dx))^{9/2}}{9de} + \frac{1}{21}\left(5ae^4\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx \\
&= -\frac{10ae^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d} \\
&\quad + \frac{2b(e\sin(c+dx))^{9/2}}{9de} + \frac{\left(5ae^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{21\sqrt{e\sin(c+dx)}} \\
&= \frac{10ae^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{10ae^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} \\
&\quad - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d} + \frac{2b(e\sin(c+dx))^{9/2}}{9de}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \frac{e^3\Big(-120a\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+(21b-138a\cos(c+dx)-28b\cos(2(c+dx))+}{252d\sqrt{\sin(c+dx)}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2),x]
[Out] (e^3*(-120*a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (21*b - 138*a*Cos[c + d*
x] - 28*b*Cos[2*(c + d*x)] + 18*a*Cos[3*(c + d*x)] + 7*b*Cos[4*(c + d*x)])*
Sqrt[Sin[c + d*x]])*Sqrt[e*Sin[c + d*x]])/(252*d*Sqrt[Sin[c + d*x]])
```

Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{9}{2}}}{9e}-\frac{e^4a\left(-6\left(\sin^5(dx+c)\right)+5\sqrt{1-\sin(dx+c)}\sqrt{2\sin(dx+c)+2}\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-4\left(\sin^3(dx+c)\right)+10\sin(dx+c)\right)}{21\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$-\frac{a\,e^4\left(-6\left(\sin^5(dx+c)\right)+5\sqrt{1-\sin(dx+c)}\sqrt{2\sin(dx+c)+2}\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-4\left(\sin^3(dx+c)\right)+10\sin(dx+c)\right)}{21\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((a+cos(d*x+c)*b)*(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/9/e*b*(e*sin(d*x+c))^(9/2)-1/21*e^4*a*(-6*sin(d*x+c)^5+5*(1-sin(d*x+c))^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2
),1/2*2^(1/2))-4*sin(d*x+c)^3+10*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(1/2
))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \frac{15\sqrt{2}a\sqrt{-i\,e}e^3\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+15\sqrt{2}a\sqrt{i\,e}e^3\text{weierstra}}{}$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/63*(15*sqrt(2)*a*sqrt(-I*e)*e^3*weierstrassPInverse(4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 15*sqrt(2)*a*sqrt(I*e)*e^3*weierstrassPInverse(4, 0, cos(
d*x + c) - I*sin(d*x + c)) + 2*(7*b*e^3*cos(d*x + c)^4 + 9*a*e^3*cos(d*x +
c)^3 - 14*b*e^3*cos(d*x + c)^2 - 24*a*e^3*cos(d*x + c) + 7*b*e^3)*sqrt(e*si
n(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(7/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \int (e\,\sin(c+d\,x))^{7/2}\,(a+b\,\cos(c+d\,x))\,dx$$

```
[In] int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x)), x)
```

3.34 $\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx=\frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

```
[Out] -2/5*a*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+2/7*b*(e*sin(d*x+c))^(7/2)/d/e-6
/5*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elli
pticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+c)^
(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2748, 2715, 2721, 2719}

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx=\frac{6ae^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

```
[In] Int[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (6*a*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[S
in[c + d*x]]) - (2*a*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d) + (2*b*(e
*Sin[c + d*x])^(7/2))/(7*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2b(e\sin(c+dx))^{7/2}}{7de} + a\int (e\sin(c+dx))^{5/2}\,dx \\
&= -\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{2b(e\sin(c+dx))^{7/2}}{7de} + \frac{1}{5}\left(3ae^2\right)\int \sqrt{e\sin(c+dx)}\,dx \\
&= -\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{2b(e\sin(c+dx))^{7/2}}{7de} \\
&\quad + \frac{\left(3ae^2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{5\sqrt{\sin(c+dx)}} \\
&= \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} \\
&\quad - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{2b(e\sin(c+dx))^{7/2}}{7de}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \frac{2(e\sin(c+dx))^{5/2}\left(-21aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+\sin^{\frac{3}{2}}(c+dx)\left(-7a\cos(c+dx)+5b\sin^2(c+dx)\right)\right)}{35d\sin^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(e*Sin[c + d*x])^(5/2)*(-21*a*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + Sin[
c + d*x]^(3/2)*(-7*a*Cos[c + d*x] + 5*b*Sin[c + d*x]^2)))/(35*d*Sin[c + d*x
]^(5/2))
```

Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.71

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{7}{2}}}{7e}-\frac{e^3a\left(6\sqrt{1-\sin(dx+c)}\sqrt{2\sin(dx+c)+2}\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\sqrt{2\sin(dx+c)+2}\left(\sqrt{\sin}(dx+c)\right)F\left(\ldots\right.}{5\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$-\frac{a\,e^3\left(6\sqrt{1-\sin(dx+c)}\sqrt{2\sin(dx+c)+2}\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\sqrt{2\sin(dx+c)+2}\left(\sqrt{\sin}(dx+c)\right)F\ldots\right.}{5\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((a+cos(d*x+c)*b)*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/7/e*b*(e*sin(d*x+c))^(7/2)-1/5*e^3*a*(6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3*
(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-s
in(d*x+c))^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^4+2*sin(d*x+c)^2)/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.27

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \frac{21i\sqrt{2}a\sqrt{-i\,e}e^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)))-\ldots}{\ldots}$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/35*(21*I*sqrt(2)*a*sqrt(-I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInver
se(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*a*sqrt(I*e)*e^2*wei
erstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))
) - 2*(5*b*e^2*cos(d*x + c)^2 + 7*a*e^2*cos(d*x + c) - 5*b*e^2)*sqrt(e*sin(
d*x + c))*sin(d*x + c))/d
```

Sympy [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \int (e\sin{(c+dx)})^{\frac{5}{2}}\,(a+b\cos{(c+dx)})\,dx$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(5/2),x)
[Out] Integral((e*sin(c + d*x))**(5/2)*(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \int (b\cos{(dx+c)}+a)(e\sin{(dx+c)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \int (b\cos{(dx+c)}+a)(e\sin{(dx+c)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \int (e\,\sin{(c+d\,x)})^{5/2}\,(a+b\,\cos{(c+d\,x)})\,dx$$

```
[In] int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x)), x)
```

3.35 $\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \frac{2ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

```
[Out] 2/5*b*(e*sin(d*x+c))^(5/2)/d/e-2/3*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*si
n(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)-2/3*a*e*cos(d*x+c)*(e*sin(d*x+c))^(1/
2)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2748, 2715, 2721, 2720}

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \frac{2ae^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}} - \frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

```
[In] Int[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*a*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*S
in[c + d*x]]) - (2*a*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d) + (2*b*(e*S
in[c + d*x])^(5/2))/(5*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(e\sin(c+dx))^{5/2}}{5de} + a\int (e\sin(c+dx))^{3/2}\,dx \\
&= -\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{2b(e\sin(c+dx))^{5/2}}{5de} + \frac{1}{3}\left(ae^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx \\
&= -\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{2b(e\sin(c+dx))^{5/2}}{5de} + \frac{\left(ae^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3\sqrt{e\sin(c+dx)}} \\
&= \frac{2ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{2b(e\sin(c+dx))^{5/2}}{5de}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \frac{2(e\sin(c+dx))^{3/2}\left(-5a\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+\sqrt{\sin(c+dx)}(-5a\cos(c+dx)+3\right.}{15d\sin^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(e*Sin[c + d*x])^(3/2)*(-5*a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + Sqrt[
Sin[c + d*x]]*(-5*a*Cos[c + d*x] + 3*b*Sin[c + d*x]^2)))/(15*d*Sin[c + d*x]
^(3/2))
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{5}{2}}}{5e}-\frac{e^2a\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\left(\sin^3(dx+c)\right)+2\sin(dx+c)\right)}{3\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$-\frac{a\,e^2\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\left(\sin^3(dx+c)\right)+2\sin(dx+c)\right)}{3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{2b(e\sin(dx+c))^{\frac{5}{2}}}{5de}$

```
[In] int((a+cos(d*x+c)*b)*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/5/e*b*(e*sin(d*x+c))^(5/2)-1/3*e^2*a*((1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*si
n(d*x+c)^3+2*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \frac{5\sqrt{2}a\sqrt{-i\,e}e\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}a\sqrt{i\,e}e\text{weierstrassP}}{}$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(5*sqrt(2)*a*sqrt(-I*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 5*sqrt(2)*a*sqrt(I*e)*e*weierstrassPInverse(4, 0, cos(d*x +
c) - I*sin(d*x + c)) - 2*(3*b*e*cos(d*x + c)^2 + 5*a*e*cos(d*x + c) - 3*b*e
)*sqrt(e*sin(d*x + c)))/d
```

Sympy [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \int (e\sin{(c+dx)})^{\frac{3}{2}}\,(a+b\cos{(c+dx)})\,\,dx$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(3/2),x)
[Out] Integral((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \int (b\cos{(dx+c)}+a)(e\sin{(dx+c)})^{\frac{3}{2}}\,\,dx$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \int (b\cos{(dx+c)}+a)(e\sin{(dx+c)})^{\frac{3}{2}}\,\,dx$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \int {\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)\,dx$$

```
[In] int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x)), x)
```

3.36 $\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

```
[Out] 2/3*b*(e*sin(d*x+c))^(3/2)/d/e-2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(
1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d
*x+c))^(1/2)/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2748, 2721, 2719}

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

```
[In] Int[(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c +
 d*x]]) + (2*b*(e*Sin[c + d*x])^(3/2))/(3*d*e)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2b(e\sin(c+dx))^{3/2}}{3de} + a\int \sqrt{e\sin(c+dx)}\,dx\\ &= \frac{2b(e\sin(c+dx))^{3/2}}{3de} + \frac{\left(a\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{\sqrt{\sin(c+dx)}}\\ &= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{2b(e\sin(c+dx))^{3/2}}{3de}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \frac{2\sqrt{e\sin(c+dx)}\left(-3aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+b\sin^{\frac{3}{2}}(c+dx)\right)}{3d\sqrt{\sin(c+dx)}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
[Out] (2*Sqrt[e*Sin[c + d*x]]*(-3*a*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + b*Sin[c
 + d*x]^(3/2)))/(3*d*Sqrt[Sin[c + d*x]])
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.72

method	result	s
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{3}{2}}}{3e}-\frac{ae\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\left(2E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$	1
parts	$-\frac{ae\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\left(2E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{2b(e\sin(dx+c))^{\frac{3}{2}}}{3de}$	1

```
[In] int((a+cos(d*x+c)*b)*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/3*b/e*(e*sin(d*x+c))^(3/2)-a*e*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/
2)*sin(d*x+c)^(1/2)*(2*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-Elliptic
F((1-sin(d*x+c))^(1/2),1/2*2^(1/2)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.31

$$\int(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx$$
$$=\frac{3i\sqrt{2}a\sqrt{-ie}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}a\sqrt{ie}\text{wei}}{3}$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*I*sqrt(2)*a*sqrt(-I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4,
0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*a*sqrt(I*e)*weierstrassZet
a(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(
e*sin(d*x + c))*b*sin(d*x + c))/d
```

Sympy [F]

$$\int(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx=\int\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))\,dx$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \int (b\cos(dx+c)+a)\sqrt{e\sin(dx+c)}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \int (b\cos(dx+c)+a)\sqrt{e\sin(dx+c)}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \frac{2\,b\,\sin(c+d\,x)\,\sqrt{e\,\sin(c+d\,x)}}{3\,d} + \frac{2\,a\,\sqrt{e\,\sin(c+d\,x)}\,\mathrm{E}\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{\sin(c+d\,x)}}$$

```
[In] int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x)),x)
[Out] (2*b*sin(c + d*x)*(e*sin(c + d*x))^(1/2))/(3*d) + (2*a*(e*sin(c + d*x))^(1/
2)*ellipticE(c/2 - pi/4 + (d*x)/2, 2))/(d*sin(c + d*x)^(1/2))
```

3.37 $\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{2b\sqrt{e\sin(c+dx)}}{de}$$

```
[Out] -2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
F(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2
)+2*b*(e*sin(d*x+c))^(1/2)/d/e
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2748, 2721, 2720}

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}} + \frac{2b\sqrt{e\sin(c+dx)}}{de}$$

```
[In] Int[(a + b*Cos[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c +
 d*x]]) + (2*b*Sqrt[e*Sin[c + d*x]])/(d*e)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b\sqrt{e\sin(c+dx)}}{de} + a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{2b\sqrt{e\sin(c+dx)}}{de} + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{\sqrt{e\sin(c+dx)}} \\
&= \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{2b\sqrt{e\sin(c+dx)}}{de}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2\left(-a\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sqrt{\sin(c+dx)}+b\sin(c+dx)\right)}{d\sqrt{e\sin(c+dx)}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
[Out] (2*(-(a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]) + b*Sin[c +
 d*x]))/(d*Sqrt[e*Sin[c + d*x]])
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

method	result
default	$-\frac{a\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)\cos(dx+c)b}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$
parts	$-\frac{a\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{2b\sqrt{e\sin(dx+c)}}{de}$
risch	$-\frac{ib\left(e^{2i(dx+c)}-1\right)\sqrt{2}\,e^{-i(dx+c)}}{d\sqrt{-ie\left(e^{2i(dx+c)}-1\right)e^{-i(dx+c)}}}-\frac{ia\sqrt{e^{i(dx+c)}+1}\,\sqrt{-2\,e^{i(dx+c)}+2}\,\sqrt{-e^{i(dx+c)}}\,F\left(\sqrt{e^{i(dx+c)}+1},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{-ie\left(e^{2i(dx+c)}-1\right)e^{i(}}}{d\sqrt{-ie\,e^{3i(dx+c)}+ie\,e^{i(dx+c)}}\,\sqrt{-ie\left(e^{2i(dx+c)}-1\right)e^{-i(dx+c)}}}$

```
[In] int((a+cos(d*x+c)*b)/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(a*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*sin(d
*x+c)*cos(d*x+c)*b)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{\sqrt{2}a\sqrt{-i\,e}\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}a\sqrt{i\,e}\text{weierstrassPInverse}(4,0,\cos(}{de}$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*a*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x +
c)) + sqrt(2)*a*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*
x + c)) + 2*sqrt(e*sin(d*x + c))*b)/(d*e)
```

Sympy [F]

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))/sqrt(e*sin(c + d*x)), x)
```

Maxima [F]

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx=\int \frac{b\cos\left(dx+c\right)+a}{\sqrt{e\sin\left(dx+c\right)}}\,dx$$

[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)

Giac [F]

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx=\int \frac{b\cos\left(dx+c\right)+a}{\sqrt{e\sin\left(dx+c\right)}}\,dx$$

[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 13.74 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx=-\frac{2\,\sqrt{\sin\left(c+d\,x\right)}\,\left(a\,\mathrm{F}\left(\frac{\pi}{4}-\frac{c}{2}-\frac{d\,x}{2}\middle|2\right)-b\,\sqrt{\sin\left(c+d\,x\right)}\right)}{d\,\sqrt{e\,\sin\left(c+d\,x\right)}}$$

[In] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(1/2),x)

[Out] -(2*sin(c + d*x)^(1/2)*(a*ellipticF(pi/4 - c/2 - (d*x)/2, 2) - b*sin(c + d*x)^(1/2)))/(d*(e*sin(c + d*x))^(1/2))

3.38 $\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2b}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}$$

```
[Out] -2*b/d/e/(e*sin(d*x+c))^(1/2)-2*a*cos(d*x+c)/d/e/(e*sin(d*x+c))^(1/2)+2*a*(
sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(
1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2748, 2716, 2721, 2719}

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2b}{de\sqrt{e\sin(c+dx)}}$$

```
[In] Int[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*b)/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c +
d*x]]) - (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*e^2
*Sqrt[Sin[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b}{de\sqrt{e\sin(c+dx)}}+a\int\frac{1}{(e\sin(c+dx))^{3/2}}\,dx\\
&= -\frac{2b}{de\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{a\int\sqrt{e\sin(c+dx)}\,dx}{e^2}\\
&= -\frac{2b}{de\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{\left(a\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{e^2\sqrt{\sin(c+dx)}}\\
&= -\frac{2b}{de\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2\left(b+a\cos(c+dx)-aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\sin(c+dx)}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x] - a*EllipticE[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c
+ d*x]]))/(d*e*Sqrt[e*Sin[c + d*x]])
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.59

method	result
default	$\frac{2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a-a\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\right.}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$
parts	$\frac{a\left(2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\right.\right.}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((a+cos(d*x+c)*b)/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a-a*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*a*cos(
d*x+c)^2-2*cos(d*x+c)*b)/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.19

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}a\sqrt{-ie}\sin(dx+c)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInv
erse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*a*sqrt(I*e)*sin(d*x
+ c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d
*x + c))) - 2*(a*cos(d*x + c) + b)*sqrt(e*sin(d*x + c)))/(d*e^2*sin(d*x + c
))
```

Sympy [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{a+b\cos{(c+dx)}}{(e\sin{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(3/2),x)
[Out] Integral((a + b*cos(c + d*x))/(e*sin(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{b\cos{(dx+c)}+a}{(e\sin{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{b\cos{(dx+c)}+a}{(e\sin{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{a+b\,\cos{(c+d\,x)}}{(e\,\sin{(c+d\,x)})^{3/2}}\,dx$$

```
[In] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(3/2),x)
[Out] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(3/2), x)
```

3.39 $\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2b}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}}$$

```
[Out] -2/3*b/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*cos(d*x+c)/d/e/(e*sin(d*x+c))^(3/2)-2
/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
F(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^2/(e*sin(d*x+c))^
(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2748, 2716, 2721, 2720}

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

```
[In] Int[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*b)/(3*d*e*(e*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x])/(3*d*e*(e*Sin[c
+ d*x])^(3/2)) + (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/
(3*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2b}{3de(e\sin(c+dx))^{3/2}} + a\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx \\
&= -\frac{2b}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2} \\
&= -\frac{2b}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3e^2\sqrt{e\sin(c+dx)}} \\
&= -\frac{2b}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2\left(b+a\cos(c+dx)+a\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3de(e\sin(c+dx))^{3/2}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
[Out] (-2*(b + a*Cos[c + d*x] + a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x
]^(3/2)))/(3*d*e*(e*Sin[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.22

method	result
default	$\frac{-\frac{2b}{3e(e\sin(dx+c))^{\frac{3}{2}}}-\frac{a\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\left(\sin^{3}(dx+c)\right)+2\sin(dx+c)\right)}{3e^{2}\sin(dx+c)^{2}\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$-\frac{a\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2(\sin^{3}(dx+c))+2\sin(dx+c)\right)}{3e^{2}\sin(dx+c)^{2}\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b}{3de(e\sin(dx+c))^{\frac{3}{2}}}$

```
[In] int((a+cos(d*x+c)*b)/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] (-2/3*b/e/(e*sin(d*x+c))^(3/2)-1/3*a/e^2*((1-sin(d*x+c))^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*s
in(d*x+c)^3+2*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.32

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \frac{\left(\sqrt{2}a\cos\left(dx+c\right)^{2}-\sqrt{2}a\right)\sqrt{-i\,e}\,\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx\right.}{}$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 1/3*((sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*a)*sqrt(-I*e)*weierstrassPInverse(
4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*
a)*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(
a*cos(d*x + c) + b)*sqrt(e*sin(d*x + c)))/(d*e^3*cos(d*x + c)^2 - d*e^3)
```

Sympy [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{a+b\cos{(c+dx)}}{(e\sin{(c+dx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(5/2),x)
[Out] Integral((a + b*cos(c + d*x))/(e*sin(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{b\cos{(dx+c)}+a}{(e\sin{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{b\cos{(dx+c)}+a}{(e\sin{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{a+b\,\cos{(c+d\,x)}}{(e\,\sin{(c+d\,x)})^{5/2}}\,dx$$

```
[In] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(5/2),x)
[Out] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(5/2), x)
```

3.40 $\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{2b}{5de(e\sin(c+dx))^{5/2}} - \frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}} - \frac{6a\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}}$$

```
[Out] -2/5*b/d/e/(e*sin(d*x+c))^(5/2)-2/5*a*cos(d*x+c)/d/e/(e*sin(d*x+c))^(5/2)-6
/5*a*cos(d*x+c)/d/e^3/(e*sin(d*x+c))^(1/2)+6/5*a*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(
1/2))*(e*sin(d*x+c))^(1/2)/d/e^4/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2748, 2716, 2721, 2719}

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{6aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}} - \frac{6a\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}} - \frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

```
[In] Int[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*b)/(5*d*e*(e*Sin[c + d*x])^(5/2)) - (2*a*Cos[c + d*x])/(5*d*e*(e*Sin[c
+ d*x])^(5/2)) - (6*a*Cos[c + d*x])/(5*d*e^3*Sqrt[e*Sin[c + d*x]]) - (6*a*E
```

```
llipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*e^4*Sqrt[Sin[c +
 d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2b}{5de(e\sin(c+dx))^{5/2}}+a\int \frac{1}{(e\sin(c+dx))^{7/2}}\,dx\\
&= -\frac{2b}{5de(e\sin(c+dx))^{5/2}}-\frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}+\frac{(3a)\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{5e^2}\\
&= -\frac{2b}{5de(e\sin(c+dx))^{5/2}}-\frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\\
&\quad -\frac{6a\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}}-\frac{(3a)\int \sqrt{e\sin(c+dx)}\,dx}{5e^4}\\
&= -\frac{2b}{5de(e\sin(c+dx))^{5/2}}-\frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\\
&\quad -\frac{6a\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}}-\frac{\left(3a\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{5e^4\sqrt{\sin(c+dx)}}
\end{aligned}
$$

$$= -\frac{2b}{5de(e\sin(c+dx))^{5/2}} - \frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}} - \frac{6a\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \frac{-4b-7a\cos(c+dx)+3a\cos(3(c+dx))+12aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{5}{2}}(c+}{10de(e\sin(c+dx))^{5/2}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-4*b - 7*a*Cos[c + d*x] + 3*a*Cos[3*(c + d*x)] + 12*a*EllipticE[(-2*c + Pi
 - 2*d*x)/4, 2]*Sin[c + d*x]^(5/2))/(10*d*e*(e*Sin[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.43

method	result
default	$\frac{-\frac{2b}{5e(e\sin(dx+c))^{\frac{5}{2}}}+\frac{a\left(6\sqrt{1-\sin(dx+c)}\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right)F}{5e^3\sin(dx+c)^3\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$\frac{a\left(6\sqrt{1-\sin(dx+c)}\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right)F\left(\sqrt{1}\right.}{5e^3\sin(dx+c)^3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((a+cos(d*x+c)*b)/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/5*b/e/(e*sin(d*x+c))^(5/2)+1/5*a/e^3*(6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3
*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1-
sin(d*x+c))^(1/2),1/2*2^(1/2))+6*sin(d*x+c)^5-4*sin(d*x+c)^3-2*sin(d*x+c))/
sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.37

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{3\left(i\sqrt{2}a\cos\left(dx+c\right)^{2}-i\sqrt{2}a\right)\sqrt{-i\,e}\sin\left(dx+c\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(dx+\right.\right.}{}$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*(I*sqrt(2)*a*cos(d*x + c)^2 - I*sqrt(2)*a)*sqrt(-I*e)*sin(d*x + c)*
weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x +
c))) + 3*(-I*sqrt(2)*a*cos(d*x + c)^2 + I*sqrt(2)*a)*sqrt(I*e)*sin(d*x + c)
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*(3*a*cos(d*x + c)^3 - 4*a*cos(d*x + c) - b)*sqrt(e*sin(d*x + c)))
/((d*e^4*cos(d*x + c)^2 - d*e^4)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{b\cos\left(dx+c\right)+a}{\left(e\sin\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx=\int \frac{b\cos{(dx+c)}+a}{(e\sin{(dx+c)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx=\int \frac{a+b\,\cos{(c+d\,x)}}{(e\,\sin{(c+d\,x)})^{7/2}}\,dx$$

```
[In] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(7/2),x)
[Out] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(7/2), x)
```

3.41 $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx=\frac{10(11a^2+2b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{231d\sqrt{e\sin(c+dx)}}-\frac{10(11a^2+2b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d}-\frac{2(11a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d}+\frac{26ab(e\sin(c+dx))^{9/2}}{99de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{11de}$$

```
[Out] -2/77*(11*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/d+26/99*a*b*(e*sin(d
*x+c))^(9/2)/d/e+2/11*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(9/2)/d/e-10/231*(1
1*a^2+2*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*sin(
d*x+c))^(1/2)-10/231*(11*a^2+2*b^2)*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{2771, 2748, 2715, 2721, 2720\}$

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \frac{10e^4(11a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{231d\sqrt{e\sin(c+dx)}} - \frac{10e^3(11a^2+2b^2)\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d} - \frac{2e(11a^2+2b^2)\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d} + \frac{26ab(e\sin(c+dx))^{9/2}}{99de} + \frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

```
[In] Int[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (10*(11*a^2 + 2*b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/(231*d*Sqrt[e*Sin[c + d*x]]) - (10*(11*a^2 + 2*b^2)*e^3*Cos[c + d*x]*Sqr
t[e*Sin[c + d*x]])/(231*d) - (2*(11*a^2 + 2*b^2)*e*Cos[c + d*x]*(e*Sin[c +
d*x])^(5/2))/(77*d) + (26*a*b*(e*Sin[c + d*x])^(9/2))/(99*d*e) + (2*b*(a +
b*Cos[c + d*x])*(e*Sin[c + d*x])^(9/2))/(11*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{11de} \\
&\quad + \frac{2}{11}\int\left(\frac{11a^2}{2}+b^2+\frac{13}{2}ab\cos(c+dx)\right)(e\sin(c+dx))^{7/2}\,dx \\
&= \frac{26ab(e\sin(c+dx))^{9/2}}{99de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{11de} \\
&\quad + \frac{1}{11}\left(11a^2+2b^2\right)\int(e\sin(c+dx))^{7/2}\,dx \\
&= -\frac{2\left(11a^2+2b^2\right)e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d} \\
&\quad + \frac{26ab(e\sin(c+dx))^{9/2}}{99de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{11de} \\
&\quad + \frac{1}{77}\left(5\left(11a^2+2b^2\right)e^2\right)\int(e\sin(c+dx))^{3/2}\,dx \\
&= -\frac{10\left(11a^2+2b^2\right)e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d} \\
&\quad - \frac{2\left(11a^2+2b^2\right)e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d} \\
&\quad + \frac{26ab(e\sin(c+dx))^{9/2}}{99de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{11de} \\
&\quad + \frac{1}{231}\left(5\left(11a^2+2b^2\right)e^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx \\
&= -\frac{10\left(11a^2+2b^2\right)e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d} \\
&\quad - \frac{2\left(11a^2+2b^2\right)e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d} \\
&\quad + \frac{26ab(e\sin(c+dx))^{9/2}}{99de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{11de} \\
&\quad + \frac{\left(5\left(11a^2+2b^2\right)e^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{231\sqrt{e\sin(c+dx)}}
\end{aligned}$$

$$= \frac{10(11a^2+2b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{231d\sqrt{e\sin(c+dx)}} - \frac{10(11a^2+2b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d} - \frac{2(11a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d} + \frac{26ab(e\sin(c+dx))^{9/2}}{99de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{11de}$$

Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \frac{\left(\frac{1}{6}(924ab-6(506a^2+71b^2)\cos(c+dx)-1232ab\cos(2(c+dx))+396a^2\cos(3(c+dx))-11\right.}{}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] ((((924*a*b - 6*(506*a^2 + 71*b^2)*Cos[c + d*x] - 1232*a*b*Cos[2*(c + d*x)]
 + 396*a^2*Cos[3*(c + d*x)] - 117*b^2*Cos[3*(c + d*x)] + 308*a*b*Cos[4*(c +
 d*x)] + 63*b^2*Cos[5*(c + d*x)])*Csc[c + d*x]^3)/6 - (40*(11*a^2 + 2*b^2)*
EllipticF[(-2*c + Pi - 2*d*x)/4, 2])/Sin[c + d*x]^(7/2))*(e*Sin[c + d*x])^(
7/2))/(924*d)
```

Maple [A] (verified)

Time = 9.72 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.31

method	result
default	$\frac{\frac{4ab(e\sin(dx+c))^{\frac{9}{2}}}{9e} - \frac{e^4\left(-42b^2\left(\cos^6(dx+c)\right)\sin(dx+c)-66a^2\left(\cos^4(dx+c)\right)\sin(dx+c)+72b^2\left(\cos^4(dx+c)\right)\sin(dx+c)+55\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c}\right.}{}}{}$
parts	$-\frac{a^2e^4\left(-6\left(\sin^5(dx+c)\right)+5\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-4\left(\sin^3(dx+c)\right)+10\sin(dx+c)\right)}{21\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((a+cos(d*x+c)*b)^2*(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4/9/e*a*b*(e*sin(d*x+c))^(9/2)-1/231*e^4*(-42*b^2*cos(d*x+c)^6*sin(d*x+c)-
66*a^2*cos(d*x+c)^4*sin(d*x+c)+72*b^2*cos(d*x+c)^4*sin(d*x+c)+55*(1-sin(d*x
+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))
^(1/2),1/2*2^(1/2))*a^2+10*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+176*a^2*cos(d*
```

```
x+c)^2*sin(d*x+c)-10*b^2*cos(d*x+c)^2*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))
^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \frac{15\sqrt{2}(11a^2+2b^2)\sqrt{-i\,e}e^3\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+15\sqrt{2}(1\ldots}{}$$

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/693*(15*sqrt(2)*(11*a^2 + 2*b^2)*sqrt(-I*e)*e^3*weierstrassPInverse(4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(11*a^2 + 2*b^2)*sqrt(I*e)*e^3
*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(63*b^2*e^3*c
os(d*x + c)^5 + 154*a*b*e^3*cos(d*x + c)^4 - 308*a*b*e^3*cos(d*x + c)^2 + 9
*(11*a^2 - 12*b^2)*e^3*cos(d*x + c)^3 + 154*a*b*e^3 - 3*(88*a^2 - 5*b^2)*e^
3*cos(d*x + c))*sqrt(e*sin(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(7/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \int (e\,\sin(c+d\,x))^{7/2}\,(a+b\,\cos(c+d\,x))^2\,dx$$

```
[In] int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2, x)
```

3.42 $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 154

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \frac{2(9a^2+2b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15d\sqrt{\sin(c+dx)}} - \frac{2(9a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{45d} + \frac{22ab(e\sin(c+dx))^{7/2}}{63de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{9de}$$

```
[Out] -2/45*(9*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+22/63*a*b*(e*sin(d*
x+c))^(7/2)/d/e+2/9*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2)/d/e-2/15*(9*a^2
+2*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+
c)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{2771, 2748, 2715, 2721, 2719\}$

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \frac{2e^2(9a^2+2b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{15d\sqrt{\sin(c+dx)}} - \frac{2e(9a^2+2b^2)\cos(c+dx)(e\sin(c+dx))^{3/2}}{45d} + \frac{22ab(e\sin(c+dx))^{7/2}}{63de} + \frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

```
[In] Int[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(9*a^2 + 2*b^2)*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]
])/(15*d*Sqrt[Sin[c + d*x]]) - (2*(9*a^2 + 2*b^2)*e*Cos[c + d*x]*(e*Sin[c +
 d*x])^(3/2))/(45*d) + (22*a*b*(e*Sin[c + d*x])^(7/2))/(63*d*e) + (2*b*(a +
 b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2))/(9*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{9de} \\
&\quad + \frac{2}{9}\int\left(\frac{9a^2}{2}+b^2+\frac{11}{2}ab\cos(c+dx)\right)(e\sin(c+dx))^{5/2}\,dx \\
&= \frac{22ab(e\sin(c+dx))^{7/2}}{63de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{9de} \\
&\quad + \frac{1}{9}\left(9a^2+2b^2\right)\int(e\sin(c+dx))^{5/2}\,dx \\
&= -\frac{2\left(9a^2+2b^2\right)e\cos(c+dx)(e\sin(c+dx))^{3/2}}{45d}+\frac{22ab(e\sin(c+dx))^{7/2}}{63de} \\
&\quad + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{9de}+\frac{1}{15}\left(\left(9a^2+2b^2\right)e^2\right)\int\sqrt{e\sin(c+dx)}\,dx \\
&= -\frac{2\left(9a^2+2b^2\right)e\cos(c+dx)(e\sin(c+dx))^{3/2}}{45d} \\
&\quad + \frac{22ab(e\sin(c+dx))^{7/2}}{63de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{9de} \\
&\quad + \frac{\left(\left(9a^2+2b^2\right)e^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{15\sqrt{\sin(c+dx)}} \\
&= \frac{2\left(9a^2+2b^2\right)e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15d\sqrt{\sin(c+dx)}} \\
&\quad - \frac{2\left(9a^2+2b^2\right)e\cos(c+dx)(e\sin(c+dx))^{3/2}}{45d} \\
&\quad + \frac{22ab(e\sin(c+dx))^{7/2}}{63de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{9de}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.75

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx =$$

$$-\frac{(e\sin(c+dx))^{5/2}\left(84(9a^2+2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)+(21(12a^2+b^2)\cos(c+dx)+5b(-36a+36a\,c\right.}{630d\sin^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/630*((e*Sin[c + d*x])^(5/2)*(84*(9*a^2 + 2*b^2)*EllipticE[(-2*c + Pi - 2
*d*x)/4, 2] + (21*(12*a^2 + b^2)*Cos[c + d*x] + 5*b*(-36*a + 36*a*Cos[2*(c
+ d*x)] + 7*b*Cos[3*(c + d*x)]))*Sin[c + d*x]^(3/2)))/(d*Sin[c + d*x]^(5/2)
)
```

Maple [A] (verified)

Time = 9.60 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.16

method	result
default	$\frac{\frac{4ab(e\sin(dx+c))^{\frac{7}{2}}}{7e}-\frac{e^3\left(10\left(\sin^6(dx+c)\right)b^2+54\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2+12\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx}\right.}{}}{}$
parts	$-\frac{a^2e^3\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\right.}{5\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((a+cos(d*x+c)*b)^2*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4/7/e*a*b*(e*sin(d*x+c))^(7/2)-1/45*e^3*(10*sin(d*x+c)^6*b^2+54*(1-sin(d*x
+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))
^(1/2),1/2*2^(1/2))*a^2+12*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-27*(1-sin(d*x+
c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^
(1/2),1/2*2^(1/2))*a^2-6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-18*a^2*sin(d*x+c
)^4-14*sin(d*x+c)^4*b^2+18*a^2*sin(d*x+c)^2+4*b^2*sin(d*x+c)^2)/cos(d*x+c)/
(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.14

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \frac{21i\sqrt{2}(9a^2+2b^2)\sqrt{-ie}e^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin$$

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(21*I*sqrt(2)*(9*a^2 + 2*b^2)*sqrt(-I*e)*e^2*weierstrassZeta(4, 0, we
ierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(9*a
^2 + 2*b^2)*sqrt(I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, c
os(d*x + c) - I*sin(d*x + c))) - 2*(35*b^2*e^2*cos(d*x + c)^3 + 90*a*b*e^2*
cos(d*x + c)^2 - 90*a*b*e^2 + 21*(3*a^2 - b^2)*e^2*cos(d*x + c))*sqrt(e*sin
(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx=\int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx=\int (e\,\sin(c+d\,x))^{5/2}\,(a+b\,\cos(c+d\,x))^2\,dx$$

```
[In] int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2, x)
```

3.43 $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 154

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{2(7a^2+2b^2)\,e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}} - \frac{2(7a^2+2b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} + \frac{18ab(e\sin(c+dx))^{5/2}}{35de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{7de}$$

```
[Out] 18/35*a*b*(e*sin(d*x+c))^(5/2)/d/e+2/7*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(5
/2)/d/e-2/21*(7*a^2+2*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(
1/2)/d/(e*sin(d*x+c))^(1/2)-2/21*(7*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^
(1/2)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{2771, 2748, 2715, 2721, 2720\}$

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{2e^2(7a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21d\sqrt{e\sin(c+dx)}} - \frac{2e(7a^2+2b^2)\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} + \frac{18ab(e\sin(c+dx))^{5/2}}{35de} + \frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

```
[In] Int[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(7*a^2 + 2*b^2)*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])
/(21*d*Sqrt[e*Sin[c + d*x]]) - (2*(7*a^2 + 2*b^2)*e*Cos[c + d*x]*Sqrt[e*Sin
[c + d*x]])/(21*d) + (18*a*b*(e*Sin[c + d*x])^(5/2))/(35*d*e) + (2*b*(a + b
*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(7*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{7de} \\
&\quad+\frac{2}{7}\int\left(\frac{7a^2}{2}+b^2+\frac{9}{2}ab\cos(c+dx)\right)(e\sin(c+dx))^{3/2}\,dx \\
&= \frac{18ab(e\sin(c+dx))^{5/2}}{35de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{7de} \\
&\quad+\frac{1}{7}\left(7a^2+2b^2\right)\int(e\sin(c+dx))^{3/2}\,dx \\
&= -\frac{2(7a^2+2b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} \\
&\quad+\frac{18ab(e\sin(c+dx))^{5/2}}{35de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{7de} \\
&\quad+\frac{1}{21}\left(\left(7a^2+2b^2\right)e^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx \\
&= -\frac{2(7a^2+2b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} \\
&\quad+\frac{18ab(e\sin(c+dx))^{5/2}}{35de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{7de} \\
&\quad+\frac{\left(\left(7a^2+2b^2\right)e^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{21\sqrt{e\sin(c+dx)}} \\
&= \frac{2(7a^2+2b^2)\,e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}} \\
&\quad-\frac{2(7a^2+2b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} \\
&\quad+\frac{18ab(e\sin(c+dx))^{5/2}}{35de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{7de}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.76

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{\left(-\frac{1}{2}(5(28a^2+5b^2)\cos(c+dx)+3b(-28a+28a\cos(2(c+dx))+5b\cos(3(c+dx))))\csc(c+\right.}{105d}$$

[In] Integrate[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]

[Out] ((-1/2*((5*(28*a^2 + 5*b^2)*Cos[c + d*x] + 3*b*(-28*a + 28*a*Cos[2*(c + d*x)] + 5*b*Cos[3*(c + d*x)]))*Csc[c + d*x]) - (10*(7*a^2 + 2*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2])/Sin[c + d*x]^(3/2))*(e*Sin[c + d*x])^(3/2))/(105*d)

Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.49

method	result
default	$-\frac{e^2\left(30b^2(\cos^4(dx+c))\sin(dx+c)+35\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2+10\sqrt{1-\sin(dx+c)}\right.}{}$
parts	$-\frac{a^2e^2\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2(\sin^3(dx+c))+2\sin(dx+c)\right)}{3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b^2e^2\left(3(\sin^5(dx\right.}{}$

[In] int((a+cos(d*x+c)*b)^2*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/105/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^2*(30*b^2*cos(d*x+c)^4*sin(d*x+c)+35*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+10*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+84*a*b*cos(d*x+c)^3*sin(d*x+c)+70*a^2*cos(d*x+c)^2*sin(d*x+c)-10*b^2*cos(d*x+c)^2*sin(d*x+c)-84*a*b*cos(d*x+c)*sin(d*x+c))/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.98

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{5\sqrt{2}(7a^2+2b^2)\sqrt{-ie}\,e\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}(7a^2+$$

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/105*(5*sqrt(2)*(7*a^2 + 2*b^2)*sqrt(-I*e)*e*weierstrassPInverse(4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(7*a^2 + 2*b^2)*sqrt(I*e)*e*weierst
rassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(15*b^2*e*cos(d*x + c
)^3 + 42*a*b*e*cos(d*x + c)^2 - 42*a*b*e + 5*(7*a^2 - b^2)*e*cos(d*x + c))*
sqrt(e*sin(d*x + c)))/d
```

Sympy [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (e\sin(c+dx))^{\frac{3}{2}}(a+b\cos(c+dx))^2\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x))**2, x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (e\,\sin(c+d\,x))^{3/2}\,(a+b\,\cos(c+d\,x))^2\,dx$$

```
[In] int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2, x)
```

3.44 $\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \frac{2(5a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} + \frac{14ab(e\sin(c+dx))^{3/2}}{15de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{5de}$$

```
[Out] 14/15*a*b*(e*sin(d*x+c))^(3/2)/d/e+2/5*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(3
/2)/d/e-2/5*(5*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1
/2)/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used

$= \{2771, 2748, 2721, 2719\}$

$$\int (a + b\cos(c+dx))^2 \sqrt{e\sin(c+dx)}\,dx$$

$$= \frac{2(5a^2+2b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} + \frac{14ab(e\sin(c+dx))^{3/2}}{15de} + \frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

```
[In] Int[(a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*(5*a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(
5*d*Sqrt[Sin[c + d*x]]) + (14*a*b*(e*Sin[c + d*x])^(3/2))/(15*d*e) + (2*b*(
a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(5*d*e)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{5de} \\
&\quad + \frac{2}{5}\int \left(\frac{5a^2}{2}+b^2+\frac{7}{2}ab\cos(c+dx)\right)\sqrt{e\sin(c+dx)}\,dx \\
&= \frac{14ab(e\sin(c+dx))^{3/2}}{15de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{5de} \\
&\quad + \frac{1}{5}(5a^2+2b^2)\int \sqrt{e\sin(c+dx)}\,dx \\
&= \frac{14ab(e\sin(c+dx))^{3/2}}{15de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{5de} \\
&\quad + \frac{\left((5a^2+2b^2)\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{5\sqrt{\sin(c+dx)}} \\
&= \frac{2(5a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} \\
&\quad + \frac{14ab(e\sin(c+dx))^{3/2}}{15de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{5de}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$
\begin{aligned}
&\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx \\
&= \frac{2\sqrt{e\sin(c+dx)}\left(-3(5a^2+2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)+b(10a+3b\cos(c+dx))\sin^{\frac{3}{2}}(c+dx)\right)}{15d\sqrt{\sin(c+dx)}}
\end{aligned}
$$

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[e*Sin[c + d*x]]*(-3*(5*a^2 + 2*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4
, 2] + b*(10*a + 3*b*Cos[c + d*x])*Sin[c + d*x]^(3/2)))/(15*d*Sqrt[Sin[c +
d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(130) = 260$.

Time = 3.29 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.39

method	result
parts	$-\frac{a^2 e\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\left(2E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d} - \frac{2b^2e\left(2\sqrt{1-\sin(dx+\right.}{}$
default	$-\frac{e\left(30\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2+12\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\right.}{}$

```
[In] int((a+cos(d*x+c)*b)^2*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*e*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*(2*Elli
pticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-sin(d*x+c))^(1/2),1/2*
2^(1/2)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d-2/5*b^2*e*(2*(1-sin(d*x+c))^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1
/2*2^(1/2))-(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*El
lipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+cos(d*x+c)^4-cos(d*x+c)^2)/cos(d*
x+c)/(e*sin(d*x+c))^(1/2)/d+4/3*a*b*(e*sin(d*x+c))^(3/2)/d/e
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = -\frac{3\sqrt{2}(-5i\,a^2-2i\,b^2)\sqrt{-i\,e}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)))\,+}{}$$

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*sqrt(2)*(-5*I*a^2 - 2*I*b^2)*sqrt(-I*e)*weierstrassZeta(4, 0, weie
rstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(5*I*a^2
+ 2*I*b^2)*sqrt(I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*
x + c) - I*sin(d*x + c))) - 2*(3*b^2*cos(d*x + c) + 10*a*b)*sqrt(e*sin(d*x
+ c))*sin(d*x + c))/d
```

Sympy [F]

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx=\int \sqrt{e\sin{(c+dx)}}(a+b\cos{(c+dx)})^2\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))**2, x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx=\int (b\cos{(dx+c)}+a)^2\sqrt{e\sin{(dx+c)}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx=\int (b\cos{(dx+c)}+a)^2\sqrt{e\sin{(dx+c)}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx=\int \sqrt{e\,\sin{(c+d\,x)}}\,(a+b\,\cos{(c+d\,x)})^2\,dx$$

```
[In] int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2, x)
```

3.45 $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2(3a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} + \frac{10ab\sqrt{e\sin(c+dx)}}{3de} + \frac{2b(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de}$$

```
[Out] -2/3*(3*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2
*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*si
n(d*x+c))^(1/2)+10/3*a*b*(e*sin(d*x+c))^(1/2)/d/e+2/3*b*(a+b*cos(d*x+c))*(e
*sin(d*x+c))^(1/2)/d/e
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2771, 2748, 2721, 2720}

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2(3a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}} + \frac{10ab\sqrt{e\sin(c+dx)}}{3de} + \frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

```
[In] Int[(a + b*Cos[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*(3*a^2 + 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*
d*Sqrt[e*Sin[c + d*x]]) + (10*a*b*Sqrt[e*Sin[c + d*x]])/(3*d*e) + (2*b*(a +
 b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*d*e)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de} + \frac{2}{3}\int \frac{\frac{3a^2}{2}+b^2+\frac{5}{2}ab\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{10ab\sqrt{e\sin(c+dx)}}{3de} + \frac{2b(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de} \\
&\quad + \frac{1}{3}\left(3a^2+2b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{10ab\sqrt{e\sin(c+dx)}}{3de} + \frac{2b(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de} \\
&\quad + \frac{\left(\left(3a^2+2b^2\right)\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3\sqrt{e\sin(c+dx)}} \\
&= \frac{2\left(3a^2+2b^2\right)\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{10ab\sqrt{e\sin(c+dx)}}{3de} + \frac{2b(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{-2(3a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sqrt{\sin(c+dx)}+2b(6a+b\cos(c+dx))\sin(c+dx)}{3d\sqrt{e\sin(c+dx)}}$$

[In] Integrate[(a + b*Cos[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]

[Out] (-2*(3*a^2 + 2*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]] + 2*b*(6*a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[e*Sin[c + d*x]])

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.49

method	result
default	$-\frac{3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2+2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{\right.}\right.}{3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$
parts	$-\frac{a^2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{b^2\left(-\frac{2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\right.}{3}\right.}{\cos(dx+c)\sqrt{e\sin}}$

[In] int((a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-2*b^2*cos(d*x+c)^2*sin(d*x+c)-12*a*b*cos(d*x+c)*sin(d*x+c))/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{\sqrt{2}(3\,a^2+2\,b^2)\sqrt{-i\,e}\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}(3\,a^2+2\,b^2)\sqrt{i\,e}\text{weierstra}}{3\,de}$$

[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")

```
[Out] 1/3*(sqrt(2)*(3*a^2 + 2*b^2)*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(2)*(3*a^2 + 2*b^2)*sqrt(I*e)*weierstrassPInver
se(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(b^2*cos(d*x + c) + 6*a*b)*sqrt
(e*sin(d*x + c)))/(d*e)
```

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
[Out] Integral((a + b*cos(c + d*x))**2/sqrt(e*sin(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{\sqrt{e\sin(dx+c)}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(e*sin(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{\sqrt{e\sin(dx+c)}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(e*sin(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^2}{\sqrt{e\,\sin(c+d\,x)}}\,dx$$

```
[In] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(1/2),x)
[Out] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(1/2), x)
```

3.46 $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}} - \frac{2(a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} - \frac{2ab(e\sin(c+dx))^{3/2}}{de^3}$$

```
[Out] -2*a*b*(e*sin(d*x+c))^(3/2)/d/e^3-2*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))/d/e/(
e*sin(d*x+c))^(1/2)+2*(a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*
x+c))^(1/2)/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2770, 2748, 2721, 2719}

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2(a^2+2b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} - \frac{2ab(e\sin(c+dx))^{3/2}}{de^3} - \frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

```
[In] Int[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x]))/(d*e*Sqrt[e*Sin[c + d*x]]) -
 (2*(a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d
*e^2*Sqrt[Sin[c + d*x]]) - (2*a*b*(e*Sin[c + d*x])^(3/2))/(d*e^3)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}} \\
&\quad -\frac{2\int\left(\frac{a^2}{2}+b^2+\frac{3}{2}ab\cos(c+dx)\right)\sqrt{e\sin(c+dx)}\,dx}{e^2} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}} \\
&\quad -\frac{2ab(e\sin(c+dx))^{3/2}}{de^3}-\frac{(a^2+2b^2)\int\sqrt{e\sin(c+dx)}\,dx}{e^2} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}-\frac{2ab(e\sin(c+dx))^{3/2}}{de^3} \\
&\quad -\frac{\left((a^2+2b^2)\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{e^2\sqrt{\sin(c+dx)}}
\end{aligned}
$$

$$= -\frac{2(b + a\cos(c+dx))(a + b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}} - \frac{2(a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} - \frac{2ab(e\sin(c+dx))^{3/2}}{de^3}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \frac{-4ab - 2(a^2+b^2)\cos(c+dx) + 2(a^2+2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)\sqrt{\sin(c}}{de\sqrt{e\sin(c+dx)}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-4*a*b - 2*(a^2 + b^2)*Cos[c + d*x] + 2*(a^2 + 2*b^2)*EllipticE[(-2*c + Pi
 - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]])/(d*e*Sqrt[e*Sin[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(140) = 280.

Time = 3.48 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.40

method	result
default	$\frac{2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2+4\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\right.}{}$
parts	$\frac{a^2\left(2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\right.\right.}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+4*
(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-s
in(d*x+c))^(1/2),1/2*2^(1/2))*b^2-(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/
2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-2*(1-si
n(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*
x+c))^(1/2),1/2*2^(1/2))*b^2-2*a^2*cos(d*x+c)^2-2*b^2*cos(d*x+c)^2-4*cos(d*
x+c)*a*b)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}(-i\,a^2-2i\,b^2)\sqrt{-i\,e}\sin{(dx+c)}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,$$

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(-I*a^2 - 2*I*b^2)*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0, w
eierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*a^2 +
 2*I*b^2)*sqrt(I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(2*a*b + (a^2 + b^2)*cos(d*x + c)
)*sqrt(e*sin(d*x + c)))/(d*e^2*sin(d*x + c))
```

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(a+b\cos{(c+dx)})^2}{(e\sin{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2/(e*sin(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(b\cos{(dx+c)}+a)^2}{(e\sin{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^2}{(e\,\sin(c+d\,x))^{3/2}}\,dx$$

```
[In] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(3/2),x)
[Out] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(3/2), x)
```

3.47 $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}} + \frac{2(a^2-2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} - \frac{2ab\sqrt{e\sin(c+dx)}}{3de^3}$$

```
[Out] -2/3*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))/d/e/(e*sin(d*x+c))^(3/2)-2/3*(a^2-2*
b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
F(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^2/(e*sin(d*x+c))^
(1/2)-2/3*a*b*(e*sin(d*x+c))^(1/2)/d/e^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2770, 2748, 2721, 2720}

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \frac{2(a^2-2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\sqrt{e\sin(c+dx)}} - \frac{2ab\sqrt{e\sin(c+dx)}}{3de^3} - \frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

```
[In] Int[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x]))/(3*d*e*(e*Sin[c + d*x])^(3/2
)) + (2*(a^2 - 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/
(3*d*e^2*Sqrt[e*Sin[c + d*x]]) - (2*a*b*Sqrt[e*Sin[c + d*x]])/(3*d*e^3)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a^2}{2}+b^2+\frac{1}{2}ab\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}} - \frac{2ab\sqrt{e\sin(c+dx)}}{3de^3} + \frac{(a^2-2b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}} - \frac{2ab\sqrt{e\sin(c+dx)}}{3de^3} \\
&\quad + \frac{\left((a^2-2b^2)\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3e^2\sqrt{e\sin(c+dx)}} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{2(a^2-2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} - \frac{2ab\sqrt{e\sin(c+dx)}}{3de^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2\left(2ab+(a^2+b^2)\cos(c+dx)+(a^2-2b^2)\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3de(e\sin(c+dx))^{3/2}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(2*a*b + (a^2 + b^2)*Cos[c + d*x] + (a^2 - 2*b^2)*EllipticF[(-2*c + Pi
- 2*d*x)/4, 2]*Sin[c + d*x]^(3/2)))/(3*d*e*(e*Sin[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.63

method	result
default	$\frac{-\frac{4ab}{3e(e\sin(dx+c))^{\frac{3}{2}}}-\frac{\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{5}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2-2b^2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{5}{2}}(dx+c)\right)}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$-\frac{a^2\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{5}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\left(\sin^3(dx+c)\right)+2\sin(dx+c)\right)}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{2b^2\left(\sqrt{1-\sin(dx+c}\right.}{}$

```
[In] int((a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-4/3*a*b/e/(e*sin(d*x+c))^(3/2)-1/3/e^2*((1-sin(d*x+c))^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2
-2*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*Ellipti
cF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+2*a^2*cos(d*x+c)^2*sin(d*x+c)+2*b^2*co
s(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.42

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \frac{\left(\sqrt{2}(a^2-2\,b^2)\cos\left(dx+c\right)^2-\sqrt{2}(a^2-2\,b^2)\right)\sqrt{-i\,e}\,\text{weierstrassPInverse}(4,0,\cos}{}$$

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((sqrt(2)*(a^2 - 2*b^2)*cos(d*x + c)^2 - sqrt(2)*(a^2 - 2*b^2))*sqrt(-I
*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(a^
2 - 2*b^2)*cos(d*x + c)^2 - sqrt(2)*(a^2 - 2*b^2))*sqrt(I*e)*weierstrassPIn
verse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(2*a*b + (a^2 + b^2)*cos(d*x
 + c))*sqrt(e*sin(d*x + c)))/(d*e^3*cos(d*x + c)^2 - d*e^3)
```

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(a+b\cos{(c+dx)})^2}{(e\sin{(c+dx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
[Out] Integral((a + b*cos(c + d*x))**2/(e*sin(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(b\cos{(dx+c)}+a)^2}{(e\sin{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(b\cos{(dx+c)}+a)^2}{(e\sin{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(a+b\,\cos\left(c+d\,x\right))^2}{\left(e\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(5/2),x)
[Out] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(5/2), x)
```

3.48 $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}} - \frac{2ab}{5de^3\sqrt{e\sin(c+dx)}} - \frac{2(3a^2-2b^2)\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{2(3a^2-2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}}$$

```
[Out] -2/5*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))/d/e/(e*sin(d*x+c))^(5/2)-2/5*a*b/d/e
^3/(e*sin(d*x+c))^(1/2)-2/5*(3*a^2-2*b^2)*cos(d*x+c)/d/e^3/(e*sin(d*x+c))^(
1/2)+2/5*(3*a^2-2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi
+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)
/d/e^4/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2770, 2748, 2716, 2721, 2719}

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{2(3a^2-2b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}} - \frac{2(3a^2-2b^2)\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{2ab}{5de^3\sqrt{e\sin(c+dx)}} - \frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

```
[In] Int[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x]))/(5*d*e*(e*Sin[c + d*x])^(5/2
)) - (2*a*b)/(5*d*e^3*Sqrt[e*Sin[c + d*x]]) - (2*(3*a^2 - 2*b^2)*Cos[c + d*
x])/(5*d*e^3*Sqrt[e*Sin[c + d*x]]) - (2*(3*a^2 - 2*b^2)*EllipticE[(c - Pi/2
 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*e^4*Sqrt[Sin[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rubi steps

$$\text{integral} = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+b^2-\frac{1}{2}ab\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{5e^2}$$

$$
\begin{aligned}
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}} \\
&\quad - \frac{2ab}{5de^3\sqrt{e\sin(c+dx)}} + \frac{(3a^2-2b^2)\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{5e^2} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}} - \frac{2ab}{5de^3\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2(3a^2-2b^2)\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{(3a^2-2b^2)\int \sqrt{e\sin(c+dx)}\,dx}{5e^4} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}} - \frac{2ab}{5de^3\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2(3a^2-2b^2)\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{\left((3a^2-2b^2)\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{5e^4\sqrt{\sin(c+dx)}} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}} - \frac{2ab}{5de^3\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2(3a^2-2b^2)\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{2(3a^2-2b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.66

$$
\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{8ab+(7a^2+2b^2)\cos(c+dx)-3a^2\cos(3(c+dx))+2b^2\cos(3(c+dx))-4(3a^2-2b^2)E\left(\frac{1}{4}(-2c+\pi-2\right.}{10de(e\sin(c+dx))^{5/2}}
$$

```
[In] Integrate[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] -1/10*(8*a*b + (7*a^2 + 2*b^2)*Cos[c + d*x] - 3*a^2*Cos[3*(c + d*x)] + 2*b^
2*Cos[3*(c + d*x)] - 4*(3*a^2 - 2*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, 2]*
Sin[c + d*x]^(5/2))/(d*e*(e*Sin[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.13

method	result
default	$\frac{-\frac{4ab}{5e(e\sin(dx+c))^{\frac{5}{2}}}+\frac{6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2-4\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right)E}{}}{}$
parts	$\frac{a^2\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right)F\left(\sqrt{}\right.\right.}{5e^3\sin(dx+c)^3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-4/5*a*b/e/(e*sin(d*x+c))^(5/2)+1/5/e^3*(6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a
^2-4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE
((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+
2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1
-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+6*a^2*cos(d*x+c)^4*sin(d*x+c)-4*b^2*cos
(d*x+c)^4*sin(d*x+c)-8*a^2*cos(d*x+c)^2*sin(d*x+c)+2*b^2*cos(d*x+c)^2*sin(d
*x+c))/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.41

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \frac{\left(\sqrt{2}(-3i\,a^2+2i\,b^2)\cos{(dx+c)}^2+\sqrt{2}(3i\,a^2-2i\,b^2)\right)\sqrt{-i\,e}\sin{(dx+c)}\,\mathrm{weierst}}{}$$

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*((sqrt(2)*(-3*I*a^2 + 2*I*b^2)*cos(d*x + c)^2 + sqrt(2)*(3*I*a^2 - 2*I*
b^2))*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4,
0, cos(d*x + c) + I*sin(d*x + c))) + (sqrt(2)*(3*I*a^2 - 2*I*b^2)*cos(d*x +
 c)^2 + sqrt(2)*(-3*I*a^2 + 2*I*b^2))*sqrt(I*e)*sin(d*x + c)*weierstrassZet
a(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*((3*a
^2 - 2*b^2)*cos(d*x + c)^3 - 2*a*b - (4*a^2 - b^2)*cos(d*x + c))*sqrt(e*sin
(d*x + c)))/((d*e^4*cos(d*x + c)^2 - d*e^4)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^2}{(e\,\sin(c+d\,x))^{7/2}}\,dx$$

```
[In] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(7/2),x)
[Out] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(7/2), x)
```

3.49 $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 242

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx=\frac{10a(11a^2+6b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{231d\sqrt{e\sin(c+dx)}}-\frac{10a(11a^2+6b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d}-\frac{2a(11a^2+6b^2)\,e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d}+\frac{2b(177a^2+44b^2)\,(e\sin(c+dx))^{9/2}}{1287de}+\frac{34ab(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{143de}+\frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{9/2}}{13de}$$

```
[Out] -2/77*a*(11*a^2+6*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/d+2/1287*b*(177*a^
2+44*b^2)*(e*sin(d*x+c))^(9/2)/d/e+34/143*a*b*(a+b*cos(d*x+c))*(e*sin(d*x+c
))^(9/2)/d/e+2/13*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(9/2)/d/e-10/231*a*(1
1*a^2+6*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*sin(
d*x+c))^(1/2)-10/231*a*(11*a^2+6*b^2)*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2771, 2941, 2748, 2715, 2721, 2720}

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx=\frac{10ae^4(11a^2+6b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{231d\sqrt{e\sin(c+dx)}}-\frac{10ae^3(11a^2+6b^2)\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d}+\frac{2b(177a^2+44b^2)\,(e\sin(c+dx))^{9/2}}{1287de}-\frac{2ae(11a^2+6b^2)\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d}+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}+\frac{34ab(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{143de}$$

```
[In] Int[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (10*a*(11*a^2 + 6*b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*
x]])/(231*d*Sqrt[e*Sin[c + d*x]]) - (10*a*(11*a^2 + 6*b^2)*e^3*Cos[c + d*x]
*Sqrt[e*Sin[c + d*x]])/(231*d) - (2*a*(11*a^2 + 6*b^2)*e*Cos[c + d*x]*(e*Si
n[c + d*x])^(5/2))/(77*d) + (2*b*(177*a^2 + 44*b^2)*(e*Sin[c + d*x])^(9/2))
/(1287*d*e) + (34*a*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(9/2))/(143*d*e
) + (2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(9/2))/(13*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{9/2}}{13de} \\
&\quad + \frac{2}{13}\int (a+b\cos(c+dx))\left(\frac{13a^2}{2}+2b^2+\frac{17}{2}ab\cos(c+dx)\right)(e\sin(c+dx))^{7/2}\,dx \\
&= \frac{34ab(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{143de}+\frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{9/2}}{13de} \\
&\quad + \frac{4}{143}\int\left(\frac{13}{4}a\left(11a^2+6b^2\right)+\frac{1}{4}b\left(177a^2+44b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{7/2}\,dx \\
&= \frac{2b\left(177a^2+44b^2\right)(e\sin(c+dx))^{9/2}}{1287de}+\frac{34ab(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{143de} \\
&\quad + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{9/2}}{13de}+\frac{1}{11}\left(a\left(11a^2+6b^2\right)\right)\int (e\sin(c+dx))^{7/2}\,dx
\end{aligned}
$$

$$= -\frac{2a(11a^2+6b^2)\,e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d} + \frac{2b(177a^2+44b^2)\,(e\sin(c+dx))^{9/2}}{1287de} + \frac{34ab(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{143de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{9/2}}{13de} + \frac{1}{77}\left(5a\left(11a^2+6b^2\right)e^2\right)\int(e\sin(c+dx))^{3/2}\,dx$$

$$= -\frac{10a(11a^2+6b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d} - \frac{2a(11a^2+6b^2)\,e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d} + \frac{2b(177a^2+44b^2)\,(e\sin(c+dx))^{9/2}}{1287de} + \frac{34ab(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{143de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{9/2}}{13de} + \frac{1}{231}\left(5a\left(11a^2+6b^2\right)e^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx$$

$$= -\frac{10a(11a^2+6b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d} - \frac{2a(11a^2+6b^2)\,e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d} + \frac{2b(177a^2+44b^2)\,(e\sin(c+dx))^{9/2}}{1287de} + \frac{34ab(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{143de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{9/2}}{13de} + \frac{\left(5a(11a^2+6b^2)\,e^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{231\sqrt{e\sin(c+dx)}}$$

$$= \frac{10a(11a^2+6b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{231d\sqrt{e\sin(c+dx)}} - \frac{10a(11a^2+6b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d} - \frac{2a(11a^2+6b^2)\,e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d} + \frac{2b(177a^2+44b^2)\,(e\sin(c+dx))^{9/2}}{1287de} + \frac{34ab(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{143de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{9/2}}{13de}$$

Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.85

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \frac{\left(154b(78a^2+11b^2)\csc^3(c+dx)+\frac{1}{3}(-156a(506a^2+213b^2)\cos(c+dx)-77b(624a^2+73b^2)\cos\right.}{}$$

[In] Integrate[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2),x]

[Out] ((154*b*(78*a^2 + 11*b^2)*Csc[c + d*x]^3 + ((-156*a*(506*a^2 + 213*b^2)*Cos[c + d*x] - 77*b*(624*a^2 + 73*b^2)*Cos[2*(c + d*x)] + 234*a*(44*a^2 - 39*b^2)*Cos[3*(c + d*x)] - 154*b*(-78*a^2 + b^2)*Cos[4*(c + d*x)] + 4914*a*b^2*Cos[5*(c + d*x)] + 693*b^3*Cos[6*(c + d*x)])*Csc[c + d*x]^3)/3 - (2080*a*(11*a^2 + 6*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2])/Sin[c + d*x]^(7/2))*(e*Sin[c + d*x])^(7/2))/(48048*d)

Maple [A] (verified)

Time = 39.30 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.14

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{9}{2}}\left(9b^2\left(\cos^2(dx+c)\right)+39a^2+4b^2\right)}{117e}-\frac{e^4a\left(-126b^2\left(\cos^6(dx+c)\right)\sin(dx+c)-66a^2\left(\cos^4(dx+c)\right)\sin(dx+c)+216b^2\left(\cos^4(dx+c)\right)\sin(dx+\right.}{}}{}$
parts	$-\frac{a^3e^4\left(-6\left(\sin^5(dx+c)\right)+5\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-4\left(\sin^3(dx+c)\right)+10\sin(dx+c)\right)}{21\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

[In] int((a+cos(d*x+c)*b)^3*(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)

[Out] (2/117/e*b*(e*sin(d*x+c))^(9/2)*(9*b^2*cos(d*x+c)^2+39*a^2+4*b^2)-1/231*e^4*a*(-126*b^2*cos(d*x+c)^6*sin(d*x+c)-66*a^2*cos(d*x+c)^4*sin(d*x+c)+216*b^2*cos(d*x+c)^4*sin(d*x+c)+55*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+30*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+176*a^2*cos(d*x+c)^2*sin(d*x+c)-30*b^2*cos(d*x+c)^2*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \frac{195\sqrt{2}(11\,a^3+6\,ab^2)\sqrt{-i\,e}e^3\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+195\sqrt{}}{}$$

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
[Out] 1/9009*(195*sqrt(2)*(11*a^3 + 6*a*b^2)*sqrt(-I*e)*e^3*weierstrassPInverse(4
, 0, cos(d*x + c) + I*sin(d*x + c)) + 195*sqrt(2)*(11*a^3 + 6*a*b^2)*sqrt(I
*e)*e^3*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(693*b
^3*e^3*cos(d*x + c)^6 + 2457*a*b^2*e^3*cos(d*x + c)^5 + 77*(39*a^2*b - 14*b
^3)*e^3*cos(d*x + c)^4 + 117*(11*a^3 - 36*a*b^2)*e^3*cos(d*x + c)^3 - 77*(7
8*a^2*b - b^3)*e^3*cos(d*x + c)^2 - 39*(88*a^3 - 15*a*b^2)*e^3*cos(d*x + c)
 + 77*(39*a^2*b + 4*b^3)*e^3)*sqrt(e*sin(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(7/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \int (e\,\sin(c+d\,x))^{7/2}\,(a+b\,\cos(c+d\,x))^3\,dx$$

```
[In] int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3, x)
```

3.50 $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx=\frac{2a(3a^2+2b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2a(3a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{15d}+\frac{2b(43a^2+12b^2)\,(e\sin(c+dx))^{7/2}}{231de}+\frac{10ab(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{33de}+\frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}{11de}$$

```
[Out] -2/15*a*(3*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+2/231*b*(43*a^2+1
2*b^2)*(e*sin(d*x+c))^(7/2)/d/e+10/33*a*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(
7/2)/d/e+2/11*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2)/d/e-2/5*a*(3*a^2+2*
b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elli
pticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+c)^
(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

$= \{2771, 2941, 2748, 2715, 2721, 2719\}$

$$\int (a + b\cos(c + dx))^3 (e\sin(c + dx))^{5/2}\,dx = \frac{2ae^2(3a^2 + 2b^2)\,E\left(\frac{1}{2}\left(c + dx - \frac{\pi}{2}\right)\middle|\,2\right)\sqrt{e\sin(c + dx)}}{5d\sqrt{\sin(c + dx)}} + \frac{2b(43a^2 + 12b^2)\,(e\sin(c + dx))^{7/2}}{231de} - \frac{2ae(3a^2 + 2b^2)\cos(c + dx)(e\sin(c + dx))^{3/2}}{15d} + \frac{2b(e\sin(c + dx))^{7/2}(a + b\cos(c + dx))^2}{11de} + \frac{10ab(e\sin(c + dx))^{7/2}(a + b\cos(c + dx))}{33de}$$

```
[In] Int[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a*(3*a^2 + 2*b^2)*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*
x]])/(5*d*Sqrt[Sin[c + d*x]]) - (2*a*(3*a^2 + 2*b^2)*e*Cos[c + d*x]*(e*Sin[
c + d*x])^(3/2))/(15*d) + (2*b*(43*a^2 + 12*b^2)*(e*Sin[c + d*x])^(7/2))/(2
31*d*e) + (10*a*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2))/(33*d*e) + (
2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2))/(11*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}{11de} \\
&\quad + \frac{2}{11}\int (a+b\cos(c+dx))\left(\frac{11a^2}{2}+2b^2+\frac{15}{2}ab\cos(c+dx)\right)(e\sin(c+dx))^{5/2}\,dx \\
&= \frac{10ab(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{33de}+\frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}{11de} \\
&\quad + \frac{4}{99}\int \left(\frac{33}{4}a\left(3a^2+2b^2\right)+\frac{3}{4}b\left(43a^2+12b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{5/2}\,dx \\
&= \frac{2b\left(43a^2+12b^2\right)(e\sin(c+dx))^{7/2}}{231de}+\frac{10ab(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{33de} \\
&\quad + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}{11de}+\frac{1}{3}\left(a\left(3a^2+2b^2\right)\right)\int (e\sin(c+dx))^{5/2}\,dx \\
&= -\frac{2a\left(3a^2+2b^2\right)e\cos(c+dx)(e\sin(c+dx))^{3/2}}{15d} \\
&\quad + \frac{2b\left(43a^2+12b^2\right)(e\sin(c+dx))^{7/2}}{231de}+\frac{10ab(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{33de} \\
&\quad + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}{11de} \\
&\quad + \frac{1}{5}\left(a\left(3a^2+2b^2\right)e^2\right)\int \sqrt{e\sin(c+dx)}\,dx
\end{aligned}
$$

$$= -\frac{2a(3a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{15d} + \frac{2b(43a^2+12b^2)\,(e\sin(c+dx))^{7/2}}{231de} + \frac{10ab(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{33de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}{11de} + \frac{\left(a(3a^2+2b^2)\,e^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{5\sqrt{\sin(c+dx)}}$$

$$= \frac{2a(3a^2+2b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2a(3a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{15d} + \frac{2b(43a^2+12b^2)\,(e\sin(c+dx))^{7/2}}{231de} + \frac{10ab(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{33de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}{11de}$$

Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.74

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = -\frac{(e\sin(c+dx))^{5/2}\left(1848(3a^3+2ab^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+(462a(4a^2+b^2)\cos(c+dx)+5b(-396a^2\right.}{4620d\sin^{\frac{5}{2}}(c}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/4620*((e*Sin[c + d*x])^(5/2)*(1848*(3*a^3 + 2*a*b^2)*EllipticE[(-2*c + P
i - 2*d*x)/4, 2] + (462*a*(4*a^2 + b^2)*Cos[c + d*x] + 5*b*(-396*a^2 - 69*b
^2 + 12*(33*a^2 + 4*b^2)*Cos[2*(c + d*x)] + 154*a*b*Cos[3*(c + d*x)] + 21*b
^2*Cos[4*(c + d*x)]))*Sin[c + d*x]^(3/2)))/(d*Sin[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 40.31 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.76

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{7}{2}}\left(7b^2\left(\cos^2(dx+c)\right)+33a^2+4b^2\right)}{77e}-\frac{e^3a\left(10\left(\sin^6(dx+c)\right)b^2+18\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)\ldots\right)}{\ldots}}{\ldots}$
parts	$-\frac{a^3e^3\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\ldots\right)\right)}{5\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}\ldots$

```
[In] int((a+cos(d*x+c)*b)^3*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/77/e*b*(e*sin(d*x+c))^(7/2)*(7*b^2*cos(d*x+c)^2+33*a^2+4*b^2)-1/15*e^3*a
*(10*sin(d*x+c)^6*b^2+18*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+12*(1-sin(d*x+c)
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1
/2),1/2*2^(1/2))*b^2-9*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-6*(1-sin(d*x+c))^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2)
,1/2*2^(1/2))*b^2-6*a^2*sin(d*x+c)^4-14*sin(d*x+c)^4*b^2+6*a^2*sin(d*x+c)^2
+4*b^2*sin(d*x+c)^2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05

$$\int(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx=\frac{231i\sqrt{2}(3a^3+2ab^2)\sqrt{-i\,e}e^2\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\mathrm{s}\ldots}{\ldots}$$

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(231*I*sqrt(2)*(3*a^3 + 2*a*b^2)*sqrt(-I*e)*e^2*weierstrassZeta(4, 0
, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 231*I*sqrt(2)
*(3*a^3 + 2*a*b^2)*sqrt(I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(105*b^3*e^2*cos(d*x + c)^4 + 385
*a*b^2*e^2*cos(d*x + c)^3 + 45*(11*a^2*b - b^3)*e^2*cos(d*x + c)^2 + 231*(a
^3 - a*b^2)*e^2*cos(d*x + c) - 15*(33*a^2*b + 4*b^3)*e^2)*sqrt(e*sin(d*x +
c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \int (e\sin(c+d\,x))^{5/2}\,(a+b\cos(c+d\,x))^3\,dx$$

```
[In] int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3, x)
```

3.51 $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx=\frac{2a(7a^2+6b^2)\,e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}}-\frac{2a(7a^2+6b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d}+\frac{2b(89a^2+28b^2)\,(e\sin(c+dx))^{5/2}}{315de}+\frac{26ab(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{63de}+\frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}{9de}$$

```
[Out] 2/315*b*(89*a^2+28*b^2)*(e*sin(d*x+c))^(5/2)/d/e+26/63*a*b*(a+b*cos(d*x+c))
*(e*sin(d*x+c))^(5/2)/d/e+2/9*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2)/d/e
-2/21*a*(7*a^2+6*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/
d/(e*sin(d*x+c))^(1/2)-2/21*a*(7*a^2+6*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(1/
2)/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

$= \{2771, 2941, 2748, 2715, 2721, 2720\}$

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \frac{2ae^2(7a^2+6b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21d\sqrt{e\sin(c+dx)}} + \frac{2b(89a^2+28b^2)\,(e\sin(c+dx))^{5/2}}{315de} - \frac{2ae(7a^2+6b^2)\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} + \frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de} + \frac{26ab(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{63de}$$

```
[In] Int[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*a*(7*a^2 + 6*b^2)*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/(21*d*Sqrt[e*Sin[c + d*x]]) - (2*a*(7*a^2 + 6*b^2)*e*Cos[c + d*x]*Sqrt[e
*Sin[c + d*x]])/(21*d) + (2*b*(89*a^2 + 28*b^2)*(e*Sin[c + d*x])^(5/2))/(31
5*d*e) + (26*a*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(63*d*e) + (2
*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2))/(9*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}{9de} \\
&\quad + \frac{2}{9}\int(a+b\cos(c+dx))\left(\frac{9a^2}{2}+2b^2+\frac{13}{2}ab\cos(c+dx)\right)(e\sin(c+dx))^{3/2}\,dx \\
&= \frac{26ab(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{63de}+\frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}{9de} \\
&\quad + \frac{4}{63}\int\left(\frac{9}{4}a\left(7a^2+6b^2\right)+\frac{1}{4}b\left(89a^2+28b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}\,dx \\
&= \frac{2b\left(89a^2+28b^2\right)(e\sin(c+dx))^{5/2}}{315de}+\frac{26ab(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{63de} \\
&\quad + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}{9de}+\frac{1}{7}\left(a\left(7a^2+6b^2\right)\right)\int(e\sin(c+dx))^{3/2}\,dx \\
&= -\frac{2a\left(7a^2+6b^2\right)e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d}+\frac{2b\left(89a^2+28b^2\right)(e\sin(c+dx))^{5/2}}{315de} \\
&\quad + \frac{26ab(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{63de} \\
&\quad + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}{9de} \\
&\quad + \frac{1}{21}\left(a\left(7a^2+6b^2\right)e^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx
\end{aligned}$$

$$= -\frac{2a(7a^2+6b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} + \frac{2b(89a^2+28b^2)\,(e\sin(c+dx))^{5/2}}{315de}$$
$$+ \frac{26ab(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{63de}$$
$$+ \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}{9de}$$
$$+ \frac{\left(a(7a^2+6b^2)\,e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{21\sqrt{e\sin(c+dx)}}$$
$$= \frac{2a(7a^2+6b^2)\,e^2\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}}$$
$$- \frac{2a(7a^2+6b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d}$$
$$+ \frac{2b(89a^2+28b^2)\,(e\sin(c+dx))^{5/2}}{315de} + \frac{26ab(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{63de}$$
$$+ \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}{9de}$$

Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \frac{\left(-20a(28a^2+15b^2)\cot(c+dx)-\frac{2}{3}b(-756a^2-147b^2+28(27a^2+4b^2)\cos(2(c+dx))+270a\right.}{}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((-20*a*(28*a^2 + 15*b^2)*Cot[c + d*x] - (2*b*(-756*a^2 - 147*b^2 + 28*(27*
a^2 + 4*b^2)*Cos[2*(c + d*x)] + 270*a*b*Cos[3*(c + d*x)] + 35*b^2*Cos[4*(c
+ d*x)])*Csc[c + d*x])/3 - (80*a*(7*a^2 + 6*b^2)*EllipticF[(-2*c + Pi - 2*d
*x)/4, 2])/Sin[c + d*x]^(3/2))*(e*Sin[c + d*x])^(3/2))/(840*d)
```

Maple [A] (verified)

Time = 3.65 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.35

method	result
parts	$-\frac{a^{3}e^{2}\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\left(\sin^{3}(dx+c)\right)+2\sin(dx+c)\right)}{3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b^{3}\left(\frac{(e\sin(dx+}{9}\right.}{}$
default	$-\frac{e^{2}\left(70b^{3}\left(\cos^{5}(dx+c)\right)\sin(dx+c)+270a\,b^{2}\left(\cos^{4}(dx+c)\right)\sin(dx+c)+105\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\mathrm{si}\right.\right.}{}$

```
[In] int((a+cos(d*x+c)*b)^3*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^3*e^2*((1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*
EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^3+2*sin(d*x+c))/co
s(d*x+c)/(e*sin(d*x+c))^(1/2)/d-2*b^3/d/e^3*(1/9*(e*sin(d*x+c))^(9/2)-1/5*e
^2*(e*sin(d*x+c))^(5/2))+6/5*a^2*b*(e*sin(d*x+c))^(5/2)/e/d-2/7*a*b^2*e^2*(
3*sin(d*x+c)^5+(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-5*sin(d*x+c)^3+2*sin(d*x+c))/c
os(d*x+c)/(e*sin(d*x+c))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \frac{15\sqrt{2}(7a^3+6ab^2)\sqrt{-i\,e}e\,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+15\sqrt{2}(7a}{}$$

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(15*sqrt(2)*(7*a^3 + 6*a*b^2)*sqrt(-I*e)*e*weierstrassPInverse(4, 0,
cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(7*a^3 + 6*a*b^2)*sqrt(I*e)*e*w
eierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(35*b^3*e*cos(d
*x + c)^4 + 135*a*b^2*e*cos(d*x + c)^3 + 7*(27*a^2*b - b^3)*e*cos(d*x + c)^
2 + 15*(7*a^3 - 3*a*b^2)*e*cos(d*x + c) - 7*(27*a^2*b + 4*b^3)*e)*sqrt(e*si
n(d*x + c)))/d
```

Sympy [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \int (e\sin{(c+dx)})^{\frac{3}{2}}\,(a+b\cos{(c+dx)})^3\,\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(3/2),x)
[Out] Integral((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x))**3, x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \int (b\cos{(dx+c)}+a)^3(e\sin{(dx+c)})^{\frac{3}{2}}\,\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \int (b\cos{(dx+c)}+a)^3(e\sin{(dx+c)})^{\frac{3}{2}}\,\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \int {\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3\,dx$$

```
[In] int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3, x)
```

3.52 $\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx = \frac{2a(5a^2+6b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}+\frac{2b(57a^2+20b^2)\,(e\sin(c+dx))^{3/2}}{105de}+\frac{22ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{35de}+\frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}{7de}$$

```
[Out] 2/105*b*(57*a^2+20*b^2)*(e*sin(d*x+c))^(3/2)/d/e+22/35*a*b*(a+b*cos(d*x+c))
*(e*sin(d*x+c))^(3/2)/d/e+2/7*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2)/d/e
-2/5*a*(5*a^2+6*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d
/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2771, 2941, 2748, 2721, 2719}

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx = \frac{2b(57a^2+20b^2)\,(e\sin(c+dx))^{3/2}}{105de}+\frac{2a(5a^2+6b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}+\frac{22ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{35de}$$

```
[In] Int[(a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*a*(5*a^2 + 6*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])
/(5*d*Sqrt[Sin[c + d*x]]) + (2*b*(57*a^2 + 20*b^2)*(e*Sin[c + d*x])^(3/2))/
(105*d*e) + (22*a*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(35*d*e) +
 (2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2))/(7*d*e)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\text{integral} = \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}{7de} + \frac{2}{7}\int (a+b\cos(c+dx))\left(\frac{7a^2}{2}+2b^2+\frac{11}{2}ab\cos(c+dx)\right)\sqrt{e\sin(c+dx)}\,dx$$

$$= \frac{22ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{35de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}{7de} + \frac{4}{35}\int \left(\frac{7}{4}a\left(5a^2+6b^2\right)+\frac{1}{4}b\left(57a^2+20b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}\,dx$$

$$= \frac{2b\left(57a^2+20b^2\right)(e\sin(c+dx))^{3/2}}{105de} + \frac{22ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{35de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}{7de} + \frac{1}{5}\left(a\left(5a^2+6b^2\right)\right)\int \sqrt{e\sin(c+dx)}\,dx$$

$$= \frac{2b\left(57a^2+20b^2\right)(e\sin(c+dx))^{3/2}}{105de} + \frac{22ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{35de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}{7de} + \frac{\left(a(5a^2+6b^2)\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{5\sqrt{\sin(c+dx)}}$$

$$= \frac{2a(5a^2+6b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} + \frac{2b\left(57a^2+20b^2\right)(e\sin(c+dx))^{3/2}}{105de} + \frac{22ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{35de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}{7de}$$

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.65

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx = \frac{\sqrt{e\sin(c+dx)}\left(-42(5a^3+6ab^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+b(210a^2+55b^2+126ab\cos(c+dx)+15b^2\right.}{105d\sqrt{\sin(c+dx)}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[e*Sin[c + d*x]]*(-42*(5*a^3 + 6*a*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/
4, 2] + b*(210*a^2 + 55*b^2 + 126*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*x)
])*Sin[c + d*x]^(3/2)))/(105*d*Sqrt[Sin[c + d*x]])
```

Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.95

method	result
parts	$-\frac{a^{3} e \sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\left(2E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b^{3}\left(\frac{(e\sin(dx+c))^{\frac{7}{2}}}{7}-\right.}{d e}$
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{3}{2}}\left(3b^{2}\left(\cos^{2}(dx+c)\right)+21a^{2}+4b^{2}\right)}{21e}-\frac{ae\left(10\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^{2}+12\sqrt{1-\sin(dx+c}\right.}{}}{}$

```
[In] int((a+cos(d*x+c)*b)^3*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*e*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*(2*Elli
pticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-sin(d*x+c))^(1/2),1/2*
2^(1/2)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d-2*b^3/d/e^3*(1/7*(e*sin(d*x+c))
^(7/2)-1/3*e^2*(e*sin(d*x+c))^(3/2))-6/5*a*b^2*e*(2*(1-sin(d*x+c))^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^
(1/2))-(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellipti
cF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+cos(d*x+c)^4-cos(d*x+c)^2)/cos(d*x+c)/
(e*sin(d*x+c))^(1/2)/d+2*a^2*b*(e*sin(d*x+c))^(3/2)/e/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx = -\frac{21\sqrt{2}(-5i\,a^3-6i\,ab^2)\sqrt{-i\,e}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))}{}$$

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/105*(21*sqrt(2)*(-5*I*a^3 - 6*I*a*b^2)*sqrt(-I*e)*weierstrassZeta(4, 0,
weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(5*I
*a^3 + 6*I*a*b^2)*sqrt(I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - 2*(15*b^3*cos(d*x + c)^2 + 63*a*b^2*cos(
d*x + c) + 105*a^2*b + 20*b^3)*sqrt(e*sin(d*x + c))*sin(d*x + c))/d
```

Sympy [F]

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx = \int \sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^3\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))**3, x)
```

Maxima [F]

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx = \int (b\cos(dx+c)+a)^3\sqrt{e\sin(dx+c)}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^3*sqrt(e*sin(d*x + c)), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx = \int (b\cos(dx+c)+a)^3\sqrt{e\sin(dx+c)}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3*sqrt(e*sin(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx = \int \sqrt{e\,\sin(c+d\,x)}\,(a+b\,\cos(c+d\,x))^3\,dx$$

```
[In] int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3, x)
```

3.53 $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2a(a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{5de} + \frac{6ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{5de} + \frac{2b(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}{5de}$$

```
[Out] -2*a*(a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*sin(
d*x+c))^(1/2)+2/5*b*(11*a^2+4*b^2)*(e*sin(d*x+c))^(1/2)/d/e+6/5*a*b*(a+b*co
s(d*x+c))*(e*sin(d*x+c))^(1/2)/d/e+2/5*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^
(1/2)/d/e
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

= {2771, 2941, 2748, 2721, 2720}

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{5de} + \frac{2a(a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}} + \frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de} + \frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{5de}$$

```
[In] Int[(a + b*Cos[c + d*x])^3/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*a*(a^2 + 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*
Sqrt[e*Sin[c + d*x]]) + (2*b*(11*a^2 + 4*b^2)*Sqrt[e*Sin[c + d*x]])/(5*d*e)
 + (6*a*b*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(5*d*e) + (2*b*(a + b*
Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]])/(5*d*e)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}{5de} \\
&\quad + \frac{2}{5}\int \frac{(a+b\cos(c+dx))\left(\frac{5a^2}{2}+2b^2+\frac{9}{2}ab\cos(c+dx)\right)}{\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{6ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{5de} + \frac{2b(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}{5de} \\
&\quad + \frac{4}{15}\int \frac{\frac{15}{4}a(a^2+2b^2)+\frac{3}{4}b(11a^2+4b^2)\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{5de} + \frac{6ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{5de} \\
&\quad + \frac{2b(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}{5de} + \left(a(a^2+2b^2)\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{5de} + \frac{6ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{5de} \\
&\quad + \frac{2b(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}{5de} \\
&\quad + \frac{\left(a(a^2+2b^2)\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{\sqrt{e\sin(c+dx)}} \\
&= \frac{2a(a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{5de} \\
&\quad + \frac{6ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{5de} + \frac{2b(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}{5de}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.62

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{-10a(a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sqrt{\sin(c+dx)}+b(30a^2+9b^2+10ab\cos(c+dx)+b^2\cos}{5d\sqrt{e\sin(c+dx)}}$$

[In] Integrate[(a + b*Cos[c + d*x])^3/Sqrt[e*Sin[c + d*x]],x]

[Out] (-10*a*(a^2 + 2*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]
 + b*(30*a^2 + 9*b^2 + 10*a*b*Cos[c + d*x] + b^2*Cos[2*(c + d*x)])*Sin[c +
d*x])/(5*d*Sqrt[e*Sin[c + d*x]])

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.34

method	result
default	$-\frac{5\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^3+10\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F}{5\cos(d}$
parts	$-\frac{a^3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b^3\left(\frac{(e\sin(dx+c))^{\frac{5}{2}}}{5}-\sqrt{e\sin(dx+c)}\,e^2\right)}{d\,e^3}+\frac{3a\,b^2}{}$

[In] int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(5*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^3+1
0*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1
-sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b^2-2*b^3*cos(d*x+c)^3*sin(d*x+c)-10*a*b^
2*cos(d*x+c)^2*sin(d*x+c)-30*a^2*b*cos(d*x+c)*sin(d*x+c)-8*b^3*cos(d*x+c)*s
in(d*x+c))/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \frac{5\sqrt{2}(a^3+2\,ab^2)\sqrt{-i\,e}\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(a^3+2\,ab^2)\sqrt{i\,e}\text{weiers}}{}$$

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 1/5*(5*sqrt(2)*(a^3 + 2*a*b^2)*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x
 + c) + I*sin(d*x + c)) + 5*sqrt(2)*(a^3 + 2*a*b^2)*sqrt(I*e)*weierstrassPI
nverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(b^3*cos(d*x + c)^2 + 5*a*b
^2*cos(d*x + c) + 15*a^2*b + 4*b^3)*sqrt(e*sin(d*x + c)))/(d*e)
```

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(1/2),x)
[Out] Integral((a + b*cos(c + d*x))**3/sqrt(e*sin(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\sqrt{e\sin(dx+c)}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(e*sin(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\sqrt{e\sin(dx+c)}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(e*sin(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^3}{\sqrt{e\,\sin(c+d\,x)}}\,dx$$

```
[In] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(1/2),x)
[Out] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(1/2), x)
```

3.54 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a(a^2+6b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} - \frac{2b(3a^2+4b^2)\,(e\sin(c+dx))^{3/2}}{3de^3} - \frac{2ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{de^3}$$

```
[Out] -2/3*b*(3*a^2+4*b^2)*(e*sin(d*x+c))^(3/2)/d/e^3-2*a*b*(a+b*cos(d*x+c))*(e*s
in(d*x+c))^(3/2)/d/e^3-2*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x
+c))^(1/2)+2*a*(a^2+6*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/
4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(
1/2)/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2941, 2748, 2721, 2719}

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2b(3a^2+4b^2)\,(e\sin(c+dx))^{3/2}}{3de^3} - \frac{2a(a^2+6b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} - \frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de^3} - \frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

```
[In] Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(d*e*Sqrt[e*Sin[c + d*x]])
 - (2*a*(a^2 + 6*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]]
)/(d*e^2*Sqrt[Sin[c + d*x]]) - (2*b*(3*a^2 + 4*b^2)*(e*Sin[c + d*x])^(3/2))
/(3*d*e^3) - (2*a*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(d*e^3)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\text{integral} = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}} - \frac{2\int(a+b\cos(c+dx))\left(\frac{a^2}{2}+2b^2+\frac{5}{2}ab\cos(c+dx)\right)\sqrt{e\sin(c+dx)}\,dx}{e^2}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}} - \frac{2ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{de^3} - \frac{4\int\left(\frac{5}{4}a(a^2+6b^2)+\frac{5}{4}b(3a^2+4b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}\,dx}{5e^2}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}} - \frac{2b(3a^2+4b^2)(e\sin(c+dx))^{3/2}}{3de^3} - \frac{2ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{de^3} - \frac{(a(a^2+6b^2))\int\sqrt{e\sin(c+dx)}\,dx}{e^2}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}} - \frac{2b(3a^2+4b^2)(e\sin(c+dx))^{3/2}}{3de^3} - \frac{2ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{de^3} - \frac{\left(a(a^2+6b^2)\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{e^2\sqrt{\sin(c+dx)}}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a(a^2+6b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} - \frac{2b(3a^2+4b^2)(e\sin(c+dx))^{3/2}}{3de^3} - \frac{2ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{de^3}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.61

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\Big(9a^2b+3b^3+3a(a^2+3b^2)\cos(c+dx)-3a(a^2+6b^2)\,E\big(\tfrac{1}{4}(-2c+\pi-2dx)\big|\,2\big)\sqrt{\sin(c+dx)}+b^3\sin^2(}{3de\sqrt{e\sin(c+dx)}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(9*a^2*b + 3*b^3 + 3*a*(a^2 + 3*b^2)*Cos[c + d*x] - 3*a*(a^2 + 6*b^2)*E
llipticE[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]] + b^3*Sin[c + d*x]^2)
)/(3*d*e*Sqrt[e*Sin[c + d*x]])
```

Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.90

method	result
default	$-\frac{3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^3+18\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F}{}$
parts	$\frac{a^3\left(2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{}\right.\right.}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/e/(e*sin(d*x+c))^(1/2)/cos(d*x+c)*(3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^3
+18*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b^2-6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^3
-36*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b^2-2*b^3*cos(d*x+c)^3+6*a^3*cos(d*x+c)
^2+18*a*b^2*cos(d*x+c)^2+18*a^2*b*cos(d*x+c)+8*b^3*cos(d*x+c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{3\,\sqrt{2}(i\,a^3+6i\,ab^2)\sqrt{-i\,e}\sin\left(dx+c\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx\right.$$

[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")

[Out] -1/3*(3*sqrt(2)*(I*a^3 + 6*I*a*b^2)*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-I*a^3 - 6*I*a*b^2)*sqrt(I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(b^3*cos(d*x + c)^2 - 9*a^2*b - 4*b^3 - 3*(a^3 + 3*a*b^2)*cos(d*x + c))*sqrt(e*sin(d*x + c)))/(d*e^2*sin(d*x + c))

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(a+b\cos\left(c+dx\right))^3}{(e\sin\left(c+dx\right))^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(3/2),x)

[Out] Integral((a + b*cos(c + d*x))**3/(e*sin(c + d*x))**(3/2), x)

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(b\cos\left(dx+c\right)+a)^3}{(e\sin\left(dx+c\right))^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(3/2), x)

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^3}{(e\,\sin(c+d\,x))^{3/2}}\,dx$$

```
[In] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(3/2),x)
[Out] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(3/2), x)
```

3.55 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 169

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}} + \frac{2a(a^2-6b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} - \frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{3de^3} - \frac{2ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de^3}$$

```
[Out] -2/3*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*(a^
2-6*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elli
pticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^2/(e*sin(d*x+
c))^(1/2)-2/3*b*(a^2+4*b^2)*(e*sin(d*x+c))^(1/2)/d/e^3-2/3*a*b*(a+b*cos(d*x
+c))*(e*sin(d*x+c))^(1/2)/d/e^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2770, 2941, 2748, 2721, 2720}

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{3de^3} + \frac{2a(a^2-6b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\sqrt{e\sin(c+dx)}} - \frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de^3} - \frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

```
[In] Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(3*d*e*(e*Sin[c + d*x])^(3
/2)) + (2*a*(a^2 - 6*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x
]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]]) - (2*b*(a^2 + 4*b^2)*Sqrt[e*Sin[c + d*x]
])/(3*d*e^3) - (2*a*b*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*d*e^3)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&& !LtQ[p, -1] && IntegerQ[2*m] && !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&=-\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}-\frac{2\int\frac{(a+b\cos(c+dx))\left(-\frac{a^2}{2}+2b^2+\frac{3}{2}ab\cos(c+dx)\right)}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2}\\
&=-\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}\\
&\quad-\frac{2ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de^3}-\frac{4\int\frac{-\frac{3}{4}a(a^2-6b^2)+\frac{3}{4}b(a^2+4b^2)\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{9e^2}\\
&=-\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{3de^3}\\
&\quad-\frac{2ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de^3}+\frac{(a(a^2-6b^2))\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2}\\
&=-\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{3de^3}\\
&\quad-\frac{2ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de^3}+\frac{\left(a(a^2-6b^2)\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3e^2\sqrt{e\sin(c+dx)}}\\
&=-\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}\\
&\quad+\frac{2a(a^2-6b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}}\\
&\quad-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{3de^3}-\frac{2ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.60

$$\int\frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx=$$
$$-\frac{6a^2b+5b^3+2a(a^2+3b^2)\cos(c+dx)-3b^3\cos(2(c+dx))+2a(a^2-6b^2)\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)}{3de(e\sin(c+dx))^{3/2}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/3*(6*a^2*b + 5*b^3 + 2*a*(a^2 + 3*b^2)*Cos[c + d*x] - 3*b^3*Cos[2*(c + d
*x)] + 2*a*(a^2 - 6*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(
3/2))/(d*e*(e*Sin[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 3.77 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.34

method	result
default	$\frac{-\frac{2b\left(-3b^2\left(\cos^2(dx+c)\right)+3a^2+4b^2\right)}{3e(e\sin(dx+c))^{\frac{3}{2}}}-\frac{a\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{5}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2-6b^2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)}\right.}{3e^2\sin(dx+c)^2\cos(}}{d}$
parts	$-\frac{a^3\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{5}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\left(\sin^3(dx+c)\right)+2\sin(dx+c)\right)}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b^3\left(\sqrt{e\sin(dx+\right.}{}$

```
[In] int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/3*b/e/(e*sin(d*x+c))^(3/2)*(-3*b^2*cos(d*x+c)^2+3*a^2+4*b^2)-1/3*a/e^2*
((1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-
sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-6*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+2*a^
2*cos(d*x+c)^2*sin(d*x+c)+6*b^2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^2/cos(d
*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \frac{\left(\sqrt{2}(a^3-6\,ab^2)\cos{(dx+c)}^2-\sqrt{2}(a^3-6\,ab^2)\right)\sqrt{-i\,e}\text{weierstrassPInverse}(4,0}{}$$

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((sqrt(2)*(a^3 - 6*a*b^2)*cos(d*x + c)^2 - sqrt(2)*(a^3 - 6*a*b^2))*sqr
t(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)
*(a^3 - 6*a*b^2)*cos(d*x + c)^2 - sqrt(2)*(a^3 - 6*a*b^2))*sqrt(I*e)*weiers
trassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(3*b^3*cos(d*x + c)^
2 - 3*a^2*b - 4*b^3 - (a^3 + 3*a*b^2)*cos(d*x + c))*sqrt(e*sin(d*x + c)))/(
d*e^3*cos(d*x + c)^2 - d*e^3)
```

Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(a+b\cos{(c+dx)})^3}{(e\sin{(c+dx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(5/2),x)
[Out] Integral((a + b*cos(c + d*x))**3/(e*sin(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(b\cos{(dx+c)}+a)^3}{(e\sin{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(b\cos{(dx+c)}+a)^3}{(e\sin{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(a+b\,\cos{(c+d\,x)})^3}{(e\,\sin{(c+d\,x)})^{5/2}}\,dx$$

```
[In] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(5/2),x)
[Out] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(5/2), x)
```

3.56 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 192

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}} + \frac{2(a+b\cos(c+dx))\left(ab-(3a^2-4b^2)\cos(c+dx)\right)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{6a(a^2-2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}} - \frac{2b(3a^2-4b^2)\,(e\sin(c+dx))^{3/2}}{5de^5}$$

```
[Out] -2/5*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x+c))^(5/2)-2/5*b*(3*
a^2-4*b^2)*(e*sin(d*x+c))^(3/2)/d/e^5+2/5*(a+b*cos(d*x+c))*(a*b-(3*a^2-4*b^
2)*cos(d*x+c))/d/e^3/(e*sin(d*x+c))^(1/2)+6/5*a*(a^2-2*b^2)*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/e^4/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2770, 2940, 2748, 2721, 2719}

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{2b(3a^2-4b^2)\,(e\sin(c+dx))^{3/2}}{5de^5} - \frac{6a(a^2-2b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}} + \frac{2(ab-(3a^2-4b^2)\cos(c+dx))\,(a+b\cos(c+dx))}{5de^3\sqrt{e\sin(c+dx)}} - \frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

```
[In] Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(5*d*e*(e*Sin[c + d*x])^(5
/2)) + (2*(a + b*Cos[c + d*x])*(a*b - (3*a^2 - 4*b^2)*Cos[c + d*x]))/(5*d*e
^3*Sqrt[e*Sin[c + d*x]]) - (6*a*(a^2 - 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2,
 2]*Sqrt[e*Sin[c + d*x]])/(5*d*e^4*Sqrt[Sin[c + d*x]]) - (2*b*(3*a^2 - 4*b^
2)*(e*Sin[c + d*x])^(3/2))/(5*d*e^5)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
```

```
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}} \\
&\quad -\frac{2\int\frac{(a+b\cos(c+dx))\left(-\frac{3a^2}{2}+2b^2+\frac{1}{2}ab\cos(c+dx)\right)}{(e\sin(c+dx))^{3/2}}\,dx}{5e^2} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}} \\
&\quad +\frac{2(a+b\cos(c+dx))\left(ab-(3a^2-4b^2)\cos(c+dx)\right)}{5de^3\sqrt{e\sin(c+dx)}} \\
&\quad +\frac{4\int\left(-\frac{3}{4}a(a^2-2b^2)-\frac{3}{4}b(3a^2-4b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}\,dx}{5e^4} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}} \\
&\quad +\frac{2(a+b\cos(c+dx))\left(ab-(3a^2-4b^2)\cos(c+dx)\right)}{5de^3\sqrt{e\sin(c+dx)}} \\
&\quad -\frac{2b(3a^2-4b^2)\,(e\sin(c+dx))^{3/2}}{5de^5}-\frac{(3a(a^2-2b^2))\int\sqrt{e\sin(c+dx)}\,dx}{5e^4}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}} \\
&\quad + \frac{2(a+b\cos(c+dx))\left(ab-(3a^2-4b^2)\cos(c+dx)\right)}{5de^3\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2b(3a^2-4b^2)\left(e\sin(c+dx)\right)^{3/2}}{5de^5} \\
&\quad - \frac{\left(3a(a^2-2b^2)\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{5e^4\sqrt{\sin(c+dx)}} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}} \\
&\quad + \frac{2(a+b\cos(c+dx))\left(ab-(3a^2-4b^2)\cos(c+dx)\right)}{5de^3\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{6a(a^2-2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}} \\
&\quad - \frac{2b(3a^2-4b^2)\left(e\sin(c+dx)\right)^{3/2}}{5de^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.68

$$
\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx =
$$

$$
-\frac{12a^2b-6b^3+a(7a^2+6b^2)\cos(c+dx)+10b^3\cos(2(c+dx))-3a^3\cos(3(c+dx))+6ab^2\cos(3(c+dx))-}{10de(e\sin(c+dx))^{5/2}}
$$

```
[In] Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] -1/10*(12*a^2*b - 6*b^3 + a*(7*a^2 + 6*b^2)*Cos[c + d*x] + 10*b^3*Cos[2*(c
+ d*x)] - 3*a^3*Cos[3*(c + d*x)] + 6*a*b^2*Cos[3*(c + d*x)] - 12*a*(a^2 - 2
*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(5/2))/(d*e*(e*Sin[c
 + d*x])^(5/2))
```

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.95

method	result
default	$-\frac{2b\left(5b^2\left(\cos^2(dx+c)\right)+3a^2-4b^2\right)}{5e(e\sin(dx+c))^{\frac{5}{2}}}+\frac{a\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2-12\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)}\right.}{}$
parts	$\frac{a^3\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)F\left(\right.\right.}{5e^3\sin(dx+c)^3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/5*b/e/(e*sin(d*x+c))^(5/2)*(5*b^2*cos(d*x+c)^2+3*a^2-4*b^2)+1/5*a/e^3*(
6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1
-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-12*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-3*
(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1-s
in(d*x+c))^(1/2),1/2*2^(1/2))*a^2+6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+6*a^2
*cos(d*x+c)^4*sin(d*x+c)-12*b^2*cos(d*x+c)^4*sin(d*x+c)-8*a^2*cos(d*x+c)^2*
sin(d*x+c)+6*b^2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*
x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{3\left(\sqrt{2}(i\,a^3-2i\,ab^2)\cos(dx+c)^2+\sqrt{2}(-i\,a^3+2i\,ab^2)\right)\sqrt{-i\,e}\sin(dx+c)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstras}}{}$$

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*(sqrt(2)*(I*a^3 - 2*I*a*b^2)*cos(d*x + c)^2 + sqrt(2)*(-I*a^3 + 2*I
*a*b^2))*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(-I*a^3 + 2*I*a*b^2)*cos
(d*x + c)^2 + sqrt(2)*(I*a^3 - 2*I*a*b^2))*sqrt(I*e)*sin(d*x + c)*weierstra
ssZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*
(5*b^3*cos(d*x + c)^2 - 3*(a^3 - 2*a*b^2)*cos(d*x + c)^3 + 3*a^2*b - 4*b^3
+ (4*a^3 - 3*a*b^2)*cos(d*x + c))*sqrt(e*sin(d*x + c)))/((d*e^4*cos(d*x + c
)^2 - d*e^4)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^3}{(e\,\sin(c+d\,x))^{7/2}}\,dx$$

```
[In] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(7/2),x)
[Out] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(7/2), x)
```

3.57 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2(a+b\cos(c+dx))\left(ab+(5a^2-4b^2)\cos(c+dx)\right)}{21de^3(e\sin(c+dx))^{3/2}} + \frac{2a(5a^2-6b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21de^4\sqrt{e\sin(c+dx)}} - \frac{2b(5a^2-4b^2)\sqrt{e\sin(c+dx)}}{21de^5}$$

```
[Out] -2/7*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x+c))^(7/2)-2/21*(a+b
*cos(d*x+c))*(a*b+(5*a^2-4*b^2)*cos(d*x+c))/d/e^3/(e*sin(d*x+c))^(3/2)-2/21
*a*(5*a^2-6*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^4/(e*
sin(d*x+c))^(1/2)-2/21*b*(5*a^2-4*b^2)*(e*sin(d*x+c))^(1/2)/d/e^5
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{2770, 2940, 2748, 2721, 2720\}$

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = -\frac{2b(5a^2-4b^2)\sqrt{e\sin(c+dx)}}{21de^5} + \frac{2a(5a^2-6b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21de^4\sqrt{e\sin(c+dx)}} - \frac{2((5a^2-4b^2)\cos(c+dx)+ab)\,(a+b\cos(c+dx))}{21de^3(e\sin(c+dx))^{3/2}} - \frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

```
[In] Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(9/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(7*d*e*(e*Sin[c + d*x])^(7
/2)) - (2*(a + b*Cos[c + d*x])*(a*b + (5*a^2 - 4*b^2)*Cos[c + d*x]))/(21*d*
e^3*(e*Sin[c + d*x])^(3/2)) + (2*a*(5*a^2 - 6*b^2)*EllipticF[(c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*d*e^4*Sqrt[e*Sin[c + d*x]]) - (2*b*(5*a^2
- 4*b^2)*Sqrt[e*Sin[c + d*x]])/(21*d*e^5)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
```

```
] || IntegerQ[m])
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2\int \frac{(a+b\cos(c+dx))\left(-\frac{5a^2}{2}+2b^2-\frac{1}{2}ab\cos(c+dx)\right)}{(e\sin(c+dx))^{5/2}}\,dx}{7e^2} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2(a+b\cos(c+dx))\left(ab+(5a^2-4b^2)\cos(c+dx)\right)}{21de^3(e\sin(c+dx))^{3/2}} + \frac{4\int \frac{\frac{1}{4}a(5a^2-6b^2)-\frac{1}{4}b(5a^2-4b^2)\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{21e^4} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2(a+b\cos(c+dx))\left(ab+(5a^2-4b^2)\cos(c+dx)\right)}{21de^3(e\sin(c+dx))^{3/2}} - \frac{2b(5a^2-4b^2)\sqrt{e\sin(c+dx)}}{21de^5} + \frac{(a(5a^2-6b^2))\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{21e^4} \\
&= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2(a+b\cos(c+dx))\left(ab+(5a^2-4b^2)\cos(c+dx)\right)}{21de^3(e\sin(c+dx))^{3/2}} - \frac{2b(5a^2-4b^2)\sqrt{e\sin(c+dx)}}{21de^5} + \frac{\left(a(5a^2-6b^2)\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{21e^4\sqrt{e\sin(c+dx)}}
\end{aligned}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2(a+b\cos(c+dx))\left(ab+(5a^2-4b^2)\cos(c+dx)\right)}{21de^3(e\sin(c+dx))^{3/2}} + \frac{2a(5a^2-6b^2)\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21de^4\sqrt{e\sin(c+dx)}} - \frac{2b(5a^2-4b^2)\sqrt{e\sin(c+dx)}}{21de^5}$$

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.75

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = -\frac{2\csc^4(c+dx)\sqrt{e\sin(c+dx)}\left(\frac{1}{4}(36a^2b-2b^3+a(17a^2+30b^2)\cos(c+dx)+14b^3\cos(2(c+dx))-5a^3\cos(\right.}{21de^5}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(9/2),x]
```

```
[Out] (-2*Csc[c + d*x]^4*Sqrt[e*Sin[c + d*x]]*((36*a^2*b - 2*b^3 + a*(17*a^2 + 30
*b^2)*Cos[c + d*x] + 14*b^3*Cos[2*(c + d*x)] - 5*a^3*Cos[3*(c + d*x)] + 6*a
*b^2*Cos[3*(c + d*x)])/4 + a*(5*a^2 - 6*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/
4, 2]*Sin[c + d*x]^(7/2)))/(21*d*e^5)
```

Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.37

method	result
default	$\frac{-\frac{2b\left(7b^2\left(\cos^2(dx+c)\right)+9a^2-4b^2\right)}{21e(e\sin(dx+c))^{\frac{7}{2}}}-\frac{a\left(5\sqrt{1-\sin(dx+c)}\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{9}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2-6\sqrt{1-\sin(dx+c)}\sqrt{2\sin(dx+c)+}\right.}{}}{}$
parts	$-\frac{a^3\left(5\sqrt{1-\sin(dx+c)}\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{9}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-10\left(\sin^5(dx+c)\right)+4\left(\sin^3(dx+c)\right)+6\sin(dx+c)\right)}{21e^4\sin(dx+c)^4\cos(dx+c)\sqrt{e\sin(dx+c)}\,d} -$

```
[In] int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/21*b/e/(e*sin(d*x+c))^(7/2)*(7*b^2*cos(d*x+c)^2+9*a^2-4*b^2)-1/21*a/e^4
*(5*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(9/2)*EllipticF(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(9/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-1
0*a^2*cos(d*x+c)^4*sin(d*x+c)+12*b^2*cos(d*x+c)^4*sin(d*x+c)+16*a^2*cos(d*x
```

```
+c)^2*sin(d*x+c)+6*b^2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^4/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.53

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \frac{\left(\sqrt{2}(5\,a^3-6\,ab^2)\cos\left(dx+c\right)^4-2\,\sqrt{2}(5\,a^3-6\,ab^2)\cos\left(dx+c\right)^2+\sqrt{2}(5\,a^3-\right.}{}$$

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*((sqrt(2)*(5*a^3 - 6*a*b^2)*cos(d*x + c)^4 - 2*sqrt(2)*(5*a^3 - 6*a*b^
2)*cos(d*x + c)^2 + sqrt(2)*(5*a^3 - 6*a*b^2))*sqrt(-I*e)*weierstrassPInver
se(4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(5*a^3 - 6*a*b^2)*cos(d*
x + c)^4 - 2*sqrt(2)*(5*a^3 - 6*a*b^2)*cos(d*x + c)^2 + sqrt(2)*(5*a^3 - 6*
a*b^2))*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))
- 2*(7*b^3*cos(d*x + c)^2 - (5*a^3 - 6*a*b^2)*cos(d*x + c)^3 + 9*a^2*b - 4*
b^3 + (8*a^3 + 3*a*b^2)*cos(d*x + c))*sqrt(e*sin(d*x + c)))/(d*e^5*cos(d*x
+ c)^4 - 2*d*e^5*cos(d*x + c)^2 + d*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \int \frac{(b\cos\left(dx+c\right)+a)^3}{(e\sin\left(dx+c\right))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^3}{(e\,\sin(c+d\,x))^{9/2}}\,dx$$

```
[In] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(9/2),x)
[Out] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(9/2), x)
```

3.58 $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 544

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx &= \frac{(-a^2+b^2)^{9/4}e^{11/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{11/2}d}\\
&+\frac{(-a^2+b^2)^{9/4}e^{11/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{11/2}d}\\
&+\frac{2a(21a^4-49a^2b^2+33b^4)e^6\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21b^6d\sqrt{e\sin(c+dx)}}\\
&-\frac{a(a^2-b^2)^3e^6\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^6\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{a(a^2-b^2)^3e^6\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^6\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{2e^5\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21b^5d}\\
&+\frac{2e^3(7(a^2-b^2)-5ab\cos(c+dx))(e\sin(c+dx))^{5/2}}{35b^3d}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}\end{aligned}$$

```
[Out] (-a^2+b^2)^(9/4)*e^(11/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(11/2)/d+(-a^2+b^2)^(9/4)*e^(11/2)*arctanh(b^(1/2)*(e*sin(d*
x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/d+2/35*e^3*(7*a^2-7*b^2-5*a*
b*cos(d*x+c))*(e*sin(d*x+c))^(5/2)/b^3/d-2/9*e*(e*sin(d*x+c))^(9/2)/b/d-2/2
1*a*(21*a^4-49*a^2*b^2+33*b^4)*e^6*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(
1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+
```

```
c)^(1/2)/b^6/d/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)^3*e^6*(sin(1/2*c+1/4*Pi+1/2
*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*
x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^6/d/(a^2-b*(b-(-a^2
+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)^3*e^6*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x
),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^6/d/(a^2-b*(b+(-a^2+
b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-2/21*e^5*(21*(a^2-b^2)^2-a*b*(7*a^2-12*b^
2)*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^5/d
```

Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2774, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \frac{e^{11/2}(b^2-a^2)^{9/4}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}d} + \frac{e^{11/2}(b^2-a^2)^{9/4}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}d} - \frac{ae^6(a^2-b^2)^3\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{b^6d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}} - \frac{ae^6(a^2-b^2)^3\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{b^6d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}} - \frac{2e^5\sqrt{e\sin(c+dx)}\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)}{21b^5d} + \frac{2e^3(e\sin(c+dx))^{5/2}\left(7(a^2-b^2)-5ab\cos(c+dx)\right)}{35b^3d} + \frac{2ae^6(21a^4-49a^2b^2+33b^4)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21b^6d\sqrt{e\sin(c+dx)}} - \frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

```
[In] Int[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-a^2 + b^2)^(9/4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 +
 b^2)^(1/4)*Sqrt[e])])/(b^(11/2)*d) + ((-a^2 + b^2)^(9/4)*e^(11/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(11/2)*d)
+ (2*a*(21*a^4 - 49*a^2*b^2 + 33*b^4)*e^6*EllipticF[(c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(21*b^6*d*Sqrt[e*Sin[c + d*x]]) - (a*(a^2 - b^2)^3*e^6*
```

```
EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (a
*(a^2 - b^2)^3*e^6*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(b^6*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*
Sin[c + d*x]]) - (2*e^5*(21*(a^2 - b^2)^2 - a*b*(7*a^2 - 12*b^2)*Cos[c + d*
x])*Sqrt[e*Sin[c + d*x]])/(21*b^5*d) + (2*e^3*(7*(a^2 - b^2) - 5*a*b*Cos[c
+ d*x])*(e*Sin[c + d*x])^(5/2))/(35*b^3*d) - (2*e*(e*Sin[c + d*x])^(9/2))/(
9*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
```

```
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
```

```
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2e(e\sin(c+dx))^{9/2}}{9bd} - \frac{e^2 \int \frac{(-b-a\cos(c+dx))(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx}{b} \\
&= \frac{2e^3(7(a^2-b^2)-5ab\cos(c+dx))\,(e\sin(c+dx))^{5/2}}{35b^3d} - \frac{2e(e\sin(c+dx))^{9/2}}{9bd} \\
&\quad - \frac{(2e^4)\int \frac{\left(\frac{1}{2}b(2a^2-7b^2)+\frac{1}{2}a(7a^2-12b^2)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx}{7b^3} \\
&= -\frac{2e^5\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21b^5d} \\
&\quad + \frac{2e^3(7(a^2-b^2)-5ab\cos(c+dx))\,(e\sin(c+dx))^{5/2}}{35b^3d} - \frac{2e(e\sin(c+dx))^{9/2}}{9bd} \\
&\quad - \frac{(4e^6)\int \frac{-\frac{1}{4}b(14a^4-30a^2b^2+21b^4)-\frac{1}{4}a(21a^4-49a^2b^2+33b^4)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{21b^5} \\
&= -\frac{2e^5\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21b^5d} \\
&\quad + \frac{2e^3(7(a^2-b^2)-5ab\cos(c+dx))\,(e\sin(c+dx))^{5/2}}{35b^3d} \\
&\quad - \frac{2e(e\sin(c+dx))^{9/2}}{9bd} - \frac{\left((a^2-b^2)^3e^6\right)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{b^6} \\
&\quad + \frac{(a(21a^4-49a^2b^2+33b^4)\,e^6)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{21b^6}
\end{aligned}$$

$$= -\frac{2e^5\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21b^5d} + \frac{2e^3(7(a^2-b^2)-5ab\cos(c+dx))\,(e\sin(c+dx))^{5/2}}{35b^3d} - \frac{2e(e\sin(c+dx))^{9/2}}{9bd} - \frac{\left(a(-a^2+b^2)^{5/2}\,e^6\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2b^6} - \frac{\left(a(-a^2+b^2)^{5/2}\,e^6\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2b^6} + \frac{\left((a^2-b^2)^3\,e^7\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx,x,e\sin(c+dx)\right)}{b^5d} + \frac{\left(a(21a^4-49a^2b^2+33b^4)\,e^6\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{21b^6\sqrt{e\sin(c+dx)}}$$

$$= \frac{2a(21a^4-49a^2b^2+33b^4)\,e^6\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21b^6d\sqrt{e\sin(c+dx)}} - \frac{2e^5\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21b^5d} + \frac{2e^3(7(a^2-b^2)-5ab\cos(c+dx))\,(e\sin(c+dx))^{5/2}}{35b^3d} - \frac{2e(e\sin(c+dx))^{9/2}}{9bd} + \frac{\left(2(a^2-b^2)^3\,e^7\right)\operatorname{Subst}\left(\int\frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{b^5d} - \frac{\left(a(-a^2+b^2)^{5/2}\,e^6\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2b^6\sqrt{e\sin(c+dx)}} - \frac{\left(a(-a^2+b^2)^{5/2}\,e^6\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2b^6\sqrt{e\sin(c+dx)}}$$

$$= \frac{2a(21a^4-49a^2b^2+33b^4)\, e^6 \,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21b^6 d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{a(-a^2+b^2)^{5/2}\, e^6 \,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^6\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{a(-a^2+b^2)^{5/2}\, e^6 \,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^6\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{2e^5\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21b^5 d}$$
$$+ \frac{2e^3(7(a^2-b^2)-5ab\cos(c+dx))\,(e\sin(c+dx))^{5/2}}{35b^3 d} - \frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$
$$+ \frac{\left((-a^2+b^2)^{5/2}\, e^6\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{b^5 d}$$
$$+ \frac{\left((-a^2+b^2)^{5/2}\, e^6\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{b^5 d}$$

$$= \frac{(-a^2+b^2)^{9/4}\, e^{11/2} \arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{11/2} d}$$
$$+ \frac{(-a^2+b^2)^{9/4}\, e^{11/2} \text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{11/2} d}$$
$$+ \frac{2a(21a^4-49a^2b^2+33b^4)\, e^6 \,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21b^6 d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{a(-a^2+b^2)^{5/2}\, e^6 \,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^6\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{a(-a^2+b^2)^{5/2}\, e^6 \,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^6\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{2e^5\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21b^5 d}$$
$$+ \frac{2e^3(7(a^2-b^2)-5ab\cos(c+dx))\,(e\sin(c+dx))^{5/2}}{35b^3 d} - \frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 48.77 (sec) , antiderivative size = 2035, normalized size of antiderivative = 3.74

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (((a*(28*a^2 - 51*b^2)*Cos[c + d*x])/(42*b^4) + ((-9*a^2 + 14*b^2)*Cos[2*(c
 + d*x)])/(45*b^3) + (a*Cos[3*(c + d*x)])/(14*b^2) - Cos[4*(c + d*x)]/(36*b
))*Csc[c + d*x]^5*(e*Sin[c + d*x])^(11/2))/d - ((e*Sin[c + d*x])^(11/2)*((2
*(392*a^3*b - 722*a*b^3)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((
a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] +
2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[
Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*
Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqr
t[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) +
 (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c +
d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^
2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(
-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Si
n[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c
+ d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-
1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(-
280*a^4 + 636*a^2*b^2 - 721*b^4)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^
2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]
])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/
(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(
1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2
 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2,
 (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d
*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c
 + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]
^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1,
 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(
a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c +
d*x]^2]) + ((840*a^4 - 1764*a^2*b^2 + 959*b^4)*Cos[c + d*x]*Cos[2*(c + d*x)
]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 -
((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 +
b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Sin[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + ((1/4 -
I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
```

```
4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - (
(1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/
4)) + (4*Sqrt[Sin[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)
) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c
 + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/
(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Si
n[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c
 + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(
-1 + Sin[c + d*x]^2))))))/((a + b*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[
1 - Sin[c + d*x]^2])))/(1680*b^4*d*Sin[c + d*x]^(11/2))
```

Maple [A] (warning: unable to verify)

Time = 6.22 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.71

method	result	size
default	Expression too large to display	930

```
[In] int((e*sin(d*x+c))^(11/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*e*b*(1/45/b^6*(e*sin(d*x+c))^(1/2)*e^4*(5*b^4*cos(d*x+c)^4+9*a^2*b^2*co
s(d*x+c)^2-19*b^4*cos(d*x+c)^2+45*a^4-99*a^2*b^2+59*b^4)-1/8*e^6*(a^6-3*a^4
*b^2+3*a^2*b^4-b^6)/b^6*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)
*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(
e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*
x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-
b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2
)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^6*a*(
-1/21/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(-6*b^4*cos(d*x+c)^4*sin(d*x+c)
+21*a^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellipt
icF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-49*a^2*b^2*(1-sin(d*x+c))^(1/2)*(2*si
n(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/
2))+33*b^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ell
ipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-14*a^2*b^2*cos(d*x+c)^2*sin(d*x+c)
+30*b^4*cos(d*x+c)^2*sin(d*x+c))+(-a^6+3*a^4*b^2-3*a^2*b^4+b^6)/b^6*(-1/2/(
-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1
/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-
sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1/2
)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c
)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1
/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)
)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(11/2)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{b\cos(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{b\cos(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx=\int \frac{\left(e\,\sin\left(c+d\,x\right)\right)^{11/2}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

```
[In] int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x)), x)
```

3.59 $\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 461

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx &= -\frac{(-a^2+b^2)^{7/4}\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{9/2}d} \\
&+ \frac{(-a^2+b^2)^{7/4}\,e^{9/2}\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{9/2}d} \\
&+ \frac{a(a^2-b^2)^2\,e^5\,\mathrm{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{a(a^2-b^2)^2\,e^5\,\mathrm{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^5\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{2a(5a^2-8b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}} \\
&+ \frac{2e^3(5(a^2-b^2)-3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15b^3d} - \frac{2e(e\sin(c+dx))^{7/2}}{7bd}
\end{aligned}
$$

```
[Out] -(-a^2+b^2)^(7/4)*e^(9/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(9/2)/d+(-a^2+b^2)^(7/4)*e^(9/2)*arctanh(b^(1/2)*(e*sin(d*x+
c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/d+2/15*e^3*(5*a^2-5*b^2-3*a*b*c
os(d*x+c))*(e*sin(d*x+c))^(3/2)/b^3/d-2/7*e*(e*sin(d*x+c))^(7/2)/b/d-a*(a^2
-b^2)^2*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d
*x+c)^(1/2)/b^5/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-a*(a^2-b^2)^2*e
^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi
(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/
```

```
2)/b^5/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2/5*a*(5*a^2-8*b^2)*e^4*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^4/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2774, 2944, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = -\frac{e^{9/2}(b^2-a^2)^{7/4}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}d} + \frac{e^{9/2}(b^2-a^2)^{7/4}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}d} + \frac{ae^5(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{b^5d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}} + \frac{ae^5(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{b^5d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}} - \frac{2ae^4(5a^2-8b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}} + \frac{2e^3(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^3d} - \frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

```
[In] Int[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -(((-a^2 + b^2)^(7/4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(b^(9/2)*d)) + ((-a^2 + b^2)^(7/4)*e^(9/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(9/2)*d) +
 (a*(a^2 - b^2)^2*e^5*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^5*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c +
 d*x]]) + (a*(a^2 - b^2)^2*e^5*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c
- Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^5*(b + Sqrt[-a^2 + b^2])*d*Sqrt[
e*Sin[c + d*x]]) - (2*a*(5*a^2 - 8*b^2)*e^4*EllipticE[(c - Pi/2 + d*x)/2, 2
]*Sqrt[e*Sin[c + d*x]])/(5*b^4*d*Sqrt[Sin[c + d*x]]) + (2*e^3*(5*(a^2 - b^2
) - 3*a*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*b^3*d) - (2*e*(e*Sin[c
+ d*x])^(7/2))/(7*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2e(e\sin(c+dx))^{7/2}}{7bd} - \frac{e^2\int\frac{(-b-a\cos(c+dx))(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx}{b}$$

$$= \frac{2e^3(5(a^2-b^2)-3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15b^3d} - \frac{2e(e\sin(c+dx))^{7/2}}{7bd} - \frac{(2e^4)\int \frac{\left(\frac{1}{2}b(2a^2-5b^2)+\frac{1}{2}a(5a^2-8b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{5b^3}$$

$$= \frac{2e^3(5(a^2-b^2)-3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15b^3d} - \frac{2e(e\sin(c+dx))^{7/2}}{7bd} - \frac{(a(5a^2-8b^2)\,e^4)\int\sqrt{e\sin(c+dx)}\,dx}{5b^4} + \frac{\left((a^2-b^2)^2\,e^4\right)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{b^4}$$

$$= \frac{2e^3(5(a^2-b^2)-3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15b^3d} - \frac{2e(e\sin(c+dx))^{7/2}}{7bd} - \frac{\left(a(a^2-b^2)^2\,e^5\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2b^5} + \frac{\left(a(a^2-b^2)^2\,e^5\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2b^5} - \frac{\left((a^2-b^2)^2\,e^5\right)\mathrm{Subst}\left(\int\frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx, x, e\sin(c+dx)\right)}{b^3d} - \frac{\left(a(5a^2-8b^2)\,e^4\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{5b^4\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a(5a^2-8b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}} + \frac{2e^3(5(a^2-b^2)-3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15b^3d} - \frac{2e(e\sin(c+dx))^{7/2}}{7bd} - \frac{\left(2(a^2-b^2)^2\,e^5\right)\mathrm{Subst}\left(\int\frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{b^3d} - \frac{\left(a(a^2-b^2)^2\,e^5\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2b^5\sqrt{e\sin(c+dx)}} + \frac{\left(a(a^2-b^2)^2\,e^5\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2b^5\sqrt{e\sin(c+dx)}}$$

$$
\begin{aligned}
&= \frac{a(a^2-b^2)^2 e^5 \,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right) \sqrt{\sin(c+dx)}}{b^5 \left(b-\sqrt{-a^2+b^2}\right) d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{a(a^2-b^2)^2 e^5 \,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right) \sqrt{\sin(c+dx)}}{b^5 \left(b+\sqrt{-a^2+b^2}\right) d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2a(5a^2-8b^2)\, e^4 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right) \middle| 2\right) \sqrt{e\sin(c+dx)}}{5b^4 d\sqrt{\sin(c+dx)}} \\
&\quad + \frac{2e^3(5(a^2-b^2)-3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15b^3 d} - \frac{2e(e\sin(c+dx))^{7/2}}{7bd} \\
&\quad + \frac{\left((a^2-b^2)^2 e^5\right) \text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{b^4 d} \\
&\quad - \frac{\left((a^2-b^2)^2 e^5\right) \text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{b^4 d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(-a^2+b^2)^{7/4} e^{9/2} \arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{9/2} d} \\
&\quad + \frac{(-a^2+b^2)^{7/4} e^{9/2} \text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{9/2} d} \\
&\quad + \frac{a(a^2-b^2)^2 e^5 \,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right) \sqrt{\sin(c+dx)}}{b^5 \left(b-\sqrt{-a^2+b^2}\right) d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{a(a^2-b^2)^2 e^5 \,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right) \sqrt{\sin(c+dx)}}{b^5 \left(b+\sqrt{-a^2+b^2}\right) d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2a(5a^2-8b^2)\, e^4 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right) \middle| 2\right) \sqrt{e\sin(c+dx)}}{5b^4 d\sqrt{\sin(c+dx)}} \\
&\quad + \frac{2e^3(5(a^2-b^2)-3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15b^3 d} - \frac{2e(e\sin(c+dx))^{7/2}}{7bd}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 36.31 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.81

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx =$$

$$-\frac{(e\sin(c+dx))^{9/2}\left(\frac{(5a^3-8ab^2)\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a}\right.\right.\right.}{}\right.}{}$$

$$+\frac{\csc^4(c+dx)(e\sin(c+dx))^{9/2}\left(-\frac{(-28a^2+37b^2)\sin(c+dx)}{42b^3}-\frac{a\sin(2(c+dx))}{5b^2}+\frac{\sin(3(c+dx))}{14b}\right)}{d}$$

```
[In] Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -1/5*((e*Sin[c + d*x])^(9/2)*(((5*a^3 - 8*a*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*
a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2
 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b
^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[S
in[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/
4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c +
 d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a
+ b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(2*a^2*b - 5*b^3)*Cos[c + d*x]
*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b
^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)
^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-
a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 +
b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*
x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Si
n[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(b^3*d*Si
n[c + d*x]^(9/2)) + (Csc[c + d*x]^4*(e*Sin[c + d*x])^(9/2)*(-1/42*((-28*a^2
 + 37*b^2)*Sin[c + d*x])/b^3 - (a*Sin[2*(c + d*x)])/(5*b^2) + Sin[3*(c + d*
x)]/(14*b)))/d
```

Maple [A] (verified)

Time = 5.37 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.85

method	result	size
default	Expression too large to display	851

```
[In] int((e*sin(d*x+c))^(9/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*e*b*(-1/21/b^4*(e*sin(d*x+c))^(3/2)*e^2*(3*b^2*cos(d*x+c)^2+7*a^2-10*b^
2)+1/8*e^4*(a^4-2*a^2*b^2+b^4)/b^6/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e
*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^
2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(
1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^
2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)
*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^5*a*(1/5/b^4
/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(10*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-16
*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-
sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-5*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+8*(1
-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin
(d*x+c))^(1/2),1/2*2^(1/2))*b^2+2*b^2*cos(d*x+c)^4-2*b^2*cos(d*x+c)^2)+(a^4
-2*a^2*b^2+b^4)/b^4*(-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*El
lipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2
*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2)
,1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(9/2)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{b\cos(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{b\cos(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\,\sin(c+d\,x))^{9/2}}{a+b\,\cos(c+d\,x)}\,dx$$

```
[In] int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x)), x)
```

3.60 $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 474

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx &= \frac{(-a^2+b^2)^{5/4}\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{7/2}d} \\ &+ \frac{(-a^2+b^2)^{5/4}\,e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{7/2}d} \\ &- \frac{2a(3a^2-4b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}} \\ &+ \frac{a(a^2-b^2)^2\,e^4\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ &+ \frac{a(a^2-b^2)^2\,e^4\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^4\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ &+ \frac{2e^3(3(a^2-b^2)-ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd}\end{aligned}$$

```
[Out] (-a^2+b^2)^(5/4)*e^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/
4)/e^(1/2))/b^(7/2)/d+(-a^2+b^2)^(5/4)*e^(7/2)*arctanh(b^(1/2)*(e*sin(d*x+c
))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/d-2/5*e*(e*sin(d*x+c))^(5/2)/b/d
+2/3*a*(3*a^2-4*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/b
^4/d/(e*sin(d*x+c))^(1/2)-a*(a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-
(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/(a^2-b*(b-(-a^2+b^2)^(1/2
)))/(e*sin(d*x+c))^(1/2)-a*(a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
```

```
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(
-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/(a^2-b*(b+(-a^2+b^2)^(1/2)
))/(e*sin(d*x+c))^(1/2)+2/3*e^3*(3*a^2-3*b^2-a*b*cos(d*x+c))*(e*sin(d*x+c))
^(1/2)/b^3/d
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2774, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx &= \frac{e^{7/2}(b^2-a^2)^{5/4}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}d} \\ &+ \frac{e^{7/2}(b^2-a^2)^{5/4}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}d} \\ &- \frac{2ae^4(3a^2-4b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3b^4d\sqrt{e\sin(c+dx)}} \\ &+ \frac{ae^4(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{b^4d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}} \\ &+ \frac{ae^4(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{b^4d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}} \\ &+ \frac{2e^3\sqrt{e\sin(c+dx)}(3(a^2-b^2)-ab\cos(c+dx))}{3b^3d} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd}\end{aligned}$$

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-a^2 + b^2)^(5/4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(7/2)*d) + ((-a^2 + b^2)^(5/4)*e^(7/2)*ArcTanh[(Sq
rt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(7/2)*d) - (2
*a*(3*a^2 - 4*b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])
/(3*b^4*d*Sqrt[e*Sin[c + d*x]]) + (a*(a^2 - b^2)^2*e^4*EllipticPi[(2*b)/(b
- Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^4*(a^2 -
 b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (a*(a^2 - b^2)^2*e^4*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (2*
e^3*(3*(a^2 - b^2) - a*b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*b^3*d) - (2
*e*(e*Sin[c + d*x])^(5/2))/(5*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2e(e\sin(c+dx))^{5/2}}{5bd} - \frac{e^2 \int \frac{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx}{b}$$

$$
\begin{aligned}
&= \frac{2e^3(3(a^2-b^2)-ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} \\
&\quad - \frac{2e(e\sin(c+dx))^{5/2}}{5bd} - \frac{(2e^4)\int \frac{\frac{1}{2}b(2a^2-3b^2)+\frac{1}{2}a(3a^2-4b^2)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3b^3} \\
&= \frac{2e^3(3(a^2-b^2)-ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd} \\
&\quad - \frac{(a(3a^2-4b^2)e^4)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3b^4} + \frac{\left((a^2-b^2)^2e^4\right)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{b^4} \\
&= \frac{2e^3(3(a^2-b^2)-ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd} \\
&\quad - \frac{\left(a(-a^2+b^2)^{3/2}e^4\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2b^4} \\
&\quad - \frac{\left(a(-a^2+b^2)^{3/2}e^4\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2b^4} \\
&\quad - \frac{\left((a^2-b^2)^2e^5\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx, x, e\sin(c+dx)\right)}{b^3d} \\
&\quad - \frac{\left(a(3a^2-4b^2)e^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3b^4\sqrt{e\sin(c+dx)}} \\
&= -\frac{2a(3a^2-4b^2)e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{2e^3(3(a^2-b^2)-ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd} \\
&\quad - \frac{\left(2(a^2-b^2)^2e^5\right)\text{Subst}\left(\int \frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{b^3d} \\
&\quad - \frac{\left(a(-a^2+b^2)^{3/2}e^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2b^4\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{\left(a(-a^2+b^2)^{3/2}e^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2b^4\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$= -\frac{2a(3a^2-4b^2)e^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}}$$
$$+\frac{a(-a^2+b^2)^{3/2}e^4 \operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^4\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{a(-a^2+b^2)^{3/2}e^4 \operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^4\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2e^3(3(a^2-b^2)-ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$
$$+\frac{\left((-a^2+b^2)^{3/2}e^4\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{b^3d}$$
$$+\frac{\left((-a^2+b^2)^{3/2}e^4\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{b^3d}$$

$$=\frac{(-a^2+b^2)^{5/4}e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{7/2}d}$$
$$+\frac{(-a^2+b^2)^{5/4}e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{7/2}d}$$
$$-\frac{2a(3a^2-4b^2)e^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}}$$
$$+\frac{a(-a^2+b^2)^{3/2}e^4 \operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^4\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{a(-a^2+b^2)^{3/2}e^4 \operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^4\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2e^3(3(a^2-b^2)-ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.54 (sec) , antiderivative size = 1955, normalized size of antiderivative = 4.12

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \frac{\left(-\frac{2a\cos(c+dx)}{3b^2}+\frac{\cos(2(c+dx))}{5b}\right)\csc^3(c+dx)(e\sin(c+dx))^{7/2}}{d}$$

$$+\frac{(e\sin(c+dx))^{7/2}\left(\frac{28ab\cos^2(c+dx)\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (((-2*a*Cos[c + d*x])/(3*b^2) + Cos[2*(c + d*x)]/(5*b))*Csc[c + d*x]^3*(e*S
in[c + d*x])^(7/2))/d + ((e*Sin[c + d*x])^(7/2)*((28*a*b*Cos[c + d*x]^2*(a
+ b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[
c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c +
d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^
2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/
4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sq
rt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c
+ d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2,
 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*A
ppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)
])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x
])*(1 - Sin[c + d*x]^2)) + (2*(-10*a^2 + 27*b^2)*Cos[c + d*x]*(a + b*Sqrt[1
 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*S
qrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt
[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b
]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2
 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c
 + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4,
 Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sq
rt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9
/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*Appel
lF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*S
in[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*S
qrt[1 - Sin[c + d*x]^2]) + ((30*a^2 - 33*b^2)*Cos[c + d*x]*Cos[2*(c + d*x)]
*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - (
```

```
(1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b
^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[S
in[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + ((1/4 - I
/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((
1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^
2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4
)) + (4*Sqrt[Sin[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x
]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2))
 + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c
+ d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a
^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(
-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin
[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c
+ d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-
1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1
 - Sin[c + d*x]^2])))/(60*b^2*d*Sin[c + d*x]^(7/2))
```

Maple [A] (verified)

Time = 5.01 (sec) , antiderivative size = 773, normalized size of antiderivative = 1.63

method	result
default	$-2eb\left(-\frac{\sqrt{e\sin(dx+c)}\,e^2\left(b^2\left(\cos^2(dx+c)\right)+5a^2-6b^2\right)}{5b^4}+\frac{e^4\left(a^4-2a^2b^2+b^4\right)\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)+\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c}}{e\sin(dx+c)-\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c}}\right.\right.}{8b^4\left(a^2\right.}\right.$

```
[In] int((e*sin(d*x+c))^(7/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*e*b*(-1/5/b^4*(e*sin(d*x+c))^(1/2)*e^2*(b^2*cos(d*x+c)^2+5*a^2-6*b^2)+1
/8*e^4*(a^4-2*a^2*b^2+b^4)/b^4*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*
2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^
(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e
*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b
^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*
e^4*a*(1/3/b^4/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(3*(1-sin(d*x+c))^(1/2)*(2
```

```
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^
(1/2))*a^2-4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*E
llipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-2*b^2*cos(d*x+c)^2*sin(d*x+c
))+(a^4-2*a^2*b^2+b^4)/b^4*(-1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-
(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b
),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/
2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))
))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{b\cos(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{b\cos(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\,\sin(c+d\,x))^{7/2}}{a+b\,\cos(c+d\,x)}\,dx$$

```
[In] int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x)), x)
```

3.61 $\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 399

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx &= -\frac{\left(-a^2+b^2\right)^{3/4}e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d}\\
&+\frac{\left(-a^2+b^2\right)^{3/4}e^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d}\\
&-\frac{a(a^2-b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{a(a^2-b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^3\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{2ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}\end{aligned}$$

```
[Out] -(-a^2+b^2)^(3/4)*e^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(5/2)/d+(-a^2+b^2)^(3/4)*e^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+
c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/d-2/3*e*(e*sin(d*x+c))^(3/2)/b/
d+a*(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))
*sin(d*x+c)^(1/2)/b^3/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+a*(a^2-b^
2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)
^(1/2)/b^3/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*a*e^2*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*P
i+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^2/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2774, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = -\frac{e^{5/2}(b^2-a^2)^{3/4}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d} + \frac{e^{5/2}(b^2-a^2)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d} - \frac{ae^3(a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}} - \frac{ae^3(a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{b^3d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}} + \frac{2ae^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}} - \frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -(((-a^2 + b^2)^(3/4)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(b^(5/2)*d)) + ((-a^2 + b^2)^(3/4)*e^(5/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(5/2)*d) -
 (a*(a^2 - b^2)*e^3*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^3*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d
*x]]) - (a*(a^2 - b^2)*e^3*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi
/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^3*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Si
n[c + d*x]]) + (2*a*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x
]])/(b^2*d*Sqrt[Sin[c + d*x]]) - (2*e*(e*Sin[c + d*x])^(3/2))/(3*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2e(e\sin(c+dx))^{3/2}}{3bd} - \frac{e^2\int\frac{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{b}$$

$$= -\frac{2e(e\sin(c+dx))^{3/2}}{3bd} + \frac{(ae^2)\int\sqrt{e\sin(c+dx)}\,dx}{b^2} - \frac{((a^2-b^2)\,e^2)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{b^2}$$

$$= -\frac{2e(e\sin(c+dx))^{3/2}}{3bd} + \frac{(a(a^2-b^2)\,e^3)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2b^3} - \frac{(a(a^2-b^2)\,e^3)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2b^3} + \frac{((a^2-b^2)\,e^3)\,\text{Subst}\left(\int\frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx, x, e\sin(c+dx)\right)}{bd} + \frac{\left(ae^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{b^2\sqrt{\sin(c+dx)}}$$

$$= \frac{2ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}} - \frac{2e(e\sin(c+dx))^{3/2}}{3bd} + \frac{(2(a^2-b^2)\,e^3)\,\text{Subst}\left(\int\frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{bd} + \frac{\left(a(a^2-b^2)\,e^3\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2b^3\sqrt{e\sin(c+dx)}} - \frac{\left(a(a^2-b^2)\,e^3\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2b^3\sqrt{e\sin(c+dx)}}$$

$$= -\frac{a(a^2-b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{a(a^2-b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^3\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$
$$-\frac{((a^2-b^2)\,e^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{b^2d}$$
$$+\frac{((a^2-b^2)\,e^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{b^2d}$$

$$= -\frac{(-a^2+b^2)^{3/4}\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d}$$
$$+\frac{(-a^2+b^2)^{3/4}\,e^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d}$$
$$-\frac{a(a^2-b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{a(a^2-b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^3\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.33 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.73

$$\int\frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \frac{(e\sin(c+dx))^{5/2}\left(-2\csc(c+dx)+\frac{\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)\left(-\frac{a\sec(c+dx)\left(3\sqrt{2}a\left(a^2-b^2\right)\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

[In] `Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x]),x]`

```
[Out] ((e*Sin[c + d*x])^(5/2)*(-2*Csc[c + d*x] + (Cos[c + d*x]*(a + b*Sqrt[Cos[c
+ d*x]^2])*(-((a*Sec[c + d*x]*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 -
(Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + L
og[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (
b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)))/(a^2 - b^2)) + ((3 +
 3*I)*b^2*(-a^2 + b^2)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(
-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^
2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)
*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]) - 8*a*b^(5
/2)*(-a^2 + b^2)^(1/4)*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[
c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/((-a^2 + b^2)^(5/4)*Sqrt[Cos[
c + d*x]^2]))))/(4*b^(3/2)*(a + b*Cos[c + d*x])*Sin[c + d*x]^(5/2))))/(3*b*d
)
```

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.60

method	result
default	$-2eb\left(\frac{(e\sin(dx+c))^{\frac{3}{2}}}{3b^2}-\frac{e^2\left(a^2-b^2\right)\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)-\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}{e\sin(dx+c)+\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}}+1\right)-\cdots\right)}{8b^4\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}}\cdots$

```
[In] int((e*sin(d*x+c))^(5/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*e*b*(1/3*(e*sin(d*x+c))^(3/2)/b^2-1/8*e^2*(a^2-b^2)/b^4/(e^2*(a^2-b^2)/
b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c
))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^
2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(
2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/
(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x
+c))^(1/2)*e^3*a*(-1/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(2*EllipticE((1-sin(d*x+c))^(1
```

```
/2),1/2*2^(1/2))-EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2)))-(a^2-b^2)/b^2
*(-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x
+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin(d*x+c))^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2
)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1
/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin\left(dx+c\right))^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\,\sin\left(c+d\,x\right))^{5/2}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

```
[In] int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x)), x)
```

3.62 $\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 410

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx &= \frac{\sqrt[4]{-a^2+b^2}e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{3/2}d}\\
&+\frac{\sqrt[4]{-a^2+b^2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{3/2}d}\\
&+\frac{2ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2d\sqrt{e\sin(c+dx)}}\\
&-\frac{a(a^2-b^2)\,e^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{a(a^2-b^2)\,e^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{2e\sqrt{e\sin(c+dx)}}{bd}\end{aligned}$$

```
[Out] (-a^2+b^2)^(1/4)*e^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/
4)/e^(1/2))/b^(3/2)/d+(-a^2+b^2)^(1/4)*e^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c
))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/d-2*a*e^2*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x)
,2^(1/2))*sin(d*x+c)^(1/2)/b^2/d/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)*e^2*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2
*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/d
/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)*e^2*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+
```

```
1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/d/(a
^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-2*e*(e*sin(d*x+c))^(1/2)/b/
d
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2774, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx &= \frac{e^{3/2}\sqrt[4]{b^2-a^2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}d} \\ &+ \frac{e^{3/2}\sqrt[4]{b^2-a^2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}d} \\ &- \frac{ae^2(a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}} \\ &- \frac{ae^2(a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{b^2d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}} \\ &+ \frac{2ae^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{b^2d\sqrt{e\sin(c+dx)}} - \frac{2e\sqrt{e\sin(c+dx)}}{bd}\end{aligned}$$

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-a^2 + b^2)^(1/4)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*d) + ((-a^2 + b^2)^(1/4)*e^(3/2)*ArcTanh[(Sq
rt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*d) + (2
*a*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^2*d*Sqrt[e*S
in[c + d*x]]) - (a*(a^2 - b^2)*e^2*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
 (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^2*(a^2 - b*(b - Sqrt[-a^2 +
b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (a*(a^2 - b^2)*e^2*EllipticPi[(2*b)/(b + S
qrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^2*(a^2 - b*
(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (2*e*Sqrt[e*Sin[c + d*x]]
)/(b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2e\sqrt{e\sin(c+dx)}}{bd}-\frac{e^2\int\frac{-b-a\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{b}\\
&= -\frac{2e\sqrt{e\sin(c+dx)}}{bd}+\frac{(ae^2)\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{b^2}+\frac{((-a^2+b^2)\,e^2)\int\frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{b^2}\\
&= -\frac{2e\sqrt{e\sin(c+dx)}}{bd}-\frac{\left(a\sqrt{-a^2+b^2}e^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2b^2}\\
&\quad-\frac{\left(a\sqrt{-a^2+b^2}e^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2b^2}\\
&\quad+\frac{((a^2-b^2)\,e^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx,x,e\sin(c+dx)\right)}{bd}\\
&\quad+\frac{\left(ae^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{b^2\sqrt{e\sin(c+dx)}}
\end{aligned}$$

$$
= \frac{2ae^2 \,\text{EllipticF}\left(\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right)\sqrt{\sin(c+dx)}}{b^2 d\sqrt{e\sin(c+dx)}} - \frac{2e\sqrt{e\sin(c+dx)}}{bd}
+ \frac{(2(a^2-b^2)\,e^3)\,\text{Subst}\left(\int \frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{bd}
- \frac{\left(a\sqrt{-a^2+b^2}e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2b^2\sqrt{e\sin(c+dx)}}
- \frac{\left(a\sqrt{-a^2+b^2}e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2b^2\sqrt{e\sin(c+dx)}}
$$

$$
= \frac{2ae^2 \,\text{EllipticF}\left(\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right)\sqrt{\sin(c+dx)}}{b^2 d\sqrt{e\sin(c+dx)}}
+ \frac{a\sqrt{-a^2+b^2}e^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{b^2\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}
- \frac{a\sqrt{-a^2+b^2}e^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{b^2\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}
- \frac{2e\sqrt{e\sin(c+dx)}}{bd} + \frac{\left(\sqrt{-a^2+b^2}e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{bd}
+ \frac{\left(\sqrt{-a^2+b^2}e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{bd}
$$

$$
= \frac{\sqrt[4]{-a^2+b^2}e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{3/2}d} + \frac{\sqrt[4]{-a^2+b^2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{3/2}d}
+ \frac{2ae^2 \,\text{EllipticF}\left(\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right)\sqrt{\sin(c+dx)}}{b^2 d\sqrt{e\sin(c+dx)}}
+ \frac{a\sqrt{-a^2+b^2}e^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{b^2\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}
- \frac{a\sqrt{-a^2+b^2}e^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{b^2\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}
- \frac{2e\sqrt{e\sin(c+dx)}}{bd}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.11 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.06

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx =$$

$$-\frac{\left(\frac{1}{20}-\frac{i}{20}\right)\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)(e\sin(c+dx))^{3/2}\left(-5(a^2-b^2)\left(2\sqrt[4]{-a^2+b^2}\arctan\left(1-\frac{(1+}{}\right.\right.\right.}{}$$

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-1/20 + I/20)*Cos[c + d*x]*(a + b*Sqrt[Cos[c + d*x]^2])*(e*Sin[c + d*x])^
(3/2)*(-5*(a^2 - b^2)*(2*(-a^2 + b^2)^(1/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqr
t[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*(-a^2 + b^2)^(1/4)*ArcTan[1 + ((1
+ I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + (-a^2 + b^2)^(1/4)*L
og[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
 + I*b*Sin[c + d*x]] - (-a^2 + b^2)^(1/4)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + (4 + 4*I)
*Sqrt[b]*Sqrt[Sin[c + d*x]]) + (4 + 4*I)*a*b^(3/2)*AppellF1[5/4, 1/2, 1, 9/
4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(5/2)))/
(b^(3/2)*(-a^2 + b^2)*d*Sqrt[Cos[c + d*x]^2]*(a + b*Cos[c + d*x])*Sin[c + d
*x]^(3/2))
```

Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.60

method	result
default	$-2eb\left(\frac{\sqrt{e\sin(dx+c)}}{b^2}-\frac{e^2\left(a^2-b^2\right)\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)+\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}{e\sin(dx+c)-\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e}}{\left(\frac{e^2(a}{b}\right.}\right.\right.}{8b^2\left(a^2e^2-b^2e^2\right)}\right.$

```
[In] int((e*sin(d*x+c))^(3/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*e*b*((e*sin(d*x+c))^(1/2)/b^2-1/8*e^2*(a^2-b^2)/b^2*(e^2*(a^2-b^2)/b^2)
^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4
)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/
2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*a
rctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)*e^2*a*(-1/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin
(d*x+c))^(1/2),1/2*2^(1/2))+(-a^2+b^2)/b^2*(-1/2/(-a^2+b^2)^(1/2)/b*(1-sin(
d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-
a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(
1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)
/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(c+dx))^{\frac{3}{2}}}{a+b\cos(c+dx)}\,dx$$

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((e*sin(c + d*x))**(3/2)/(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin\left(dx+c\right))^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin\left(dx+c\right))^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\,\sin\left(c+d\,x\right))^{3/2}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

```
[In] int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x)), x)
```

3.63 $\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 302

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}d}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}d}+\frac{ae\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{ae\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$

```
[Out] -arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^
2+b^2)^(1/4)/d/b^(1/2)+arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4
)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(1/4)/d/b^(1/2)-a*e*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2
*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/d/(b-(-a^2+b^2)^(1/2))/
(e*sin(d*x+c))^(1/2)-a*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2
^(1/2))*sin(d*x+c)^(1/2)/b/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2780, 2886, 2884, 335, 304, 211, 214}

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}d\sqrt[4]{b^2-a^2}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}d\sqrt[4]{b^2-a^2}} + \frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}} + \frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}$$

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] -((Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(Sqrt[b]*(-a^2 + b^2)^(1/4)*d)) + (Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Sin
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(Sqrt[b]*(-a^2 + b^2)^(1/4)*d) +
 (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*Ell
ipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ae)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2b} + \frac{(ae)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2b} \\
&\quad - \frac{(be)\text{Subst}\left(\int \frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx, x, e\sin(c+dx)\right)}{d} \\
&= -\frac{(2be)\text{Subst}\left(\int \frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} \\
&\quad - \frac{\left(ae\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2b\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{\left(ae\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2b\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$= \frac{ae\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{ae\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{e\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{d} - \frac{e\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{d}$$

$$= -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}d} + \frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}d} + \frac{ae\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{ae\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 1.65 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.20

$$\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx$$

$$= \frac{2\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)\sqrt{e\sin(c+dx)}\left(\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(2\arctan\left(1-\frac{(1+i)\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-2\arctan\left(1+\frac{(1+i)\sqrt{b}\sqrt{\sin(c\cdots}}{\sqrt[4]{-a^2+\cdots}}\right.\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[c + d*x]*(a + b*Sqrt[Cos[c + d*x]^2])*Sqrt[e*Sin[c + d*x]]*(((1/8 +
I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] -
Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]
] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)
^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)
) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^
2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2))))/(d*Sqrt[Cos[c + d*x]^2]*(a
+ b*Cos[c + d*x])*Sqrt[Sin[c + d*x]])
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.64

method	result
default	$-\frac{e\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)-\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2(a^2-b^2)}{b^2}}}{e\sin(dx+c)+\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2(a^2-b^2)}{b^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}-1\right.\right.}{4b\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}$

```
[In] int((e*sin(d*x+c))^(1/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*e/b/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2
)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin
(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^
2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(
1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))
+1/2*a*e*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/b*(El
lipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b
^2)^(1/2)+EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1
/2))*b-EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/2)
)*(-a^2+b^2)^(1/2)+EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b
,1/2*2^(1/2))*b)/(-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2))/cos(d*x+c)/(e*s
in(d*x+c))^(1/2))/d
```

Fricas [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx=\int \frac{\sqrt{e\sin\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{e\sin{(c+dx)}}}{a+b\cos{(c+dx)}}\,dx$$

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*cos(d*x+c)),x)
[Out] Integral(sqrt(e*sin(c + d*x))/(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{e\sin{(dx+c)}}}{b\cos{(dx+c)}+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{e\sin{(dx+c)}}}{b\cos{(dx+c)}+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{e\,\sin{(c+d\,x)}}}{a+b\,\cos{(c+d\,x)}}\,dx$$

```
[In] int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x)), x)
```

3.64 $\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 307

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}\,d\sqrt{e}}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}\,d\sqrt{e}}+\frac{a\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}+\frac{a\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}$$

```
[Out] arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2
+b^2)^(3/4)/d/e^(1/2)+arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)
/e^(1/2))*b^(1/2)/(-a^2+b^2)^(3/4)/d/e^(1/2)-a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2
)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/
(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/d/(a^2-b*(b-(-a^2+b^2)^(1/2)
))/(e*sin(d*x+c))^(1/2)-a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),
2^(1/2))*sin(d*x+c)^(1/2)/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/
2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2781, 2886, 2884, 335, 218, 214, 211}

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\,(b^2-a^2)^{3/4}} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\,(b^2-a^2)^{3/4}}$$
$$+ \frac{a\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}$$
$$+ \frac{a\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}$$

```
[In] Int[1/((a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/((-a^2 + b^2)^(3/4)*d*Sqrt[e]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(3/4)*d*Sqrt[e]) + (a
*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
 + d*x]])/((a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (a*El
lipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
d*x]])/((a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule steps

$$\text{integral} = -\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2\sqrt{-a^2+b^2}} - \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2\sqrt{-a^2+b^2}} - \frac{(be)\text{Subst}\left(\int \frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx, x, e\sin(c+dx)\right)}{d}$$

$$= -\frac{(2be)\text{Subst}\left(\int \frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} - \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2\sqrt{-a^2+b^2}\sqrt{e\sin(c+dx)}} - \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2\sqrt{-a^2+b^2}\sqrt{e\sin(c+dx)}}$$

$$= \frac{a\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} + \frac{a\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{\sqrt{-a^2+b^2}d} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{\sqrt{-a^2+b^2}d}$$

$$= \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}d\sqrt{e}} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}d\sqrt{e}} + \frac{a\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} + \frac{a\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 1.62 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$

$$= \frac{10(a+b)\,\text{AppellF1}}{de(a+b\cos(c+dx))\left(5(a+b)\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(-a+b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)+2\left(-2(a\right.\right.}$$

```
[In] Integrate[1/((a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (10*(a + b)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[
(c + d*x)/2]^2)/(a + b)]*Sqrt[e*Sin[c + d*x]])/(d*e*(a + b*Cos[c + d*x])*(5
*(a + b)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c
+ d*x)/2]^2)/(a + b)] + 2*(-2*(a - b)*AppellF1[5/4, -1/2, 2, 9/4, -Tan[(c +
 d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[5/4,
1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])*T
an[(c + d*x)/2]^2))
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.69

method	result
default	$-\frac{b e \left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}} \sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)+\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}{e\sin(dx+c)-\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right.\right.}{4\left(a^{2}e^{2}-b^{2}e^{2}\right)}$

```
[In] int(1/(a+cos(d*x+c)*b)/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*b*e*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*
x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/
b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^
(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4
)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin
(d*x+c))^(1/2)-1))+1/2*a*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(1/2)*(EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^
(1/2))*(-a^2+b^2)^(1/2)+EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(
1/2)),1/2*2^(1/2))*b+EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))
*b,1/2*2^(1/2))*(-a^2+b^2)^(1/2)-EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2
+b^2)^(1/2))*b,1/2*2^(1/2))*b)/(-a^2+b^2)^(1/2)/(-b+(-a^2+b^2)^(1/2))/(b+(-
a^2+b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sin{(c+dx)}}\,(a+b\cos{(c+dx)})}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)\sqrt{e\sin{(dx+c)}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)\sqrt{e\sin{(dx+c)}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\,\sin{(c+d\,x)}}\,\left(a+b\,\cos{(c+d\,x)}\right)}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))),x)
[Out] int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))), x)
```

3.65 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 426

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = &-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}\,de^{3/2}} \\ &+\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}\,de^{3/2}}+\frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} \\ &-\frac{ab\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\ &-\frac{ab\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\ &-\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}}\end{aligned}$$

```
[Out] -b^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^
2+b^2)^(5/4)/d/e^(3/2)+b^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(5/4)/d/e^(3/2)+2*(b-a*cos(d*x+c))/(a^2-b^2)/
d/e/(e*sin(d*x+c))^(1/2)+a*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+
1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2
)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+
c))^(1/2)+a*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin
(d*x+c)^(1/2)/(a^2-b^2)/d/e/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*a*(
sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(
1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)/d/e^2/sin(d*x
+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2775, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = -\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{3/2}\left(b^2-a^2\right)^{5/4}} -\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\left(a^2-b^2\right)\sqrt{\sin(c+dx)}}+\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}} -\frac{ab\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{de\left(a^2-b^2\right)\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}} -\frac{ab\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{de\left(a^2-b^2\right)\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}$$

```
[In] Int[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -((b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/((-a^2 + b^2)^(5/4)*d*e^(3/2))) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(5/4)*d*e^(3/2)) +
 (2*(b - a*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) - (a*b*Ell
ipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (a*b*
EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (2
*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)*d*e^
2*Sqrt[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a /b, 0]

Rule 335

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)* (c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x]) ^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ [-1, n, 1] && IntegerQ[2*n]

Rule 2775

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x _)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^ (m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2* (a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[2*m, 2*p]

Rule 2780

Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst [Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c , d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,

```
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}}-\frac{2\int\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{1}{2}ab\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{(a^2-b^2)\,e^2}\\
&= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}}-\frac{a\int\sqrt{e\sin(c+dx)}\,dx}{(a^2-b^2)\,e^2}-\frac{b^2\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{(a^2-b^2)\,e^2}\\
&= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}}+\frac{(ab)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2\,(a^2-b^2)\,e}\\
&\quad-\frac{(ab)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2\,(a^2-b^2)\,e}\\
&\quad+\frac{b^3\text{Subst}\left(\int\frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx,x,e\sin(c+dx)\right)}{(a^2-b^2)\,de}\\
&\quad-\frac{\left(a\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{(a^2-b^2)\,e^2\sqrt{\sin(c+dx)}}
\end{aligned}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}}$$
$$+ \frac{(2b^3)\,\text{Subst}\left(\int \frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)\,de}$$
$$+ \frac{\left(ab\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2\,(a^2-b^2)\,e\sqrt{e\sin(c+dx)}}$$
$$- \frac{\left(ab\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2\,(a^2-b^2)\,e\sqrt{e\sin(c+dx)}}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}}$$
$$- \frac{ab\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$- \frac{ab\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$- \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}}$$
$$- \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)\,de}$$
$$+ \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)\,de}$$

$$= -\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}\,de^{3/2}} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}\,de^{3/2}}$$
$$+ \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}}$$
$$- \frac{ab\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$- \frac{ab\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$- \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.03 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = -\frac{2(-b+a\cos(c+dx))\sin(c+dx)}{(a^2-b^2)\,d(e\sin(c+dx))^{3/2}}$$

$$-\frac{\sin^{\frac{3}{2}}(c+dx)\left(\frac{a\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\sqrt[4]{a\right.\right.\right.}{}\right.}{}$$

```
[In] Integrate[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-2*(-b + a*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)*d*(e*Sin[c + d*x])^(3/
2)) - (Sin[c + d*x]^(3/2)*((a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)
*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*
ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sq
rt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Si
n[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[
Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Si
n[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b
*Sqrt[1 - Sin[c + d*x]^2]))/(12*Sqrt[b]*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(
1 - Sin[c + d*x]^2)) + (2*(a^2 + b^2)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[
1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 +
 ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 +
 b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c +
 d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin
[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1
[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c
 + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*C
os[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/((a - b)*(a + b)*d*(e*Sin[c + d*x]
)^(3/2))
```

Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.82

method	result
default	$\frac{-be\left(-\frac{2}{e^{2}\left(a^{2}-b^{2}\right)\sqrt{e\sin(dx+c)}}-\frac{\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)-\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}{e\sin(dx+c)+\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\sin(dx+c)}}{\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}}+1\right)}{4e^{2}(a-b)(a+b)\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}}\right)}{}$

```
[In] int(1/(a+cos(d*x+c)*b)/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*e*(-2/e^2/(a^2-b^2)/(e*sin(d*x+c))^(1/2)-1/4/e^2/(a-b)/(a+b)/(e^2*(a^2-
b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(
d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*ar
ctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(
1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-1/2*(4*(1-sin(d*x+
c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^
(1/2),1/2*2^(1/2))*a^2-2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+(1-sin(d*x+c))^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2
),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b+(1-sin(d*x+c))^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2
),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2-(1-sin(d*x+c))^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^
2)^(1/2))*b,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b+(1-sin(d*x+c))^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^
2)^(1/2))*b,1/2*2^(1/2))*b^2-4*a^2*cos(d*x+c)^2)*a/e/(b+(-a^2+b^2)^(1/2))/(
-b+(-a^2+b^2)^(1/2))/(a+b)/(a-b)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\sin{(c+dx)})^{\frac{3}{2}}\,(a+b\cos{(c+dx)})}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(3/2),x)
[Out] Integral(1/((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)\,(e\sin{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)\,(e\sin{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))),x)
[Out] int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))), x)
```

3.66 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 447

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{7/4}\,de^{5/2}} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{7/4}\,de^{5/2}} + \frac{2(b-a\cos(c+dx))}{3(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3(a^2-b^2)\,de^2\sqrt{e\sin(c+dx)}} - \frac{ab^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}} - \frac{ab^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}}$$

```
[Out] b^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2
+b^2)^(7/4)/d/e^(5/2)+b^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/(-a^2+b^2)^(7/4)/d/e^(5/2)+2/3*(b-a*cos(d*x+c))/(a^2-b^2)
/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^
(1/2)/(a^2-b^2)/d/e^2/(e*sin(d*x+c))^(1/2)+a*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*
b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b*(b-
(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+a*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(
b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b*(b+(-a
^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2775, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{5/2}\left(b^2-a^2\right)^{7/4}} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{5/2}\left(b^2-a^2\right)^{7/4}} + \frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}} - \frac{ab^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{de^2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}} - \frac{ab^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{de^2\left(a^2-b^2\right)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}} + \frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

```
[In] Int[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/((-a^2 + b^2)^(7/4)*d*e^(5/2)) + (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(7/4)*d*e^(5/2)) + (2
*(b - a*Cos[c + d*x]))/(3*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + (2*a*El
lipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*(a^2 - b^2)*d*e^2*Sqr
t[e*Sin[c + d*x]]) - (a*b^2*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - P
i/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 +
 b^2]))*d*e^2*Sqrt[e*Sin[c + d*x]]) - (a*b^2*EllipticPi[(2*b)/(b + Sqrt[-a^
2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b
*(b + Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(b-a\cos(c+dx))}{3\,(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a^2}{2}+\frac{3b^2}{2}-\frac{1}{2}ab\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3\,(a^2-b^2)\,e^2} \\
&= \frac{2(b-a\cos(c+dx))}{3\,(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3\,(a^2-b^2)\,e^2} - \frac{b^2\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{(a^2-b^2)\,e^2} \\
&= \frac{2(b-a\cos(c+dx))}{3\,(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} - \frac{(ab^2)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2\,(-a^2+b^2)^{3/2}\,e^2} \\
&\quad - \frac{(ab^2)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2\,(-a^2+b^2)^{3/2}\,e^2} \\
&\quad + \frac{b^3\text{Subst}\left(\int \frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx, x, e\sin(c+dx)\right)}{(a^2-b^2)\,de} \\
&\quad + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3\,(a^2-b^2)\,e^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$= \frac{2(b-a\cos(c+dx))}{3(a^2-b^2)de(e\sin(c+dx))^{3/2}}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3(a^2-b^2)de^2\sqrt{e\sin(c+dx)}}$$
$$+\frac{(2b^3)\,\text{Subst}\left(\int\frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)de}$$
$$-\frac{\left(ab^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2(-a^2+b^2)^{3/2}e^2\sqrt{e\sin(c+dx)}}$$
$$-\frac{\left(ab^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2(-a^2+b^2)^{3/2}e^2\sqrt{e\sin(c+dx)}}$$

$$= \frac{2(b-a\cos(c+dx))}{3(a^2-b^2)de(e\sin(c+dx))^{3/2}}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3(a^2-b^2)de^2\sqrt{e\sin(c+dx)}}$$
$$+\frac{ab^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(-a^2+b^2)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$
$$-\frac{ab^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(-a^2+b^2)^{3/2}\left(b+\sqrt{-a^2+b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$
$$+\frac{b^3\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{(-a^2+b^2)^{3/2}de^2}$$
$$+\frac{b^3\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{(-a^2+b^2)^{3/2}de^2}$$

$$= \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{7/4}de^{5/2}}+\frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{7/4}de^{5/2}}$$
$$+\frac{2(b-a\cos(c+dx))}{3(a^2-b^2)de(e\sin(c+dx))^{3/2}}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3(a^2-b^2)de^2\sqrt{e\sin(c+dx)}}$$
$$+\frac{ab^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(-a^2+b^2)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$
$$-\frac{ab^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(-a^2+b^2)^{3/2}\left(b+\sqrt{-a^2+b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.45 (sec) , antiderivative size = 1192, normalized size of antiderivative = 2.67

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = -\frac{2(-b+a\cos(c+dx))\sin(c+dx)}{3(a^2-b^2)\,d(e\sin(c+dx))^{5/2}}$$

$$+ \frac{\sin^{\frac{5}{2}}(c+dx)\left(\frac{2ab\cos^2(c+dx)\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\right.\right.}{4\sqrt{2}}\right.}{}\right.}{}$$

[In] Integrate[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]

```
[Out] (-2*(-b + a*Cos[c + d*x])*Sin[c + d*x])/(3*(a^2 - b^2)*d*(e*Sin[c + d*x])^(
5/2)) + (Sin[c + d*x]^(5/2)*((2*a*b*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c +
d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2
)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1
/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c +
 d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^
2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b
^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2
])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c
+ d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]
^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1,
9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a
^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^
2)) + (2*(a^2 - 3*b^2)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/
8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 +
 b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^
2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[
Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*
(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3
/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[
c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)
/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*S
in[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[
c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*
(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))
/(3*(a - b)*(a + b)*d*(e*Sin[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.59

method	result
default	$-2eb\left(-\frac{b^{2}\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)+\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}{e\sin(dx+c)-\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arc}\ldots}{8e^{2}(a-b)(a+b)\left(a^{2}e^{2}-b^{2}e^{2}\right)}\right.$

[In] int(1/(a+cos(d*x+c)*b)/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] (-2*e*b*(-1/8/e^2/(a-b)/(a+b)*b^2*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))-1/3/e^2/(a^2-b^2)/(e*sin(d*x+c))^(3/2))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*a/e^2*(1/3/(a^2-b^2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*((1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+c))-1/(a-b)/(a+b)*b^2*(-1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d

Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx=\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)\left(e\sin\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral(-sqrt(e*sin(d*x + c))/((b*e^3*cos(d*x + c)^3 + a*e^3*cos(d*x + c)^2 - b*e^3*cos(d*x + c) - a*e^3)*sin(d*x + c)), x)

Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(e\sin(c+dx))^{\frac{5}{2}}\,(a+b\cos(c+dx))}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(5/2),x)
[Out] Integral(1/((e*sin(c + d*x))**(5/2)*(a + b*cos(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\,(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\,(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{{(e\,\sin(c+d\,x))}^{5/2}\,(a+b\,\cos(c+d\,x))}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))),x)
[Out] int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))), x)
```

3.67 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 501

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = -\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{9/4}de^{7/2}} + \frac{b^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{9/4}de^{7/2}} + \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} - \frac{2(5b^3+a(3a^2-8b^2)\cos(c+dx))}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+dx)}} + \frac{ab^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} + \frac{ab^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b^2\right)^2\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} - \frac{2a(3a^2-8b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5\left(a^2-b^2\right)^2de^4\sqrt{\sin(c+dx)}}$$

```
[Out] -b^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^
2+b^2)^(9/4)/d/e^(7/2)+b^(7/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(9/4)/d/e^(7/2)+2/5*(b-a*cos(d*x+c))/(a^2-b^2
)/d/e/(e*sin(d*x+c))^(5/2)-2/5*(5*b^3+a*(3*a^2-8*b^2)*cos(d*x+c))/(a^2-b^2)
^2/d/e^3/(e*sin(d*x+c))^(1/2)-a*b^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^
2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(b-(-a^2+b^2)^(1/2))/
(e*sin(d*x+c))^(1/2)-a*b^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/
4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2))
,2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(b+(-a^2+b^2)^(1/2))/(e*sin(d*
```

```
x+c))^(1/2)+2/5*a*(3*a^2-8*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+
c))^(1/2)/(a^2-b^2)^2/d/e^4/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2775, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = &-\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{7/2}\left(b^2-a^2\right)^{9/4}}\\ &+\frac{b^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{7/2}\left(b^2-a^2\right)^{9/4}}-\frac{2a(3a^2-8b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\left(a^2-b^2\right)^2\sqrt{\sin(c+dx)}}\\ &+\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{2(a(3a^2-8b^2)\cos(c+dx)+5b^3)}{5de^3\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}}\\ &+\frac{ab^3\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{de^3\left(a^2-b^2\right)^2\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}\\ &+\frac{ab^3\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{de^3\left(a^2-b^2\right)^2\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}\end{aligned}$$

```
[In] Int[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] -((b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/((-a^2 + b^2)^(9/4)*d*e^(7/2))) + (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(9/4)*d*e^(7/2)) +
 (2*(b - a*Cos[c + d*x]))/(5*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(5/2)) - (2*(
5*b^3 + a*(3*a^2 - 8*b^2)*Cos[c + d*x]))/(5*(a^2 - b^2)^2*d*e^3*Sqrt[e*Sin[
c + d*x]]) + (a*b^3*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*e^3*Sq
rt[e*Sin[c + d*x]]) + (a*b^3*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(b + Sqrt[-a^2 + b^2])
*d*e^3*Sqrt[e*Sin[c + d*x]]) - (2*a*(3*a^2 - 8*b^2)*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*(a^2 - b^2)^2*d*e^4*Sqrt[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2(b-a\cos(c+dx))}{5\,(a^2-b^2)\,de(e\sin(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+\frac{5b^2}{2}-\frac{3}{2}ab\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx}{5\,(a^2-b^2)\,e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{5\,(a^2-b^2)\,de(e\sin(c+dx))^{5/2}} - \frac{2(5b^3+a(3a^2-8b^2)\cos(c+dx))}{5\,(a^2-b^2)^2\,de^3\sqrt{e\sin(c+dx)}} + \frac{4\int \frac{\left(\frac{1}{4}\left(-3a^4+8a^2b^2+5b^4\right)-\frac{1}{4}ab\left(3a^2-8b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{5\,(a^2-b^2)^2\,e^4}$$

$$
\begin{aligned}
&= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} - \frac{2(5b^3+a(3a^2-8b^2)\cos(c+dx))}{5\left(a^2-b^2\right)^2 de^3\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{b^4\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{\left(a^2-b^2\right)^2e^4} - \frac{(a(3a^2-8b^2))\int\sqrt{e\sin(c+dx)}\,dx}{5\left(a^2-b^2\right)^2e^4} \\
&= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} - \frac{2(5b^3+a(3a^2-8b^2)\cos(c+dx))}{5\left(a^2-b^2\right)^2 de^3\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{(ab^3)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2\left(a^2-b^2\right)^2e^3} \\
&\quad + \frac{(ab^3)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2\left(a^2-b^2\right)^2e^3} \\
&\quad - \frac{b^5\text{Subst}\left(\int\frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx,x,e\sin(c+dx)\right)}{\left(a^2-b^2\right)^2de^3} \\
&\quad - \frac{\left(a(3a^2-8b^2)\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{5\left(a^2-b^2\right)^2e^4\sqrt{\sin(c+dx)}} \\
&= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} - \frac{2(5b^3+a(3a^2-8b^2)\cos(c+dx))}{5\left(a^2-b^2\right)^2 de^3\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2a(3a^2-8b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5\left(a^2-b^2\right)^2de^4\sqrt{\sin(c+dx)}} \\
&\quad - \frac{(2b^5)\,\text{Subst}\left(\int\frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{\left(a^2-b^2\right)^2de^3} \\
&\quad - \frac{\left(ab^3\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2\left(a^2-b^2\right)^2e^3\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{\left(ab^3\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{2\left(a^2-b^2\right)^2e^3\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$= \frac{2(b-a\cos(c+dx))}{5(a^2-b^2)de(e\sin(c+dx))^{5/2}} - \frac{2(5b^3+a(3a^2-8b^2)\cos(c+dx))}{5(a^2-b^2)^2de^3\sqrt{e\sin(c+dx)}}$$
$$+ \frac{ab^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}}$$
$$+ \frac{ab^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}}$$
$$- \frac{2a(3a^2-8b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5(a^2-b^2)^2de^4\sqrt{\sin(c+dx)}}$$
$$+ \frac{b^4\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)^2de^3}$$
$$- \frac{b^4\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)^2de^3}$$

$$= -\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{9/4}de^{7/2}} + \frac{b^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{9/4}de^{7/2}}$$
$$+ \frac{2(b-a\cos(c+dx))}{5(a^2-b^2)de(e\sin(c+dx))^{5/2}} - \frac{2(5b^3+a(3a^2-8b^2)\cos(c+dx))}{5(a^2-b^2)^2de^3\sqrt{e\sin(c+dx)}}$$
$$+ \frac{ab^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}}$$
$$+ \frac{ab^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}}$$
$$- \frac{2a(3a^2-8b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5(a^2-b^2)^2de^4\sqrt{\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.66 (sec) , antiderivative size = 811, normalized size of antiderivative = 1.62

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \frac{\sin^{\frac{7}{2}}(c+dx)\left(\frac{4a^2b-14b^3+(-7a^3+12ab^2)\cos(c+dx)+10b^3\cos(2(c+dx))+3a^3}{2(a^2-b^2)^2\sin^{\frac{5}{2}}(c+dx)}\right.}{}$$

```
[In] Integrate[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] (Sin[c + d*x]^(7/2)*((4*a^2*b - 14*b^3 + (-7*a^3 + 12*a*b^2)*Cos[c + d*x] +
 10*b^3*Cos[2*(c + d*x)] + 3*a^3*Cos[3*(c + d*x)] - 8*a*b^2*Cos[3*(c + d*x)
])/(2*(a^2 - b^2)^2*Sin[c + d*x]^(5/2)) - (Cos[c + d*x]*(a + b*Sqrt[Cos[c +
 d*x]^2])*((a*(3*a^2 - 8*b^2)*Sec[c + d*x]*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(
2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*Ar
cTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt
[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[
c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[
c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)))/(Sqrt[b
]*(-a^2 + b^2)) + (24*(3*a^4 - 8*a^2*b^2 - 5*b^4)*(((1/8 + I/8)*(2*ArcTan[1
 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 +
((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 +
b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c +
d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[
c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[
3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c
+ d*x]^(3/2))/(3*(a^2 - b^2))))/Sqrt[Cos[c + d*x]^2]))/(12*(a - b)^2*(a + b
)^2*(a + b*Cos[c + d*x]))))/(5*d*(e*Sin[c + d*x])^(7/2))
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 1007, normalized size of antiderivative = 2.01

method	result	size
default	Expression too large to display	1007

```
[In] int(1/(a+cos(d*x+c)*b)/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*e*b*(1/8*b^2/e^4/(a-b)^2/(a+b)^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln(
(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(
a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))
^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/
b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)-1))-1/5/e^2/(a+b)/(a-b)/(e*sin(d*x+c))^(5/2)+1/e^4/
(a-b)^2/(a+b)^2*b^2/(e*sin(d*x+c))^(1/2))-1/10/e^3*(12*(1-sin(d*x+c))^(1/2)
*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2
*2^(1/2))*a^4-32*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/
2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2*b^2-6*(1-sin(d*x+c))^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c))^(1/2),1
/2*2^(1/2))*a^4+16*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
7/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2*b^2-5*(1-sin(d*x+c))^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi((1-sin(d*x+c))^(1/2
),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^3-5*(1-sin(d*x+c
))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi((1-sin(d*x+c))^
(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^4+5*(1-sin(d*x+c))^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-
a^2+b^2)^(1/2))*b,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^3-5*(1-sin(d*x+c))^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(
b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/2))*b^4+12*a^4*cos(d*x+c)^4*sin(d*x+c)-32*a^
2*b^2*cos(d*x+c)^4*sin(d*x+c)-16*a^4*cos(d*x+c)^2*sin(d*x+c)+36*a^2*b^2*cos
(d*x+c)^2*sin(d*x+c))*a/(b+(-a^2+b^2)^(1/2))/(-b+(-a^2+b^2)^(1/2))/(a+b)^2/
(a-b)^2/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\,(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{{(e\,\sin(c+d\,x))}^{7/2}\,(a+b\,\cos(c+d\,x))}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))),x)
[Out] int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))), x)
```

3.68 $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 557

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx &= \frac{9a(-a^2+b^2)^{5/4}\,e^{11/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{11/2}d} \\ &+ \frac{9a(-a^2+b^2)^{5/4}\,e^{11/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{11/2}d} \\ &- \frac{3(21a^4-28a^2b^2+5b^4)\,e^6\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{7b^6d\sqrt{e\sin(c+dx)}} \\ &+ \frac{9a^2(a^2-b^2)^2\,e^6\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^6\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ &+ \frac{9a^2(a^2-b^2)^2\,e^6\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^6\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ &+ \frac{3e^5(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{7b^5d} \\ &- \frac{9e^3(7a-5b\cos(c+dx))(e\sin(c+dx))^{5/2}}{35b^3d} + \frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}\end{aligned}$$

```
[Out] 9/2*a*(-a^2+b^2)^(5/4)*e^(11/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/b^(11/2)/d+9/2*a*(-a^2+b^2)^(5/4)*e^(11/2)*arctanh(b^(1/
2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/d-9/35*e^3*(7*a-
5*b*cos(d*x+c))*(e*sin(d*x+c))^(5/2)/b^3/d+e*(e*sin(d*x+c))^(9/2)/b/d/(a+b*
cos(d*x+c))+3/7*(21*a^4-28*a^2*b^2+5*b^4)*e^6*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))*sin(d*x+c)^(1/2)/b^6/d/(e*sin(d*x+c))^(1/2)-9/2*a^2*(a^2-b^2)^2*e^6*(sin
```

```
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/
2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^6/
d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-9/2*a^2*(a^2-b^2)^2*e^6
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(c
os(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)
/b^6/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+3/7*e^5*(21*a*(a^2
-b^2)-b*(7*a^2-5*b^2)*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^5/d
```

Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{9ae^{11/2}(b^2-a^2)^{5/4}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{11/2}d}$$
$$+\frac{9ae^{11/2}(b^2-a^2)^{5/4}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{11/2}d}$$
$$+\frac{9a^2e^6(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2b^6d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}$$
$$+\frac{9a^2e^6(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2b^6d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}$$
$$+\frac{3e^5\sqrt{e\sin(c+dx)}(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))}{7b^5d}$$
$$-\frac{3e^6(21a^4-28a^2b^2+5b^4)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{7b^6d\sqrt{e\sin(c+dx)}}$$
$$-\frac{9e^3(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^3d}+\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}$$

```
[In] Int[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (9*a*(-a^2 + b^2)^(5/4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a
^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(11/2)*d) + (9*a*(-a^2 + b^2)^(5/4)*e^(11/2
)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*
b^(11/2)*d) - (3*(21*a^4 - 28*a^2*b^2 + 5*b^4)*e^6*EllipticF[(c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(7*b^6*d*Sqrt[e*Sin[c + d*x]]) + (9*a^2*(a^2 -
 b^2)^2*e^6*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]
*Sqrt[Sin[c + d*x]])/(2*b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c
 + d*x]]) + (9*a^2*(a^2 - b^2)^2*e^6*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]
), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^6*(a^2 - b*(b + Sqrt[-a^
```

```
2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (3*e^5*(21*a*(a^2 - b^2) - b*(7*a^2 -
5*b^2)*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(7*b^5*d) - (9*e^3*(7*a - 5*b*Co
s[c + d*x])*(e*Sin[c + d*x])^(5/2))/(35*b^3*d) + (e*(e*Sin[c + d*x])^(9/2))
/(b*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
```

tegersQ[2*m, 2*p]

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
```

^2, 0]

Rubi steps

$$\text{integral} = \frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} - \frac{(9e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx}{2b}$$

$$= -\frac{9e^3(7a-5b\cos(c+dx))(e\sin(c+dx))^{5/2}}{35b^3d} + \frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} - \frac{(9e^4)\int \frac{\left(-ab-\frac{1}{2}(7a^2-5b^2)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx}{7b^3}$$

$$= \frac{3e^5(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{7b^5d} - \frac{9e^3(7a-5b\cos(c+dx))(e\sin(c+dx))^{5/2}}{35b^3d} + \frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} - \frac{(6e^6)\int \frac{\frac{1}{2}ab(7a^2-8b^2)+\frac{1}{4}(21a^4-28a^2b^2+5b^4)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{7b^5}$$

$$= \frac{3e^5(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{7b^5d} - \frac{9e^3(7a-5b\cos(c+dx))(e\sin(c+dx))^{5/2}}{35b^3d} + \frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} + \frac{\left(9a(a^2-b^2)^2e^6\right)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2b^6} - \frac{(3(21a^4-28a^2b^2+5b^4)e^6)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{14b^6}$$

$$= \frac{3e^5(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{7b^5d} - \frac{9e^3(7a-5b\cos(c+dx))(e\sin(c+dx))^{5/2}}{35b^3d} + \frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} - \frac{\left(9a^2(-a^2+b^2)^{3/2}e^6\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b^6} - \frac{\left(9a^2(-a^2+b^2)^{3/2}e^6\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4b^6} - \frac{\left(9a(a^2-b^2)^2e^7\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx, x, e\sin(c+dx)\right)}{2b^5d} - \frac{\left(3(21a^4-28a^2b^2+5b^4)e^6\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{14b^6\sqrt{e\sin(c+dx)}}$$

$$
\begin{aligned}
&= -\frac{3(21a^4-28a^2b^2+5b^4)\,e^6\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{7b^6d\sqrt{e\sin(c+dx)}} \\
&\quad+\frac{3e^5(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{7b^5d} \\
&\quad-\frac{9e^3(7a-5b\cos(c+dx))(e\sin(c+dx))^{5/2}}{35b^3d}+\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} \\
&\quad-\frac{\left(9a(a^2-b^2)^2e^7\right)\text{Subst}\left(\int\frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{b^5d} \\
&\quad-\frac{\left(9a^2(-a^2+b^2)^{3/2}e^6\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b^6\sqrt{e\sin(c+dx)}} \\
&\quad-\frac{\left(9a^2(-a^2+b^2)^{3/2}e^6\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4b^6\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3(21a^4-28a^2b^2+5b^4)\,e^6\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{7b^6d\sqrt{e\sin(c+dx)}} \\
&\quad+\frac{9a^2(-a^2+b^2)^{3/2}e^6\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^6\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad-\frac{9a^2(-a^2+b^2)^{3/2}e^6\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^6\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad+\frac{3e^5(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{7b^5d} \\
&\quad-\frac{9e^3(7a-5b\cos(c+dx))(e\sin(c+dx))^{5/2}}{35b^3d}+\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} \\
&\quad+\frac{\left(9a(-a^2+b^2)^{3/2}e^6\right)\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2b^5d} \\
&\quad+\frac{\left(9a(-a^2+b^2)^{3/2}e^6\right)\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2b^5d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9a(-a^2+b^2)^{5/4}\, e^{11/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{11/2}d} \\
&+ \frac{9a(-a^2+b^2)^{5/4}\, e^{11/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{11/2}d} \\
&- \frac{3(21a^4-28a^2b^2+5b^4)\, e^6\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{7b^6d\sqrt{e\sin(c+dx)}} \\
&+ \frac{9a^2(-a^2+b^2)^{3/2}\, e^6\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^6\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{9a^2(-a^2+b^2)^{3/2}\, e^6\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^6\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{3e^5(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{7b^5d} \\
&- \frac{9e^3(7a-5b\cos(c+dx))(e\sin(c+dx))^{5/2}}{35b^3d} + \frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.67 (sec) , antiderivative size = 2029, normalized size of antiderivative = 3.64

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((((-28*a^2 + 17*b^2)*Cos[c + d*x])/(14*b^4) + (-a^2 + b^2)^2/(b^5*(a + b*C
os[c + d*x])) + (2*a*Cos[2*(c + d*x)])/(5*b^3) - Cos[3*(c + d*x)]/(14*b^2))
*Csc[c + d*x]^5*(e*Sin[c + d*x])^(11/2))/d - ((e*Sin[c + d*x])^(11/2)*((2*(
35*a^4 - 126*a^2*b^2 + 75*b^4)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^
2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/
4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)]
- Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]
] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/
4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3
/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Si
n[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((
-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x
]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (
b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4,
Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 +
```

```
b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) +
 (2*(70*a^3*b - 93*a*b^3)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((
-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^
2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 +
 b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sq
rt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + b^2)
^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*S
in[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*
(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]
^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^
2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, S
in[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b
^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2]
) + ((-140*a^3*b + 147*a*b^3)*Cos[c + d*x]*Cos[2*(c + d*x)]*(a + b*Sqrt[1 -
 Sin[c + d*x]^2])*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*
Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1
/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-
a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + ((1/4 - I/4)*(-2*a^2 + b^
2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d
*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a
^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Sin
[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c
+ d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 -
b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*Appell
F1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2
*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a
^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*S
in[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]
^2)))))/((a + b*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x]^
2])))/(70*b^5*d*Sin[c + d*x]^(11/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1657 vs. $2(579) = 1158$.

Time = 18.68 (sec) , antiderivative size = 1658, normalized size of antiderivative = 2.98

method	result	size
default	Expression too large to display	1658

```
[In] int((e*sin(d*x+c))^(11/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-4*e^3*a*b*(-1/5/b^6*(e*sin(d*x+c))^(1/2)*e^2*(b^2*cos(d*x+c)^2+10*a^2-11*
```

b^2)+e^4/b^6*((-1/4*a^4+1/2*a^2*b^2-1/4*b^4)*(e*sin(d*x+c))^(1/2)/(-b^2*cos
(d*x+c)^2*e^2+a^2*e^2)+9/32*(a^4-2*a^2*b^2+b^4)*(e^2*(a^2-b^2)/b^2)^(1/4)/(
a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin
(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b
^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*a
rctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^
(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))+(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)*e^6*(1/7/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(-2*b^4*co
s(d*x+c)^4*sin(d*x+c)+35*a^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*si
n(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-49*a^2*b^2*(1-si
n(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*
x+c))^(1/2),1/2*2^(1/2))+11*b^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-14*a^2*b^2*co
s(d*x+c)^2*sin(d*x+c)+10*b^4*cos(d*x+c)^2*sin(d*x+c))-(-7*a^6+15*a^4*b^2-9*
a^2*b^4+b^6)/b^6*(-1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)
^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1
/2))+1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*Elli
pticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))-2*a^2*(a
^6-3*a^4*b^2+3*a^2*b^4-b^6)/b^6*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*si
n(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/(-a^2+b^2)^(1/
2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(
1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/
2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(
1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b
*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2)
,1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b
*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2)
,1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(11/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\,\sin\left(c+d\,x\right))^{11/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^2, x)
```

3.69 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 473

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx &= -\frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{9/2}d} \\
&+ \frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{9/2}d} \\
&- \frac{7a^2(a^2-b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{7a^2(a^2-b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^5\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{7(5a^2-3b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}} \\
&- \frac{7e^3(5a-3b\cos(c+dx))(e\sin(c+dx))^{3/2}}{15b^3d} + \frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}
\end{aligned}$$

```
[Out] -7/2*a*(-a^2+b^2)^(3/4)*e^(9/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/b^(9/2)/d+7/2*a*(-a^2+b^2)^(3/4)*e^(9/2)*arctanh(b^(1/2)
*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/d-7/15*e^3*(5*a-3*b
*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/b^3/d+e*(e*sin(d*x+c))^(7/2)/b/d/(a+b*cos
(d*x+c))+7/2*a^2*(a^2-b^2)*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1
/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^5/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1
/2)+7/2*a^2*(a^2-b^2)*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),
```

```
2^(1/2))*sin(d*x+c)^(1/2)/b^5/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-7
/5*(5*a^2-3*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b
^4/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2944, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}dx = &-\frac{7ae^{9/2}(b^2-a^2)^{3/4}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}\\ &+\frac{7ae^{9/2}(b^2-a^2)^{3/4}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}\\ &-\frac{7a^2e^5(a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2b^5d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}\\ &-\frac{7a^2e^5(a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2b^5d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}\\ &+\frac{7e^4(5a^2-3b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}}\\ &-\frac{7e^3(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^3d}+\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}\end{aligned}$$

```
[In] Int[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-7*a*(-a^2 + b^2)^(3/4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a
^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(9/2)*d) + (7*a*(-a^2 + b^2)^(3/4)*e^(9/2)*
ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^
(9/2)*d) - (7*a^2*(a^2 - b^2)*e^5*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^5*(b - Sqrt[-a^2 + b^2])*d*
Sqrt[e*Sin[c + d*x]]) - (7*a^2*(a^2 - b^2)*e^5*EllipticPi[(2*b)/(b + Sqrt[-
a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^5*(b + Sqrt[-a
^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (7*(5*a^2 - 3*b^2)*e^4*EllipticE[(c -
Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*b^4*d*Sqrt[Sin[c + d*x]]) - (7*e
^3*(5*a - 3*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*b^3*d) + (e*(e*Sin[
c + d*x])^(7/2))/(b*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = \frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} - \frac{(7e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx}{2b}$$

$$= -\frac{7e^3(5a-3b\cos(c+dx))(e\sin(c+dx))^{3/2}}{15b^3d} + \frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} - \frac{(7e^4)\int\frac{\left(-ab-\frac{1}{2}(5a^2-3b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{5b^3}$$

$$= -\frac{7e^3(5a-3b\cos(c+dx))(e\sin(c+dx))^{3/2}}{15b^3d} + \frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} + \frac{(7(5a^2-3b^2)\,e^4)\int\sqrt{e\sin(c+dx)}\,dx}{10b^4} - \frac{(7a(a^2-b^2)\,e^4)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{2b^4}$$

$$= -\frac{7e^3(5a-3b\cos(c+dx))(e\sin(c+dx))^{3/2}}{15b^3d} + \frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} + \frac{(7a^2(a^2-b^2)\,e^5)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b^5} - \frac{(7a^2(a^2-b^2)\,e^5)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4b^5} + \frac{(7a(a^2-b^2)\,e^5)\,\text{Subst}\left(\int\frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx, x, e\sin(c+dx)\right)}{2b^3d} + \frac{\left(7(5a^2-3b^2)\,e^4\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{10b^4\sqrt{\sin(c+dx)}}$$

$$= \frac{7(5a^2-3b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}} - \frac{7e^3(5a-3b\cos(c+dx))(e\sin(c+dx))^{3/2}}{15b^3d} + \frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} + \frac{(7a(a^2-b^2)\,e^5)\,\text{Subst}\left(\int\frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{b^3d} + \frac{\left(7a^2(a^2-b^2)\,e^5\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b^5\sqrt{e\sin(c+dx)}} - \frac{\left(7a^2(a^2-b^2)\,e^5\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4b^5\sqrt{e\sin(c+dx)}}$$

$$= -\frac{7a^2(a^2-b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{7a^2(a^2-b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^5\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{7(5a^2-3b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}}$$
$$-\frac{7e^3(5a-3b\cos(c+dx))(e\sin(c+dx))^{3/2}}{15b^3d}+\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}$$
$$-\frac{(7a(a^2-b^2)\,e^5)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2b^4d}$$
$$+\frac{(7a(a^2-b^2)\,e^5)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2b^4d}$$

$$= -\frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{9/2}d}$$
$$+\frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{9/2}d}$$
$$-\frac{7a^2(a^2-b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{7a^2(a^2-b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^5\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{7(5a^2-3b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}}$$
$$-\frac{7e^3(5a-3b\cos(c+dx))(e\sin(c+dx))^{3/2}}{15b^3d}+\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 13.43 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.56

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{(e\sin(c+dx))^{9/2}\left(8b^{3/2}(-35a^2+18b^2-14ab\cos(c+dx)+3b^2\cos(2(c+dx)\right.}{}$$

```
[In] Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((e*Sin[c + d*x])^(9/2)*(8*b^(3/2)*(-35*a^2 + 18*b^2 - 14*a*b*Cos[c + d*x]
+ 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x]^(3/2) + 7*Cos[c + d*x]*(a + b*Sqrt[C
os[c + d*x]^2])*(-(((5*a^2 - 3*b^2)*Sec[c + d*x]*(3*Sqrt[2]*a*(a^2 - b^2)^(
3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)]
- 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Lo
g[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*S
qrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4
, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)))/(
a^2 - b^2)) + (48*a*b^(5/2)*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sq
rt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]
*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2
+ b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c
+ d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[
c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2
- b^2))))/Sqrt[Cos[c + d*x]^2])))/(120*b^(9/2)*d*(a + b*Cos[c + d*x])*Sin[c
 + d*x]^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1627 vs. 2(499) = 998.

Time = 17.78 (sec) , antiderivative size = 1628, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	1628

```
[In] int((e*sin(d*x+c))^(9/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-4*e^3*a*b*(1/3*(e*sin(d*x+c))^(3/2)/b^4-e^2/b^4*((-1/4*a^2+1/4*b^2)*(e*si
n(d*x+c))^(3/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)+1/8*(7/4*a^2-7/4*b^2)/b^2/(
```

```
e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4
)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/
2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*a
rctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))+(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)*e^5*(-1/5/b^4/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(
30*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-16*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-1
5*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1
-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+8*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+2*b
^2*cos(d*x+c)^4-2*b^2*cos(d*x+c)^2)-(5*a^4-6*a^2*b^2+b^4)/b^4*(-1/2/b^2*(1-
sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(
1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2
)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(
1/2)))+2*a^2*(a^4-2*a^2*b^2+b^4)/b^4*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)-1/2/a^2/(a^2-b^2)*(
1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+1/4/a^2/(a^2
-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/8/(
a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d
*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin
(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(
-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-
a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),
1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*E
llipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))))/cos(
d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(9/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\,\sin(c+d\,x))^{9/2}}{(a+b\,\cos(c+d\,x))^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^2, x)
```

3.70 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 487

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx &= \frac{5a\sqrt[4]{-a^2+b^2}e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{7/2}d}\\ &+\frac{5a\sqrt[4]{-a^2+b^2}e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{7/2}d}\\ &+\frac{5(3a^2-b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}}\\ &-\frac{5a^2(a^2-b^2)\,e^4\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\ &-\frac{5a^2(a^2-b^2)\,e^4\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^4\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\ &-\frac{5e^3(3a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d}+\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}\end{aligned}$$

```
[Out] 5/2*a*(-a^2+b^2)^(1/4)*e^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/b^(7/2)/d+5/2*a*(-a^2+b^2)^(1/4)*e^(7/2)*arctanh(b^(1/2)*
(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/d+e*(e*sin(d*x+c))^(
5/2)/b/d/(a+b*cos(d*x+c))-5/3*(3*a^2-b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))*sin(d*x+c)^(1/2)/b^4/d/(e*sin(d*x+c))^(1/2)+5/2*a^2*(a^2-b^2)*e^4*(sin(1
/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*
c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/
(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+5/2*a^2*(a^2-b^2)*e^4*(si
```

```
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1
/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4
/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-5/3*e^3*(3*a-b*cos(d*x
+c))*(e*sin(d*x+c))^(1/2)/b^3/d
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d} + \frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d} + \frac{5e^4(3a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3b^4d\sqrt{e\sin(c+dx)}} - \frac{5a^2e^4(a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2b^4d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}} - \frac{5a^2e^4(a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2b^4d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}} - \frac{5e^3\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^3d} + \frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}$$

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (5*a*(-a^2 + b^2)^(1/4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(7/2)*d) + (5*a*(-a^2 + b^2)^(1/4)*e^(7/2)*A
rcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(
7/2)*d) + (5*(3*a^2 - b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(3*b^4*d*Sqrt[e*Sin[c + d*x]]) - (5*a^2*(a^2 - b^2)*e^4*EllipticPi
[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(
2*b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (5*a^2*(a^
2 - b^2)*e^4*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2
]*Sqrt[Sin[c + d*x]])/(2*b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[
c + d*x]]) - (5*e^3*(3*a - b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*b^3*d)
+ (e*(e*Sin[c + d*x])^(5/2))/(b*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = \frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} - \frac{(5e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx}{2b}$$

$$= -\frac{5e^3(3a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} + \frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} - \frac{(5e^4)\int \frac{-ab-\frac{1}{2}(3a^2-b^2)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3b^3}$$

$$= -\frac{5e^3(3a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} + \frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} - \frac{(5a(a^2-b^2)e^4)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2b^4} + \frac{(5(3a^2-b^2)e^4)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{6b^4}$$

$$= -\frac{5e^3(3a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} + \frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} - \frac{\left(5a^2\sqrt{-a^2+b^2}e^4\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b^4} - \frac{\left(5a^2\sqrt{-a^2+b^2}e^4\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4b^4} + \frac{(5a(a^2-b^2)e^5)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx, x, e\sin(c+dx)\right)}{2b^3d} + \frac{\left(5(3a^2-b^2)e^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{6b^4\sqrt{e\sin(c+dx)}}$$

$$= \frac{5(3a^2-b^2)e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}} - \frac{5e^3(3a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} + \frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} + \frac{(5a(a^2-b^2)e^5)\,\text{Subst}\left(\int \frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{b^3d} - \frac{\left(5a^2\sqrt{-a^2+b^2}e^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b^4\sqrt{e\sin(c+dx)}} - \frac{\left(5a^2\sqrt{-a^2+b^2}e^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4b^4\sqrt{e\sin(c+dx)}}$$

$$= \frac{5(3a^2-b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}}$$
$$+\frac{5a^2\sqrt{-a^2+b^2}e^4\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^4\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{5a^2\sqrt{-a^2+b^2}e^4\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^4\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{5e^3(3a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d}+\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}$$
$$+\frac{\left(5a\sqrt{-a^2+b^2}e^4\right)\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2b^3d}$$
$$+\frac{\left(5a\sqrt{-a^2+b^2}e^4\right)\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2b^3d}$$

$$= \frac{5a\sqrt[4]{-a^2+b^2}e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{7/2}d}$$
$$+\frac{5a\sqrt[4]{-a^2+b^2}e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{7/2}d}$$
$$+\frac{5(3a^2-b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}}$$
$$+\frac{5a^2\sqrt{-a^2+b^2}e^4\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^4\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{5a^2\sqrt{-a^2+b^2}e^4\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^4\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{5e^3(3a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d}+\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 13.65 (sec) , antiderivative size = 1956, normalized size of antiderivative = 4.02

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{\left(\frac{2\cos(c+dx)}{3b^2}+\frac{-a^2+b^2}{b^3(a+b\cos(c+dx))}\right)\csc^3(c+dx)(e\sin(c+dx))^{7/2}}{d}$$

$$+\frac{(e\sin(c+dx))^{7/2}\left(\frac{2(3a^2-5b^2)\cos^2(c+dx)\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\mathrm{lo}\right.}{}\right.}{}\right.}{}$$

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (((2*Cos[c + d*x])/(3*b^2) + (-a^2 + b^2)/(b^3*(a + b*Cos[c + d*x])))*Csc[c
 + d*x]^3*(e*Sin[c + d*x])^(7/2))/d + ((e*Sin[c + d*x])^(7/2)*((2*(3*a^2 -
5*b^2)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (
Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt
[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Lo
g[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*
AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^
2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[
1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(
2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^
2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin
[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2
)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (8*a*b*Cos[c + d*x]*(a
+ b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)
*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sq
rt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log
[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/
2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c +
d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4,
Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4,
 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 +
b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c
 + d*x])*Sqrt[1 - Sin[c + d*x]^2]) - (6*a*b*Cos[c + d*x]*Cos[2*(c + d*x)]*(
a + b*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1
```

```
 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2
)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin
[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + ((1/4 - I/4
)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*
Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/
4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)
^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4))
 + (4*Sqrt[Sin[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^
2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) +
 (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c +
d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a
^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c
 + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c +
d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1
+ Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 -
 Sin[c + d*x]^2])))/(6*b^3*d*Sin[c + d*x]^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1500 vs. $2(513) = 1026$.

Time = 17.46 (sec) , antiderivative size = 1501, normalized size of antiderivative = 3.08

method	result	size
default	Expression too large to display	1501

```
[In] int((e*sin(d*x+c))^(7/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-4*e^3*a*b*((e*sin(d*x+c))^(1/2)/b^4-e^2/b^4*((-1/4*a^2+1/4*b^2)*(e*sin(d*
x+c))^(1/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)+5/32*(a^2-b^2)*(e^2*(a^2-b^2)/b
^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(
1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-
(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^
(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+
2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))+(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)*e^4*(-1/3/b^4/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2
)*(9*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF
((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-
2*b^2*cos(d*x+c)^2*sin(d*x+c))-1/b^4*(5*a^4-6*a^2*b^2+b^4)*(-1/2/(-a^2+b^2)
^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c
))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1/2)/b*(1-si
n(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin
(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+
```

```
(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+2*a^2*(a^4-2*a^2*b^2+b^4)/b^4*(1/2*b^2/e/
a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+1/4
/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/
2))-5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2
)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+
1/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2
)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+
5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)
*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/4/
a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)
*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/co
s(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(c+d\,x))^{7/2}}{(a+b\cos(c+d\,x))^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^2, x)
```

3.71 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 404

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx &= -\frac{3ae^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}d}\\
&+\frac{3ae^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}d}\\
&+\frac{3a^2e^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{3a^2e^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^3\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{3e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}}+\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}\end{aligned}$$

```
[Out] -3/2*a*e^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/b^(5/2)/(-a^2+b^2)^(1/4)/d+3/2*a*e^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(
1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(1/4)/d+e*(e*sin(d*x+c))^
(3/2)/b/d/(a+b*cos(d*x+c))-3/2*a^2*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2
+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^3/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*
x+c))^(1/2)-3/2*a^2*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^
(1/2))*sin(d*x+c)^(1/2)/b^3/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+3*e
^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(
cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^2/d/sin(d*x+c)^(1
/2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2772, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = -\frac{3ae^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}} + \frac{3ae^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}} + \frac{3a^2e^3\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}} + \frac{3a^2e^3\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2b^3d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}} + \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))} - \frac{3e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}}$$

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-3*a*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*b^(5/2)*(-a^2 + b^2)^(1/4)*d) + (3*a*e^(5/2)*ArcTanh[(Sqrt[b]*Sq
rt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(5/2)*(-a^2 + b^2)^
(1/4)*d) + (3*a^2*e^3*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^3*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c
 + d*x]]) + (3*a^2*e^3*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^3*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[
c + d*x]]) - (3*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/
(b^2*d*Sqrt[Sin[c + d*x]]) + (e*(e*Sin[c + d*x])^(3/2))/(b*d*(a + b*Cos[c +
 d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))} - \frac{(3e^2)\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{2b} \\
&= \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))} - \frac{(3e^2)\int \sqrt{e\sin(c+dx)}\,dx}{2b^2} + \frac{(3ae^2)\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{2b^2} \\
&= \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))} - \frac{(3a^2e^3)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b^3} \\
&\quad + \frac{(3a^2e^3)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4b^3} \\
&\quad - \frac{(3ae^3)\operatorname{Subst}\left(\int \frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx, x, e\sin(c+dx)\right)}{2bd} \\
&\quad - \frac{\left(3e^2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{2b^2\sqrt{\sin(c+dx)}}
\end{aligned}$$

$$= -\frac{3e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}} + \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}$$
$$- \frac{(3ae^3)\,\text{Subst}\left(\int \frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{bd}$$
$$- \frac{\left(3a^2e^3\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b^3\sqrt{e\sin(c+dx)}}$$
$$+ \frac{\left(3a^2e^3\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4b^3\sqrt{e\sin(c+dx)}}$$

$$= \frac{3a^2e^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{3a^2e^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2b^3\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{3e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}} + \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}$$
$$+ \frac{(3ae^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{2b^2d}$$
$$- \frac{(3ae^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{2b^2d}$$

$$= -\frac{3ae^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}d} + \frac{3ae^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}d}$$
$$+ \frac{3a^2e^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{3a^2e^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2b^3\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{3e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}} + \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.90 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.91

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{(e\sin(c+dx))^{5/2}\left(8b^{3/2}\csc(c+dx)+\frac{\left(a+b\sqrt{\cos^2(c+dx)}\right)\left(3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(2\arctan\left(1-\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((e*Sin[c + d*x])^(5/2)*(8*b^(3/2)*Csc[c + d*x] + ((a + b*Sqrt[Cos[c + d*x]
^2])*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c +
 d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(
5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
 + b^2)]*Sin[c + d*x]^(3/2)))/((a^2 - b^2)*Sin[c + d*x]^(5/2))))/(8*b^(5/2)
*d*(a + b*Cos[c + d*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1667 vs. 2(436) = 872.

Time = 15.83 (sec) , antiderivative size = 1668, normalized size of antiderivative = 4.13

method	result	size
default	Expression too large to display	1668

```
[In] int((e*sin(d*x+c))^(5/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-2*e^3*a*b*(-1/2*(e*sin(d*x+c))^(3/2)/b^2/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)+
3/16/b^4/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(
d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1
/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))
+1/4*e^3*a^2*(3*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(5/2)*EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2))
,1/2*2^(1/2))*b^2-3*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(5/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2
))*b,1/2*2^(1/2))*b^2-12*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(5/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^3+6*(1-sin(d*x+c))
```

```
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/
2),1/2*2^(1/2))*b^3+3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(5/2)*EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2
))*b^3+3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*Ellip
ticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/2))*b^3+3*(-a^
2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*E
llipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2-3*
(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)*EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^
2-3*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/2)
)*a^2+3*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(
1/2))*b^2-12*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*E
llipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2*b+12*(1-sin(d*x+c))^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^
(1/2))*b^3+6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*E
llipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2*b-6*(1-sin(d*x+c))^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(
1/2))*b^3+3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*El
lipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2*b-3
*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1
-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3+3*(1-sin(d*x+c
))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^
(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/2))*a^2*b-3*(1-sin(d*x+c))^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+
(-a^2+b^2)^(1/2))*b,1/2*2^(1/2))*b^3-4*b^3*sin(d*x+c)^4+4*b^3*sin(d*x+c)^2)
/b^3/(b+(-a^2+b^2)^(1/2))/(-b+(-a^2+b^2)^(1/2))/(-b^2*cos(d*x+c)^2+a^2)/cos
(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\,\sin(c+d\,x))^{5/2}}{(a+b\,\cos(c+d\,x))^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2, x)
```

3.72 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 418

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx &= \frac{ae^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} \\
&+ \frac{ae^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2d\sqrt{e\sin(c+dx)}} \\
&+ \frac{a^2e^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{a^2e^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^2\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} + \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}
\end{aligned}$$

```
[Out] 1/2*a*e^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))
/b^(3/2)/(-a^2+b^2)^(3/4)/d+1/2*a*e^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1
/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(3/4)/d+e^2*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+
1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/b^2/d/(e*sin(d*x+c))^(1/2)-1/2*a^2*e^2*(
sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos
(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b
^2/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-1/2*a^2*e^2*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+
1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/d/(a
^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+e*(e*sin(d*x+c))^(1/2)/b/d/
(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2772, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{ae^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}d\left(b^2-a^2\right)^{3/4}} + \frac{ae^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}d\left(b^2-a^2\right)^{3/4}} + \frac{a^2e^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}} + \frac{a^2e^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2b^2d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}} + \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))} - \frac{e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{b^2d\sqrt{e\sin(c+dx)}}$$

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (a*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(2*b^(3/2)*(-a^2 + b^2)^(3/4)*d) + (a*e^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e*
Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(3/2)*(-a^2 + b^2)^(3/4)
*d) - (e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^2*d*Sqrt
[e*Sin[c + d*x]]) + (a^2*e^2*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]
))*d*Sqrt[e*Sin[c + d*x]]) + (a^2*e^2*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2
]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^2*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (e*Sqrt[e*Sin[c + d*x]])/(b*d*(a + b*
Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))} - \frac{e^2\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2b}\\
&= \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))} - \frac{e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{2b^2} + \frac{(ae^2)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2b^2}\\
&= \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))} - \frac{(a^2e^2)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b^2\sqrt{-a^2+b^2}}\\
&\quad - \frac{(a^2e^2)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4b^2\sqrt{-a^2+b^2}}\\
&\quad - \frac{(ae^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx, x, e\sin(c+dx)\right)}{2bd}\\
&\quad - \frac{\left(e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{2b^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$= -\frac{e^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2 d\sqrt{e\sin(c+dx)}} + \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))} - \frac{(ae^3)\operatorname{Subst}\left(\int \frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{bd} - \frac{\left(a^2e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b^2\sqrt{-a^2+b^2}\sqrt{e\sin(c+dx)}} - \frac{\left(a^2e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4b^2\sqrt{-a^2+b^2}\sqrt{e\sin(c+dx)}}$$

$$= -\frac{e^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2 d\sqrt{e\sin(c+dx)}} + \frac{a^2e^2 \operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} - \frac{a^2e^2 \operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^2\sqrt{-a^2+b^2}\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))} + \frac{(ae^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{2b\sqrt{-a^2+b^2}d} + \frac{(ae^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{2b\sqrt{-a^2+b^2}d}$$

$$= \frac{ae^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} + \frac{ae^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{e^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2 d\sqrt{e\sin(c+dx)}} + \frac{a^2e^2 \operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} - \frac{a^2e^2 \operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^2\sqrt{-a^2+b^2}\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.49 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.33

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{(e\sin(c+dx))^{3/2}\left(\csc(c+dx)-\frac{\left(a+b\sqrt{\cos^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\,\mathrm{arc}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((e*Sin[c + d*x])^(3/2)*(Csc[c + d*x] - ((a + b*Sqrt[Cos[c + d*x]^2])*((a*(
-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*A
rcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqr
t[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin
[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[S
in[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5
*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x
]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2 + b
^2*Sin[c + d*x]^2)*(-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]
^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/
4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF
1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin
[c + d*x]^2))))/Sin[c + d*x]^(3/2)))/(b*d*(a + b*Cos[c + d*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1369 vs. 2(449) = 898.

Time = 4.17 (sec) , antiderivative size = 1370, normalized size of antiderivative = 3.28

method	result	size
default	Expression too large to display	1370

```
[In] int((e*sin(d*x+c))^(3/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-4*e^3*a*b*(-1/4/b^2*(e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)+
1/32/b^2*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x
+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b
^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(
```

```
1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)
*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(
d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^2*(1/b^2*(1-sin(d*x+
c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-(-3*a^2+b^2)/b^2*(-1/2
/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((
1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1
/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^
(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))-2*a^2*(a^2-b^2)/b^2*(1/2*b^2/e
/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+1/
4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1
/2))-5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/
2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))
+1/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/
2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))
+5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/
2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b
)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/4
/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/
2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b
)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/c
os(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\,\sin(c+d\,x))^{3/2}}{(a+b\,\cos(c+d\,x))^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2, x)
```

3.73 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 438

$$
\begin{aligned}
\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx &= \frac{a\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\left(-a^2+b^2\right)^{5/4}d} - \frac{a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\left(-a^2+b^2\right)^{5/4}d} \\
&\quad + \frac{a^2e\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b\left(a^2-b^2\right)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{a^2e\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b\left(a^2-b^2\right)\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)d\sqrt{\sin(c+dx)}} \\
&\quad - \frac{b(e\sin(c+dx))^{3/2}}{\left(a^2-b^2\right)de(a+b\cos(c+dx))}
\end{aligned}
$$

```
[Out] -b*(e*sin(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*cos(d*x+c))+1/2*a*arctan(b^(1/2)
*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(5/4)/d/
b^(1/2)-1/2*a*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)*e^(1/2)/(-a^2+b^2)^(5/4)/d/b^(1/2)-1/2*a^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2
)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/
(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b-(-a^2+b^2)^
(1/2))/(e*sin(d*x+c))^(1/2)-1/2*a^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+
b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b+(-a^2+b^2)^(1/2))/(e
*sin(d*x+c))^(1/2)-(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2
*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^
2-b^2)/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2773, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \frac{a\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}d\,(b^2-a^2)^{5/4}} - \frac{a\sqrt{e}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}d\,(b^2-a^2)^{5/4}} - \frac{b(e\sin(c+dx))^{3/2}}{de\,(a^2-b^2)\,(a+b\cos(c+dx))} + \frac{E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\,(a^2-b^2)\sqrt{\sin(c+dx)}} + \frac{a^2e\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2bd\,(a^2-b^2)\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}} + \frac{a^2e\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2bd\,(a^2-b^2)\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}$$

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (a*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(2*Sqrt[b]*(-a^2 + b^2)^(5/4)*d) - (a*Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*
Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*Sqrt[b]*(-a^2 + b^2)^(5/4)
*d) + (a^2*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2
]*Sqrt[Sin[c + d*x]])/(2*b*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[
c + d*x]]) + (a^2*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt
[e*Sin[c + d*x]]) + (EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])
/((a^2 - b^2)*d*Sqrt[Sin[c + d*x]]) - (b*(e*Sin[c + d*x])^(3/2))/((a^2 - b^
2)*d*e*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b(e\sin(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\cos(c+dx))}+\frac{\int \frac{\left(-a-\frac{1}{2}b\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{-a^2+b^2}\\
&= -\frac{b(e\sin(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\cos(c+dx))}+\frac{\int \sqrt{e\sin(c+dx)}\,dx}{2\,(a^2-b^2)}+\frac{a\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{2\,(a^2-b^2)}\\
&= -\frac{b(e\sin(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\cos(c+dx))}-\frac{(a^2e)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b\,(a^2-b^2)}\\
&\quad+\frac{(a^2e)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4b\,(a^2-b^2)}\\
&\quad-\frac{(abe)\text{Subst}\left(\int \frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx,x,e\sin(c+dx)\right)}{2\,(a^2-b^2)\,d}\\
&\quad+\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}\,dx}{2\,(a^2-b^2)\sqrt{\sin(c+dx)}}
\end{aligned}$$

$$= \frac{E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{\sin(c+dx)}} - \frac{b(e\sin(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\cos(c+dx))} - \frac{(abe)\text{Subst}\left(\int \frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)\,d} - \frac{\left(a^2e\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b\,(a^2-b^2)\sqrt{e\sin(c+dx)}} + \frac{\left(a^2e\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4b\,(a^2-b^2)\sqrt{e\sin(c+dx)}}$$

$$= \frac{a^2e\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2b\,(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{a^2e\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2b\,(a^2-b^2)\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{\sin(c+dx)}} - \frac{b(e\sin(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\cos(c+dx))} + \frac{(ae)\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{2\,(a^2-b^2)\,d} - \frac{(ae)\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{2\,(a^2-b^2)\,d}$$

$$= \frac{a\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\,(-a^2+b^2)^{5/4}\,d} - \frac{a\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\,(-a^2+b^2)^{5/4}\,d} + \frac{a^2e\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2b\,(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{a^2e\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2b\,(a^2-b^2)\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{\sin(c+dx)}} - \frac{b(e\sin(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\cos(c+dx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 13.70 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \frac{b\sin(c+dx)\sqrt{e\sin(c+dx)}}{(-a^2+b^2)\,d(a+b\cos(c+dx))}$$

$$+\frac{\sqrt{e\sin(c+dx)}\left(\frac{\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{\ldots}\right.\right.\right.}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (b*Sin[c + d*x]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)*d*(a + b*Cos[c + d*x]))
 + (Sqrt[e*Sin[c + d*x]]*((Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2
*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*Arc
Tan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[
a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c
 + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin
[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c
 + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sq
rt[1 - Sin[c + d*x]^2]))/(12*Sqrt[b]*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 -
 Sin[c + d*x]^2)) + (4*a*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*
Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqr
t[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 +
I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[
Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1,
 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)
)/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*
Sqrt[1 - Sin[c + d*x]^2])))/(2*(a - b)*(a + b)*d*Sqrt[Sin[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. 2(471) = 942.

Time = 4.09 (sec) , antiderivative size = 1306, normalized size of antiderivative = 2.98

method	result	size
default	Expression too large to display	1306

```
[In] int((e*sin(d*x+c))^(1/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-4*e^3*a*b*(1/4*(e*sin(d*x+c))^(3/2)/(a^2*e^2-b^2*e^2)/(-b^2*cos(d*x+c)^2*
e^2+a^2*e^2)+1/32/(a^2*e^2-b^2*e^2)/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(
ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^
2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+
c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^
(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e*(1/2/b^
2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2
),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^
2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/
2*2^(1/2))+2*a^2*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*
x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)-1/2/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*E
llipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^
(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(1-sin(
d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-
a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*si
n(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a
^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1
/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*El
lipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2
/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x
+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e \sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{e \sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{e \sin{(c+dx)}}}{(a+b\cos{(c+dx)})^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*cos(d*x+c))**2,x)
[Out] Integral(sqrt(e*sin(c + d*x))/(a + b*cos(c + d*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{e \sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{e \sin{(dx+c)}}}{(b\cos{(dx+c)}+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{e \sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{e \sin{(dx+c)}}}{(b\cos{(dx+c)}+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{e\,\sin\left(c+d\,x\right)}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2, x)
```

3.74 $\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 445

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx &= -\frac{3a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{7/4}d\sqrt{e}}-\frac{3a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{7/4}d\sqrt{e}}\\
&\quad-\frac{\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{e\sin(c+dx)}}\\
&\quad+\frac{3a^2\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&\quad+\frac{3a^2\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&\quad-\frac{b\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de(a+b\cos(c+dx))}\end{aligned}$$

```
[Out] -3/2*a*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2
)/(-a^2+b^2)^(7/4)/d/e^(1/2)-3/2*a*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a
^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(7/4)/d/e^(1/2)+(sin(1/2*c+1/4*Pi
+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2
*d*x),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/(e*sin(d*x+c))^(1/2)-3/2*a^2*(s
in(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(
1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a
^2-b^2)/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-3/2*a^2*(sin(1/
2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c
+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^
```

```
2)/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-b*(e*sin(d*x+c))^(1/
2)/(a^2-b^2)/d/e/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2773, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\begin{aligned}&\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx\\&=-\frac{3a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\left(b^2-a^2\right)^{7/4}}-\frac{3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\left(b^2-a^2\right)^{7/4}}\\&\quad-\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\\&\quad+\frac{3a^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2d\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}\\&\quad+\frac{3a^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2d\left(a^2-b^2\right)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}\end{aligned}$$

```
[In] Int[1/((a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (-3*a*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*(-a^2 + b^2)^(7/4)*d*Sqrt[e]) - (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(7/4)*d*
Sqrt[e]) - (EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^
2)*d*Sqrt[e*Sin[c + d*x]]) + (3*a^2*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2])
, (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)*(a^2 - b*(b - S
qrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (3*a^2*EllipticPi[(2*b)/(b + Sq
rt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)*
(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (b*Sqrt[e*Sin[c
+ d*x]])/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de(a+b\cos(c+dx))} + \frac{\int \frac{-a+\frac{1}{2}b\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{-a^2+b^2} \\
&= -\frac{b\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de(a+b\cos(c+dx))} - \frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{2\,(a^2-b^2)} + \frac{(3a)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2\,(a^2-b^2)} \\
&= -\frac{b\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de(a+b\cos(c+dx))} + \frac{(3a^2)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4\,(-a^2+b^2)^{3/2}} \\
&\quad + \frac{(3a^2)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4\,(-a^2+b^2)^{3/2}} \\
&\quad - \frac{(3abe)\text{Subst}\left(\int \frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx, x, e\sin(c+dx)\right)}{2\,(a^2-b^2)\,d} \\
&\quad - \frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{2\,(a^2-b^2)\,\sqrt{e\sin(c+dx)}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{e\sin(c+dx)}} - \frac{b\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de(a+b\cos(c+dx))} \\
&\quad - \frac{(3abe)\text{Subst}\left(\int \frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)\,d} \\
&\quad + \frac{\left(3a^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4\left(-a^2+b^2\right)^{3/2}\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{\left(3a^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4\left(-a^2+b^2\right)^{3/2}\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{3a^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(-a^2+b^2\right)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{3a^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(-a^2+b^2\right)^{3/2}\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{b\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de(a+b\cos(c+dx))} \\
&\quad - \frac{(3ab)\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{2\left(-a^2+b^2\right)^{3/2}d} \\
&\quad - \frac{(3ab)\text{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{2\left(-a^2+b^2\right)^{3/2}d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{7/4}d\sqrt{e}} - \frac{3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{7/4}d\sqrt{e}} \\
&\quad - \frac{\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{3a^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(-a^2+b^2\right)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{3a^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(-a^2+b^2\right)^{3/2}\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{b\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de(a+b\cos(c+dx))}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 9.67 (sec) , antiderivative size = 1182, normalized size of antiderivative = 2.66

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = -\frac{b\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{\sqrt{\sin(c+dx)}\left(-\frac{2b\cos^2(c+dx)\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] -((b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]
)) + (Sqrt[Sin[c + d*x]]*((-2*b*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]
^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1
/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)]
 - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x
]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*S
in[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/(
(-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*
x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2,
(b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4,
 Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 +
 b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2))
+ (4*a*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]
*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2
*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[
Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2
- b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^
2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*Appe
llF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] -
 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(
-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2
*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*
```

```
x]^2)))))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(2*(a - b)*(a +
 b)*d*Sqrt[e*Sin[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1279 vs. $2(476) = 952$.

Time = 4.31 (sec) , antiderivative size = 1280, normalized size of antiderivative = 2.88

method	result	size
default	Expression too large to display	1280

```
[In] int(1/(a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-4*a*b*e^3*(1/4*(e*sin(d*x+c))^(1/2)/(a^2*e^2-b^2*e^2)/(-b^2*cos(d*x+c)^2*
e^2+a^2*e^2)+3/32/(a^2*e^2-b^2*e^2)^2*(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln
((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*
(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c)
)^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1
/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(1/2/(-a^2+
b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d
*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/(-a^2+b^2)^(1/2)/b*(
1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1
/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+2*a^2*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*
x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+1/4/a^2/(a^2-b^2)*(1-sin
(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/(-a
^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-si
n(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)/(-a
^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-si
n(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+5/8/(a^2-b^2)/(-a^2+b
^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*
x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/4/a^2/(a^2-b^2)/(-a^2+b
^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*
x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c
))^(1/2))/d
```

Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^2\sqrt{e\sin\left(dx+c\right)}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*sin(d*x + c))/((b^2*e*cos(d*x + c)^2 + 2*a*b*e*cos(d*x + c)
 + a^2*e)*sin(d*x + c)), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sin\left(c+dx\right)}\left(a+b\cos\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^2\sqrt{e\sin\left(dx+c\right)}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sqrt(e*sin(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\,\sin\left(c+d\,x\right)}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2),x)
[Out] int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2), x)
```

3.75 $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 507

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx &= \frac{5ab^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}} \\ &- \frac{5ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}} \\ &- \frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5ab-(2a^2+3b^2)\cos(c+dx)}{(a^2-b^2)^2\,de\sqrt{e\sin(c+dx)}} \\ &- \frac{5a^2b\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\ &- \frac{5a^2b\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\ &- \frac{(2a^2+3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)^2de^2\sqrt{\sin(c+dx)}}\end{aligned}$$

```
[Out] 5/2*a*b^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))
/(-a^2+b^2)^(9/4)/d/e^(3/2)-5/2*a*b^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1
/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(9/4)/d/e^(3/2)-b/(a^2-b^2)/d/e/(a
+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2)+(5*a*b-(2*a^2+3*b^2)*cos(d*x+c))/(a^2-b
^2)^2/d/e/(e*sin(d*x+c))^(1/2)+5/2*a^2*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a
^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e/(b-(-a^2+b^2)^(1/2
))/(e*sin(d*x+c))^(1/2)+5/2*a^2*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
```

```
/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)
^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e/(b+(-a^2+b^2)^(1/2))/(e*s
in(d*x+c))^(1/2)+(2*a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c
))^(1/2)/(a^2-b^2)^2/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx &= \frac{5ab^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{3/2}\left(b^2-a^2\right)^{9/4}} \\
&- \frac{5ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{3/2}\left(b^2-a^2\right)^{9/4}} \\
&- \frac{(2a^2+3b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\left(a^2-b^2\right)^2\sqrt{\sin(c+dx)}} \\
&+ \frac{5ab-(2a^2+3b^2)\cos(c+dx)}{de\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}} - \frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))} \\
&- \frac{5a^2b\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2de\left(a^2-b^2\right)^2\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}} \\
&- \frac{5a^2b\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2de\left(a^2-b^2\right)^2\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}
\end{aligned}$$

```
[In] Int[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (5*a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*(-a^2 + b^2)^(9/4)*d*e^(3/2)) - (5*a*b^(3/2)*ArcTanh[(Sqrt[b]*Sqr
t[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(9/4)*d*e
^(3/2)) - b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]) + (
5*a*b - (2*a^2 + 3*b^2)*Cos[c + d*x])/((a^2 - b^2)^2*d*e*Sqrt[e*Sin[c + d*x
]]) - (5*a^2*b*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2,
 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*e*Sqrt[e*
Sin[c + d*x]]) - (5*a^2*b*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)^2*(b + Sqrt[-a^2 + b^2])*
d*e*Sqrt[e*Sin[c + d*x]]) - ((2*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2,
2]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^2*d*e^2*Sqrt[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{\int \frac{-a+\frac{3}{2}b\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx}{-a^2+b^2}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5ab-(2a^2+3b^2)\cos(c+dx)}{(a^2-b^2)^2\,de\sqrt{e\sin(c+dx)}} + \frac{2\int \frac{\left(-\frac{1}{2}a(a^2+4b^2)-\frac{1}{4}b(2a^2+3b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{(a^2-b^2)^2\,e^2}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5ab-(2a^2+3b^2)\cos(c+dx)}{(a^2-b^2)^2\,de\sqrt{e\sin(c+dx)}} - \frac{(5ab^2)\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{2\,(a^2-b^2)^2\,e^2} - \frac{(2a^2+3b^2)\int \sqrt{e\sin(c+dx)}\,dx}{2\,(a^2-b^2)^2\,e^2}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5ab-(2a^2+3b^2)\cos(c+dx)}{(a^2-b^2)^2\,de\sqrt{e\sin(c+dx)}} + \frac{(5a^2b)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4\,(a^2-b^2)^2\,e} - \frac{(5a^2b)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4\,(a^2-b^2)^2\,e} + \frac{(5ab^3)\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx, x, e\sin(c+dx)\right)}{2\,(a^2-b^2)^2\,de} - \frac{\left((2a^2+3b^2)\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{2\,(a^2-b^2)^2\,e^2\sqrt{\sin(c+dx)}}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5ab-(2a^2+3b^2)\cos(c+dx)}{(a^2-b^2)^2\,de\sqrt{e\sin(c+dx)}} - \frac{(2a^2+3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)^2\,de^2\sqrt{\sin(c+dx)}} + \frac{(5ab^3)\,\mathrm{Subst}\left(\int \frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)^2\,de} + \frac{\left(5a^2b\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4\,(a^2-b^2)^2\,e\sqrt{e\sin(c+dx)}} - \frac{\left(5a^2b\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4\,(a^2-b^2)^2\,e\sqrt{e\sin(c+dx)}}$$

$$
\begin{aligned}
&= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5ab-(2a^2+3b^2)\cos(c+dx)}{(a^2-b^2)^2\,de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{5a^2b\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{5a^2b\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{(2a^2+3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)^2 de^2\sqrt{\sin(c+dx)}} \\
&\quad - \frac{(5ab^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2\left(a^2-b^2\right)^2 de} \\
&\quad + \frac{(5ab^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2\left(a^2-b^2\right)^2 de}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{5ab^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}} - \frac{5ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}} \\
&\quad - \frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5ab-(2a^2+3b^2)\cos(c+dx)}{(a^2-b^2)^2\,de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{5a^2b\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{5a^2b\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{(2a^2+3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)^2 de^2\sqrt{\sin(c+dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.54 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.53

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx =$$

$$-\frac{\sin^{\frac{3}{2}}(c+dx)\left(\frac{12(-6a^2b+b^3+4a(a^2-b^2)\cos(c+dx)+b(2a^2+3b^2)\cos(2(c+dx)))}{(a^2-b^2)^2\sqrt{\sin(c+dx)}}+\frac{\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)\left(\frac{(2a^2+3b^2)\sec(c+dx)}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -1/24*(Sin[c + d*x]^(3/2)*((12*(-6*a^2*b + b^3 + 4*a*(a^2 - b^2)*Cos[c + d*
x] + b*(2*a^2 + 3*b^2)*Cos[2*(c + d*x)]))/((a^2 - b^2)^2*Sqrt[Sin[c + d*x]]
) + (Cos[c + d*x]*(a + b*Sqrt[Cos[c + d*x]^2])*(((2*a^2 + 3*b^2)*Sec[c + d*
x]*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c
 + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d
*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/
2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 +
 b^2)]*Sin[c + d*x]^(3/2)))/(Sqrt[b]*(-a^2 + b^2)) + (48*a*(a^2 + 4*b^2)*((
(1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)
^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1
/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c
 + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2
 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2
)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^
2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2))))/Sqrt[Cos[c + d*x]^2]
))/((a - b)^2*(a + b)^2)))/(d*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. 2(536) = 1072.

Time = 4.67 (sec) , antiderivative size = 2002, normalized size of antiderivative = 3.95

method	result	size
default	Expression too large to display	2002

```
[In] int(1/(a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-4*e^3*a*b*(-b^2/e^4/(a-b)^2/(a+b)^2*(1/4*(e*sin(d*x+c))^(3/2)/(-b^2*cos(d
*x+c)^2*e^2+a^2*e^2)+5/32/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(
d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*
2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1
/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*s
in(d*x+c))^(1/2)-1)))-1/e^4/(a^2-b^2)^2/(e*sin(d*x+c))^(1/2))-1/4/e*a^2*(5*
(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)*EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^
2*b-5*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/
2))*a^2*b-2*a^2*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+5*(-a^2+b^2)^(1/2)*(1-sin
(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticPi((1-sin(d*
x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3-5*(-a^2+b^2)^(1/2)*(1
-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticPi((1-si
n(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/2))*b^3+8*(1-sin(d*x+c))^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticE((1-sin(d*x+c))^(1/2
),1/2*2^(1/2))*a^2*b^2-4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2*b^2-5*(-a^2+b^2)
^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellipti
cPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3+5*(-a^2+
b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ell
ipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/2))*b^3+4*(1
-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin
(d*x+c))^(1/2),1/2*2^(1/2))*a^2*b^2+5*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(
1/2)),1/2*2^(1/2))*a^2*b^2+5*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*si
n(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/2
*2^(1/2))*a^2*b^2-12*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^4-5*(1-sin(d*x+c))^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),
-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^4-5*(1-sin(d*x+c))^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^
2)^(1/2))*b,1/2*2^(1/2))*b^4+12*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)
```

```
*sin(d*x+c)^(5/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^4-6*(1-sin(
d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+
c))^(1/2),1/2*2^(1/2))*b^4+5*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*si
n(d*x+c)^(5/2)*EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2
*2^(1/2))*b^4+5*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2
)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/2))*b^4
+8*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^4-4*a^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+6*
b^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))+8*a^2*b^2*cos(d*x+c)^4-8*a^2*b^2*cos(d*x+
c)^2-8*a^4*cos(d*x+c)^2+12*b^4*cos(d*x+c)^4-4*b^4*cos(d*x+c)^2)/(b+(-a^2+b^
2)^(1/2))/(-b+(-a^2+b^2)^(1/2))/(-b^2*cos(d*x+c)^2+a^2)/(a+b)^2/(a-b)^2/cos
(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2),x)
[Out] int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2), x)
```

3.76 $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 530

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = -\frac{7ab^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}} - \frac{7ab^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}} - \frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{7ab-(2a^2+5b^2)\cos(c+dx)}{3\left(a^2-b^2\right)^2de(e\sin(c+dx))^{3/2}} + \frac{(2a^2+5b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3\left(a^2-b^2\right)^2de^2\sqrt{e\sin(c+dx)}} - \frac{7a^2b^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}} - \frac{7a^2b^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}}$$

```
[Out] -7/2*a*b^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/(-a^2+b^2)^(11/4)/d/e^(5/2)-7/2*a*b^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^
(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(11/4)/d/e^(5/2)-b/(a^2-b^2)/d/e
/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2)+1/3*(7*a*b-(2*a^2+5*b^2)*cos(d*x+c))
/(a^2-b^2)^2/d/e/(e*sin(d*x+c))^(3/2)-1/3*(2*a^2+5*b^2)*(sin(1/2*c+1/4*Pi+1
/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d
*x),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(e*sin(d*x+c))^(1/2)+7/2*a^
2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
```

```
cPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/(a^2-b^2)^2/d/e^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+7
/2*a^2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*
x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1
/2)
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2945, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = -\frac{7ab^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{5/2}\left(b^2-a^2\right)^{11/4}}-\frac{7ab^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{5/2}\left(b^2-a^2\right)^{11/4}}+\frac{\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}}-\frac{7a^2b^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2de^2\left(a^2-b^2\right)^2\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}-\frac{7a^2b^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2de^2\left(a^2-b^2\right)^2\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}+\frac{7ab-\left(2a^2+5b^2\right)\cos(c+dx)}{3de\left(a^2-b^2\right)^2(e\sin(c+dx))^{3/2}}$$

```
[In] Int[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-7*a*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*(-a^2 + b^2)^(11/4)*d*e^(5/2)) - (7*a*b^(5/2)*ArcTanh[(Sqrt[b]*S
qrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(11/4)*
d*e^(5/2)) - b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)
) + (7*a*b - (2*a^2 + 5*b^2)*Cos[c + d*x])/(3*(a^2 - b^2)^2*d*e*(e*Sin[c +
d*x])^(3/2)) + ((2*a^2 + 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
 + d*x]])/(3*(a^2 - b^2)^2*d*e^2*Sqrt[e*Sin[c + d*x]]) - (7*a^2*b^2*Ellipti
cPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(2*(a^2 - b^2)^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Sin[c + d*
x]]) - (7*a^2*b^2*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)
```

```
/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)^2*(a^2 - b*(b + Sqrt[-a^2 + b^2])
)*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{\int \frac{-a+\frac{5}{2}b\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx}{-a^2+b^2}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{7ab-(2a^2+5b^2)\cos(c+dx)}{3\,(a^2-b^2)^2\,de(e\sin(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}a(a^2-8b^2)+\frac{1}{4}b(2a^2+5b^2)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3\,(a^2-b^2)^2\,e^2}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{7ab-(2a^2+5b^2)\cos(c+dx)}{3\,(a^2-b^2)^2\,de(e\sin(c+dx))^{3/2}} - \frac{(7ab^2)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2\,(a^2-b^2)^2\,e^2} + \frac{(2a^2+5b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{6\,(a^2-b^2)^2\,e^2}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{7ab-(2a^2+5b^2)\cos(c+dx)}{3\,(a^2-b^2)^2\,de(e\sin(c+dx))^{3/2}} + \frac{(7a^2b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4\,(-a^2+b^2)^{5/2}\,e^2} + \frac{(7a^2b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4\,(-a^2+b^2)^{5/2}\,e^2} + \frac{(7ab^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx, x, e\sin(c+dx)\right)}{2\,(a^2-b^2)^2\,de} + \frac{\left((2a^2+5b^2)\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{6\,(a^2-b^2)^2\,e^2\sqrt{e\sin(c+dx)}}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{7ab-(2a^2+5b^2)\cos(c+dx)}{3\,(a^2-b^2)^2\,de(e\sin(c+dx))^{3/2}} + \frac{(2a^2+5b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3\,(a^2-b^2)^2\,de^2\sqrt{e\sin(c+dx)}} + \frac{(7ab^3)\,\text{Subst}\left(\int\frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)^2\,de} + \frac{\left(7a^2b^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4\,(-a^2+b^2)^{5/2}\,e^2\sqrt{e\sin(c+dx)}} + \frac{\left(7a^2b^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4\,(-a^2+b^2)^{5/2}\,e^2\sqrt{e\sin(c+dx)}}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{7ab-(2a^2+5b^2)\cos(c+dx)}{3\,(a^2-b^2)^2\,de(e\sin(c+dx))^{3/2}} + \frac{(2a^2+5b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3\,(a^2-b^2)^2\,de^2\sqrt{e\sin(c+dx)}} - \frac{7a^2b^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\,(-a^2+b^2)^{5/2}\left(b-\sqrt{-a^2+b^2}\right)de^2\sqrt{e\sin(c+dx)}} + \frac{7a^2b^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\,(-a^2+b^2)^{5/2}\left(b+\sqrt{-a^2+b^2}\right)de^2\sqrt{e\sin(c+dx)}} - \frac{(7ab^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2\,(-a^2+b^2)^{5/2}\,de^2} - \frac{(7ab^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2\,(-a^2+b^2)^{5/2}\,de^2}$$

$$
\begin{aligned}
&= -\frac{7ab^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}} - \frac{7ab^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}} \\
&\quad - \frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{7ab-(2a^2+5b^2)\cos(c+dx)}{3\left(a^2-b^2\right)^2de(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{(2a^2+5b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3\left(a^2-b^2\right)^2de^2\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{7a^2b^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(-a^2+b^2\right)^{5/2}\left(b-\sqrt{-a^2+b^2}\right)de^2\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{7a^2b^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(-a^2+b^2\right)^{5/2}\left(b+\sqrt{-a^2+b^2}\right)de^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.81 (sec) , antiderivative size = 1257, normalized size of antiderivative = 2.37

$$
\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \frac{\left(\frac{b^3}{(a^2-b^2)^2(a+b\cos(c+dx))} - \frac{2(-2ab+a^2\cos(c+dx)+b^2\cos(c+dx))\csc^2(c+dx)}{3(a^2-b^2)^2}\right.}{d(e\sin(c+dx))^{5/2}}
$$

$$
+ \frac{\sin^{\frac{5}{2}}(c+dx)\left(\frac{2(2a^2b+5b^3)\cos^2(c+dx)\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a\right.}\right.}{}\right.}{}\right.}{}
$$

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] ((b^3/((a^2 - b^2)^2*(a + b*Cos[c + d*x])) - (2*(-2*a*b + a^2*Cos[c + d*x]
+ b^2*Cos[c + d*x])*Csc[c + d*x]^2)/(3*(a^2 - b^2)^2))*Sin[c + d*x]^3)/(d*(
e*Sin[c + d*x])^(5/2)) + (Sin[c + d*x]^(5/2)*((2*(2*a^2*b + 5*b^3)*Cos[c +
d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]
*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqr
t[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]
*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^
2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]
```

```
))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -
1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c
+ d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5
/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[
5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^
2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-
a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*C
os[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(2*a^3 - 16*a*b^2)*Cos[c + d*x]*(a
+ b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)
*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sq
rt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log
[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/
2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c +
d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4,
Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4,
 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 +
b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c
 + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(6*(a - b)^2*(a + b)^2*d*(e*Sin[c + d*
x])^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1473 vs. 2(556) = 1112.

Time = 6.30 (sec) , antiderivative size = 1474, normalized size of antiderivative = 2.78

method	result	size
default	Expression too large to display	1474

```
[In] int(1/(a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-4*e^3*a*b*(-1/e^4/(a-b)^2/(a+b)^2*b^2*(1/4*(e*sin(d*x+c))^(1/2)/(-b^2*cos
(d*x+c)^2*e^2+a^2*e^2)+7/32*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(
1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/
2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*si
n(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(
a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-1/3/e^4/(a^2-b^2)^2/(e*sin(d*x+c))^(3
/2))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e^2*(1/3*(-a^2-b^2)/(a^2-b^2)^2/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*((1-sin(d*x+c))^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2
))+2*cos(d*x+c)^2*sin(d*x+c))+2*a^2*b^2/(a-b)/(a+b)*(1/2*b^2/e/a^2/(a^2-b^2
)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+1/4/a^2/(a^2-b^
```

```
2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)
^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-5/8/(a^2
-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*Ellipti
cPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2
-b^2)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*Ellipti
cPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+5/8/(a^2-b^2
)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi(
(1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/4/a^2/(a^2-b^2
)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi(
(1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+b^2*(a^2+b^2)/(
a-b)^2/(a+b)^2*(-1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(
1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2
))+1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*Ellipt
icPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c
)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^2\left(e\sin\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(e\,\sin\left(c+d\,x\right)\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2),x)
[Out] int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2), x)
```

3.77 $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 590

$$\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx &= \frac{9ab^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2(-a^2+b^2)^{13/4}de^{7/2}} \\
&- \frac{9ab^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2(-a^2+b^2)^{13/4}de^{7/2}} \\
&- \frac{b}{(a^2-b^2)de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} \\
&+ \frac{9ab-(2a^2+7b^2)\cos(c+dx)}{5(a^2-b^2)^2de(e\sin(c+dx))^{5/2}} - \frac{3(15ab^3+(2a^4-10a^2b^2-7b^4)\cos(c+dx))}{5(a^2-b^2)^3de^3\sqrt{e\sin(c+dx)}} \\
&+ \frac{9a^2b^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2(a^2-b^2)^3\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
&+ \frac{9a^2b^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2(a^2-b^2)^3\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
&- \frac{3(2a^4-10a^2b^2-7b^4)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5(a^2-b^2)^3de^4\sqrt{\sin(c+dx)}}
\end{aligned}$$

```
[Out] 9/2*a*b^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))
/(-a^2+b^2)^(13/4)/d/e^(7/2)-9/2*a*b^(7/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(
1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/e^(7/2)-b/(a^2-b^2)/d/e/
(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2)+1/5*(9*a*b-(2*a^2+7*b^2)*cos(d*x+c))/
(a^2-b^2)^2/d/e/(e*sin(d*x+c))^(5/2)-3/5*(15*a*b^3+(2*a^4-10*a^2*b^2-7*b^4)
*cos(d*x+c))/(a^2-b^2)^3/d/e^3/(e*sin(d*x+c))^(1/2)-9/2*a^2*b^3*(sin(1/2*c+
```

```
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4
*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3
/d/e^3/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-9/2*a^2*b^3*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi
+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3/d/
e^3/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+3/5*(2*a^4-10*a^2*b^2-7*b^4)*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^3/d/e^4/sin(
d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx &= \frac{9ab^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{7/2}\left(b^2-a^2\right)^{13/4}} \\ &- \frac{9ab^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{7/2}\left(b^2-a^2\right)^{13/4}} \\ &- \frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))} \\ &+ \frac{9ab-\left(2a^2+7b^2\right)\cos(c+dx)}{5de\left(a^2-b^2\right)^2(e\sin(c+dx))^{5/2}} \\ &+ \frac{9a^2b^3\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2de^3\left(a^2-b^2\right)^3\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}} \\ &+ \frac{9a^2b^3\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{2de^3\left(a^2-b^2\right)^3\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}} \\ &- \frac{3\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\left(a^2-b^2\right)^3\sqrt{\sin(c+dx)}} \\ &- \frac{3\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{5de^3\left(a^2-b^2\right)^3\sqrt{e\sin(c+dx)}}\end{aligned}$$

```
[In] Int[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] (9*a*b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*(-a^2 + b^2)^(13/4)*d*e^(7/2)) - (9*a*b^(7/2)*ArcTanh[(Sqrt[b]*Sq
rt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(13/4)*d
*e^(7/2)) - b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))
```

```
 + (9*a*b - (2*a^2 + 7*b^2)*Cos[c + d*x])/(5*(a^2 - b^2)^2*d*e*(e*Sin[c + d
*x])^(5/2)) - (3*(15*a*b^3 + (2*a^4 - 10*a^2*b^2 - 7*b^4)*Cos[c + d*x]))/(5
*(a^2 - b^2)^3*d*e^3*Sqrt[e*Sin[c + d*x]]) + (9*a^2*b^3*EllipticPi[(2*b)/(b
 - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 -
b^2)^3*(b - Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Sin[c + d*x]]) + (9*a^2*b^3*Elli
pticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*
x]])/(2*(a^2 - b^2)^3*(b + Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Sin[c + d*x]]) -
(3*(2*a^4 - 10*a^2*b^2 - 7*b^4)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin
[c + d*x]])/(5*(a^2 - b^2)^3*d*e^4*Sqrt[Sin[c + d*x]])
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 214</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

<u>Rule 304</u>

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

<u>Rule 335</u>

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

<u>Rule 2719</u>

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

<u>Rule 2721</u>

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

<u>Rule 2773</u>

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
```

```
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}+\frac{\int \frac{-a+\frac{7}{2}b\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx}{-a^2+b^2}\\
&= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\\
&\quad+\frac{9ab-(2a^2+7b^2)\cos(c+dx)}{5\,(a^2-b^2)^2\,de(e\sin(c+dx))^{5/2}}+\frac{2\int \frac{\frac{3}{2}a(a^2-4b^2)+\frac{3}{4}b(2a^2+7b^2)\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx}{5\,(a^2-b^2)^2\,e^2}\\
&= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\\
&\quad+\frac{9ab-(2a^2+7b^2)\cos(c+dx)}{5\,(a^2-b^2)^2\,de(e\sin(c+dx))^{5/2}}-\frac{3(15ab^3+(2a^4-10a^2b^2-7b^4)\cos(c+dx))}{5\,(a^2-b^2)^3\,de^3\sqrt{e\sin(c+dx)}}\\
&\quad-\frac{4\int \frac{\left(\frac{3}{4}a(a^4-5a^2b^2-11b^4)+\frac{3}{8}b(2a^4-10a^2b^2-7b^4)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{5\,(a^2-b^2)^3\,e^4}\\
&= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\\
&\quad+\frac{9ab-(2a^2+7b^2)\cos(c+dx)}{5\,(a^2-b^2)^2\,de(e\sin(c+dx))^{5/2}}-\frac{3(15ab^3+(2a^4-10a^2b^2-7b^4)\cos(c+dx))}{5\,(a^2-b^2)^3\,de^3\sqrt{e\sin(c+dx)}}\\
&\quad+\frac{(9ab^4)\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{2\,(a^2-b^2)^3\,e^4}-\frac{(3(2a^4-10a^2b^2-7b^4))\int \sqrt{e\sin(c+dx)}\,dx}{10\,(a^2-b^2)^3\,e^4}
\end{aligned}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9ab-(2a^2+7b^2)\cos(c+dx)}{5\,(a^2-b^2)^2\,de(e\sin(c+dx))^{5/2}} - \frac{3(15ab^3+(2a^4-10a^2b^2-7b^4)\cos(c+dx))}{5\,(a^2-b^2)^3\,de^3\sqrt{e\sin(c+dx)}} - \frac{(9a^2b^3)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4\,(a^2-b^2)^3\,e^3} + \frac{(9a^2b^3)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4\,(a^2-b^2)^3\,e^3} - \frac{(9ab^5)\,\mathrm{Subst}\left(\int\frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx,x,e\sin(c+dx)\right)}{2\,(a^2-b^2)^3\,de^3} - \frac{\left(3(2a^4-10a^2b^2-7b^4)\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{10\,(a^2-b^2)^3\,e^4\sqrt{\sin(c+dx)}}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9ab-(2a^2+7b^2)\cos(c+dx)}{5\,(a^2-b^2)^2\,de(e\sin(c+dx))^{5/2}} - \frac{3(15ab^3+(2a^4-10a^2b^2-7b^4)\cos(c+dx))}{5\,(a^2-b^2)^3\,de^3\sqrt{e\sin(c+dx)}} - \frac{3(2a^4-10a^2b^2-7b^4)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5\,(a^2-b^2)^3\,de^4\sqrt{\sin(c+dx)}} - \frac{(9ab^5)\,\mathrm{Subst}\left(\int\frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)^3\,de^3} - \frac{\left(9a^2b^3\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4\,(a^2-b^2)^3\,e^3\sqrt{e\sin(c+dx)}} + \frac{\left(9a^2b^3\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{4\,(a^2-b^2)^3\,e^3\sqrt{e\sin(c+dx)}}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9ab-(2a^2+7b^2)\cos(c+dx)}{5\,(a^2-b^2)^2\,de(e\sin(c+dx))^{5/2}} - \frac{3(15ab^3+(2a^4-10a^2b^2-7b^4)\cos(c+dx))}{5\,(a^2-b^2)^3\,de^3\sqrt{e\sin(c+dx)}} + \frac{9a^2b^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\,(a^2-b^2)^3\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} + \frac{9a^2b^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\,(a^2-b^2)^3\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} - \frac{3(2a^4-10a^2b^2-7b^4)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5\,(a^2-b^2)^3\,de^4\sqrt{\sin(c+dx)}} + \frac{(9ab^4)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2\,(a^2-b^2)^3\,de^3} - \frac{(9ab^4)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2\,(a^2-b^2)^3\,de^3}$$

$$= \frac{9ab^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\,(-a^2+b^2)^{13/4}\,de^{7/2}} - \frac{9ab^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\,(-a^2+b^2)^{13/4}\,de^{7/2}} - \frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9ab-(2a^2+7b^2)\cos(c+dx)}{5\,(a^2-b^2)^2\,de(e\sin(c+dx))^{5/2}} - \frac{3(15ab^3+(2a^4-10a^2b^2-7b^4)\cos(c+dx))}{5\,(a^2-b^2)^3\,de^3\sqrt{e\sin(c+dx)}} + \frac{9a^2b^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\,(a^2-b^2)^3\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} + \frac{9a^2b^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\,(a^2-b^2)^3\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} - \frac{3(2a^4-10a^2b^2-7b^4)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5\,(a^2-b^2)^3\,de^4\sqrt{\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.95 (sec) , antiderivative size = 950, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \frac{\sin^4(c+dx)\left(-\frac{2(20ab^3+3a^4\cos(c+dx)-15a^2b^2\cos(c+dx)-8b^4\cos(c+dx))\csc}{5(a^2-b^2)^3}\right.}{d(e\sin}$$

$$-\frac{3\sin^{\frac{7}{2}}(c+dx)\left(\frac{(2a^4b-10a^2b^3-7b^5)\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\right.\right.}{}\right.}{}$$

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] (Sin[c + d*x]^4*((-2*(20*a*b^3 + 3*a^4*Cos[c + d*x] - 15*a^2*b^2*Cos[c + d*
x] - 8*b^4*Cos[c + d*x])*Csc[c + d*x])/(5*(a^2 - b^2)^3) - (2*(-2*a*b + a^2
*Cos[c + d*x] + b^2*Cos[c + d*x])*Csc[c + d*x]^3)/(5*(a^2 - b^2)^2) - (b^5*
Sin[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x]))))/(d*(e*Sin[c + d*x])^(7
/2)) - (3*Sin[c + d*x]^(7/2)*(((2*a^4*b - 10*a^2*b^3 - 7*b^5)*Cos[c + d*x]^
2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c
+ d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*
x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]
*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2
)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 +
b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a
^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(2*a^5 - 10*a^3*b
^2 - 22*a*b^4)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sq
rt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]
*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2
+ b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c
+ d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[
c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2
- b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - S
in[c + d*x]^2])))/(10*(a - b)^3*(a + b)^3*d*(e*Sin[c + d*x])^(7/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1748 vs. 2(612) = 1224.

Time = 6.69 (sec) , antiderivative size = 1749, normalized size of antiderivative = 2.96

method	result	size
default	Expression too large to display	1749

```
[In] int(1/(a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*e^3*a*b*(2*b^4/e^6/(a-b)^3/(a+b)^3*(1/4*(e*sin(d*x+c))^(3/2)/(-b^2*cos(
d*x+c)^2*e^2+a^2*e^2)+9/32/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin
(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^
2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(
1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*
sin(d*x+c))^(1/2)-1)))-2/5/e^4/(a+b)^2/(a-b)^2/(e*sin(d*x+c))^(5/2)+4/e^6/(
a-b)^3/(a+b)^3*b^2/(e*sin(d*x+c))^(1/2))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/
e^3*(-1/5*(-a^2-b^2)/(a^2-b^2)^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/sin(d*x+
c)/(cos(d*x+c)^2-1)*(6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3*(1-sin(d*x+c))^(1/2)
*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2
*2^(1/2))+6*sin(d*x+c)*cos(d*x+c)^4-8*cos(d*x+c)^2*sin(d*x+c))+b^2*(3*a^2+b
^2)/(a^2-b^2)^3*(2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-(1-sin(d*x+c))^(1/2)*(2*si
n(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/
2))-2*cos(d*x+c)^2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)-2*a^2*b^4/(a-b)^2/(a+
b)^2*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/
(-b^2*cos(d*x+c)^2+a^2)-1/2/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-
sin(d*x+c))^(1/2),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*si
n(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Ellipt
icF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)
/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/
2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2
)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-
3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1
/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-
sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(
1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1
/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))-b^4*(3*a^2+b^2)/(a-b)^3/(a+b)^3*(-1/2
/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+
```

```
c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(
1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(
-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b)
,1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(e\,\sin(c+d\,x))^{7/2}\,(a+b\,\cos(c+d\,x))^2}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2),x)
[Out] int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2), x)
```

3.78 $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 590

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx &= \frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}d}\\
&- \frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}d}\\
&- \frac{11a(9a^4-11a^2b^2+2b^4)\,e^7\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^7\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&- \frac{11a(9a^4-11a^2b^2+2b^4)\,e^7\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^7\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&+ \frac{11a(45a^2-37b^2)\,e^6E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20b^6d\sqrt{\sin(c+dx)}}\\
&- \frac{11e^5(5(9a^2-2b^2)-27ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{60b^5d}\\
&+ \frac{11e^3(9a+2b\cos(c+dx))(e\sin(c+dx))^{7/2}}{28b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}\end{aligned}$$

```
[Out] 11/8*(9*a^4-11*a^2*b^2+2*b^4)*e^(13/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/
(-a^2+b^2)^(1/4)/e^(1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d-11/8*(9*a^4-11*a^2*b^
2+2*b^4)*e^(13/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(
1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d-11/60*e^5*(45*a^2-10*b^2-27*a*b*cos(d*x+c
))*(e*sin(d*x+c))^(3/2)/b^5/d+11/28*e^3*(9*a+2*b*cos(d*x+c))*(e*sin(d*x+c))
^(7/2)/b^3/d/(a+b*cos(d*x+c))+1/2*e*(e*sin(d*x+c))^(11/2)/b/d/(a+b*cos(d*x+
```

```
c))^2+11/8*a*(9*a^4-11*a^2*b^2+2*b^4)*e^7*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-
a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^7/d/(b-(-a^2+b^2)^(1/2))/(e*sin
(d*x+c))^(1/2)+11/8*a*(9*a^4-11*a^2*b^2+2*b^4)*e^7*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),
2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^7/d/(b+(-a^2+b^2)^(1/2
))/(e*sin(d*x+c))^(1/2)-11/20*a*(45*a^2-37*b^2)*e^6*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),
2^(1/2))*(e*sin(d*x+c))^(1/2)/b^6/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2772, 2942, 2944, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{11ae^6(45a^2-37b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20b^6d\sqrt{\sin(c+dx)}}$$
$$-\frac{11e^5(e\sin(c+dx))^{3/2}\left(5(9a^2-2b^2)-27ab\cos(c+dx)\right)}{60b^5d}$$
$$+\frac{11e^{13/2}(9a^4-11a^2b^2+2b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{13/2}d\sqrt[4]{b^2-a^2}}$$
$$-\frac{11e^{13/2}(9a^4-11a^2b^2+2b^4)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{13/2}d\sqrt[4]{b^2-a^2}}$$
$$-\frac{11ae^7(9a^4-11a^2b^2+2b^4)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8b^7d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}$$
$$-\frac{11ae^7(9a^4-11a^2b^2+2b^4)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8b^7d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}$$
$$+\frac{11e^3(e\sin(c+dx))^{7/2}(9a+2b\cos(c+dx))}{28b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}$$

```
[In] Int[(e*Sin[c + d*x])^(13/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (11*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^(13/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*
x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(13/2)*(-a^2 + b^2)^(1/4)*d) - (11
*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^(13/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]
])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(13/2)*(-a^2 + b^2)^(1/4)*d) - (11*a
*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^7*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*b^7*(b - Sqrt[-a^2 + b^2])*d*
Sqrt[e*Sin[c + d*x]]) - (11*a*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^7*EllipticPi[(
```

```
2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*
b^7*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (11*a*(45*a^2 - 37*b^2
)*e^6*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(20*b^6*d*Sqrt
[Sin[c + d*x]]) - (11*e^5*(5*(9*a^2 - 2*b^2) - 27*a*b*Cos[c + d*x])*(e*Sin[
c + d*x])^(3/2))/(60*b^5*d) + (11*e^3*(9*a + 2*b*Cos[c + d*x])*(e*Sin[c + d
*x])^(7/2))/(28*b^3*d*(a + b*Cos[c + d*x])) + (e*(e*Sin[c + d*x])^(11/2))/(
2*b*d*(a + b*Cos[c + d*x])^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
```

```
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
```

```
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(11e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx}{4b} \\
&= \frac{11e^3(9a+2b\cos(c+dx))(e\sin(c+dx))^{7/2}}{28b^3d(a+b\cos(c+dx))} \\
&\quad + \frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2} + \frac{(11e^4)\int \frac{\left(-b-\frac{9}{2}a\cos(c+dx)\right)(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx}{4b^3} \\
&= -\frac{11e^5(5(9a^2-2b^2)-27ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{60b^5d} \\
&\quad + \frac{11e^3(9a+2b\cos(c+dx))(e\sin(c+dx))^{7/2}}{28b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2} \\
&\quad + \frac{(11e^6)\int \frac{\left(\frac{1}{2}b(9a^2-5b^2)+\frac{1}{4}a(45a^2-37b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{10b^5} \\
&= -\frac{11e^5(5(9a^2-2b^2)-27ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{60b^5d} \\
&\quad + \frac{11e^3(9a+2b\cos(c+dx))(e\sin(c+dx))^{7/2}}{28b^3d(a+b\cos(c+dx))} \\
&\quad + \frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2} + \frac{(11a(45a^2-37b^2)\,e^6)\int \sqrt{e\sin(c+dx)}\,dx}{40b^6} \\
&\quad - \frac{(11(9a^4-11a^2b^2+2b^4)\,e^6)\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{8b^6}
\end{aligned}
$$

$$= -\frac{11e^5\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{60b^5d}$$
$$+\frac{11e^3(9a+2b\cos(c+dx))(e\sin(c+dx))^{7/2}}{28b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}$$
$$+\frac{\left(11a\left(9a^4-11a^2b^2+2b^4\right)e^7\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16b^7}$$
$$-\frac{\left(11a\left(9a^4-11a^2b^2+2b^4\right)e^7\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16b^7}$$
$$+\frac{\left(11\left(9a^4-11a^2b^2+2b^4\right)e^7\right)\operatorname{Subst}\left(\int\frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx,x,e\sin(c+dx)\right)}{8b^5d}$$
$$+\frac{\left(11a\left(45a^2-37b^2\right)e^6\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{40b^6\sqrt{\sin(c+dx)}}$$

$$=\frac{11a\left(45a^2-37b^2\right)e^6E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20b^6d\sqrt{\sin(c+dx)}}$$
$$-\frac{11e^5\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{60b^5d}$$
$$+\frac{11e^3(9a+2b\cos(c+dx))(e\sin(c+dx))^{7/2}}{28b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}$$
$$+\frac{\left(11\left(9a^4-11a^2b^2+2b^4\right)e^7\right)\operatorname{Subst}\left(\int\frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{4b^5d}$$
$$+\frac{\left(11a\left(9a^4-11a^2b^2+2b^4\right)e^7\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16b^7\sqrt{e\sin(c+dx)}}$$
$$-\frac{\left(11a\left(9a^4-11a^2b^2+2b^4\right)e^7\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16b^7\sqrt{e\sin(c+dx)}}$$

$$=-\frac{11a(9a^4-11a^2b^2+2b^4)\,e^7\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^7\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{11a(9a^4-11a^2b^2+2b^4)\,e^7\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^7\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{11a(45a^2-37b^2)\,e^6E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20b^6d\sqrt{\sin(c+dx)}}$$
$$-\frac{11e^5(5(9a^2-2b^2)-27ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{60b^5d}$$
$$+\frac{11e^3(9a+2b\cos(c+dx))(e\sin(c+dx))^{7/2}}{28b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}$$
$$-\frac{(11(9a^4-11a^2b^2+2b^4)\,e^7)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8b^6d}$$
$$+\frac{(11(9a^4-11a^2b^2+2b^4)\,e^7)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8b^6d}$$

$$=\frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}d}$$
$$-\frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}d}$$
$$-\frac{11a(9a^4-11a^2b^2+2b^4)\,e^7\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^7\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{11a(9a^4-11a^2b^2+2b^4)\,e^7\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^7\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{11a(45a^2-37b^2)\,e^6E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20b^6d\sqrt{\sin(c+dx)}}$$
$$-\frac{11e^5(5(9a^2-2b^2)-27ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{60b^5d}$$
$$+\frac{11e^3(9a+2b\cos(c+dx))(e\sin(c+dx))^{7/2}}{28b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.73 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.58

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{11(e\sin(c+dx))^{13/2}\left(\frac{(45a^3-37ab^2)\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right.\right.\right.}{}\right.}{} + \frac{\csc^6(c+dx)(e\sin(c+dx))^{13/2}\left(\frac{(-168a^2+65b^2)\sin(c+dx)}{42b^5}-\frac{19(a^3\sin(c+dx)-ab^2\sin(c+dx))}{4b^5(a+b\cos(c+dx))}+\frac{a^4\sin(c+dx)-2a^2b^2\sin(c+dx)-}{2b^5(a+b\cos(c+dx))}\right.}{d}$$

```
[In] Integrate[(e*Sin[c + d*x])^(13/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (11*(e*Sin[c + d*x])^(13/2)*(((45*a^3 - 37*a*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]
*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^
2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 -
b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[
Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^
2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3
/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c
+ d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a
 + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(18*a^2*b - 10*b^3)*Cos[c + d
*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2
+ b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b
^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt
[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]
*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2
 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c +
 d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 -
 Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(40*b^
5*d*Sin[c + d*x]^(13/2)) + (Csc[c + d*x]^6*(e*Sin[c + d*x])^(13/2)*(((-168*
a^2 + 65*b^2)*Sin[c + d*x])/(42*b^5) - (19*(a^3*Sin[c + d*x] - a*b^2*Sin[c
+ d*x]))/(4*b^5*(a + b*Cos[c + d*x])) + (a^4*Sin[c + d*x] - 2*a^2*b^2*Sin[c
 + d*x] + b^4*Sin[c + d*x])/(2*b^5*(a + b*Cos[c + d*x])^2) + (3*a*Sin[2*(c
+ d*x)])/(5*b^4) - Sin[3*(c + d*x)]/(14*b^3)))/d
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2994 vs. $2(607) = 1214$.

Time = 100.32 (sec) , antiderivative size = 2995, normalized size of antiderivative = 5.08

method	result	size
default	Expression too large to display	2995

```
[In] int((e*sin(d*x+c))^(13/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2*e^3*b*(-1/21/b^6*(e*sin(d*x+c))^(3/2)*e^2*(3*b^2*cos(d*x+c)^2+42*a^2-17*
b^2)+e^4/b^6*(-1/8*(e*sin(d*x+c))^(3/2)*e^2*(-21*a^4*b^2*cos(d*x+c)^2+23*a^
2*b^4*cos(d*x+c)^2-2*b^6*cos(d*x+c)^2+17*a^6-15*a^4*b^2-2*a^2*b^4)/(-b^2*co
s(d*x+c)^2*e^2+a^2*e^2)^2+1/8*(99/8*a^4-121/8*a^2*b^2+11/4*b^4)/b^2/(e^2*(a
^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*s
in(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2
-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2
*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(
2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))-(cos(d*x+c)^2*
e*sin(d*x+c))^(1/2)*e^7*a*(1/5/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(100*(
1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-si
n(d*x+c))^(1/2),1/2*2^(1/2))*a^2-78*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-50*(1
-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin
(d*x+c))^(1/2),1/2*2^(1/2))*a^2+39*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+6*b^2*
cos(d*x+c)^4-6*b^2*cos(d*x+c)^2)+3*(7*a^4-10*a^2*b^2+3*b^4)/b^6*(-1/2/b^2*(
1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1
/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b
^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2
^(1/2)))-3*(5*a^6-11*a^4*b^2+7*a^2*b^4-b^6)/b^6*(1/2*b^2/e/a^2/(a^2-b^2)*si
n(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)-1/2/a^2/
(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+1
/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(
1/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*Elliptic
Pi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-
b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(
1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^
```

```
(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2
)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)
^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1
/2)))+4*a^2*(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/b^6*(1/4*b^2/e/a^2/(a^2-b^2)*sin(
d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)^2+1/16*b^2
*(11*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)/(-b^2*cos(d*x+c)^2+a^2)-11/16/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Ellip
ticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+3/8/a^4/(a^2-b^2)^2*(1-sin(d*x+c))^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+11/32/a^2/(a^2-b^2)^2*(
1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/16/a^4/(a^
2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^
2-21/64/(a^2-b^2)^2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*Elliptic
Pi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+7/16/a^2/(a^2
-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c)
)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^2*(1-s
in(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*si
n(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1
-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-21/64/(a^2-b^2)^2/b^2*(1-sin(d*x+c))^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)
/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/
2)/b),1/2*2^(1/2))+7/16/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(
1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2
))-3/16/a^4/(a^2-b^2)^2*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*Elli
pticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x
+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(13/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{13}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(13/2)/(b*cos(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\,\sin(c+d\,x))^{13/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

```
[In] int((e*sin(c + d*x))^(13/2)/(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(13/2)/(a + b*cos(c + d*x))^3, x)
```

3.79 $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 604

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = -\frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{11/2}\,(-a^2+b^2)^{3/4}\,d}$$
$$-\frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{11/2}\,(-a^2+b^2)^{3/4}\,d}$$
$$+\frac{3a(21a^2-13b^2)\,e^6\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^6d\sqrt{e\sin(c+dx)}}$$
$$-\frac{9a(7a^4-9a^2b^2+2b^4)\,e^6\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^6\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{9a(7a^4-9a^2b^2+2b^4)\,e^6\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^6\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{3e^5(3(7a^2-2b^2)-7ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^5d}$$
$$+\frac{9e^3(7a+2b\cos(c+dx))(e\sin(c+dx))^{5/2}}{20b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}$$

```
[Out] -9/8*(7*a^4-9*a^2*b^2+2*b^4)*e^(11/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(
-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/(-a^2+b^2)^(3/4)/d-9/8*(7*a^4-9*a^2*b^2+2
*b^4)*e^(11/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2
))/b^(11/2)/(-a^2+b^2)^(3/4)/d+9/20*e^3*(7*a+2*b*cos(d*x+c))*(e*sin(d*x+c))
^(5/2)/b^3/d/(a+b*cos(d*x+c))+1/2*e*(e*sin(d*x+c))^(9/2)/b/d/(a+b*cos(d*x+c
))^2-3/4*a*(21*a^2-13*b^2)*e^6*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
```

```
c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(
1/2)/b^6/d/(e*sin(d*x+c))^(1/2)+9/8*a*(7*a^4-9*a^2*b^2+2*b^4)*e^6*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1
/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^6/d/(a^
2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+9/8*a*(7*a^4-9*a^2*b^2+2*b^4
)*e^6*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/b^6/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-3/4*e^5*(21*a
^2-6*b^2-7*a*b*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^5/d
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2772, 2942, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = {} & \frac{3ae^6(21a^2-13b^2)\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{4b^6d\sqrt{e\sin(c+dx)}} \\
& - \frac{3e^5\sqrt{e\sin(c+dx)}(3(7a^2-2b^2)-7ab\cos(c+dx))}{4b^5d} \\
& - \frac{9e^{11/2}(7a^4-9a^2b^2+2b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{11/2}d\left(b^2-a^2\right)^{3/4}} \\
& - \frac{9e^{11/2}(7a^4-9a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{11/2}d\left(b^2-a^2\right)^{3/4}} \\
& - \frac{9ae^6(7a^4-9a^2b^2+2b^4)\sqrt{\sin(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8b^6d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}} \\
& - \frac{9ae^6(7a^4-9a^2b^2+2b^4)\sqrt{\sin(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8b^6d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}} \\
& + \frac{9e^3(e\sin(c+dx))^{5/2}(7a+2b\cos(c+dx))}{20b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}
\end{aligned}$$

```
[In] Int[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-9*(7*a^4 - 9*a^2*b^2 + 2*b^4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x
]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(11/2)*(-a^2 + b^2)^(3/4)*d) - (9*(
7*a^4 - 9*a^2*b^2 + 2*b^4)*e^(11/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/
((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(11/2)*(-a^2 + b^2)^(3/4)*d) + (3*a*(21
*a^2 - 13*b^2)*e^6*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(4*
b^6*d*Sqrt[e*Sin[c + d*x]]) - (9*a*(7*a^4 - 9*a^2*b^2 + 2*b^4)*e^6*Elliptic
```

```
Pi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])
/(8*b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (9*a*(7*
a^4 - 9*a^2*b^2 + 2*b^4)*e^6*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*b^6*(a^2 - b*(b + Sqrt[-a^2 + b^2]
))*d*Sqrt[e*Sin[c + d*x]]) - (3*e^5*(3*(7*a^2 - 2*b^2) - 7*a*b*Cos[c + d*x]
)*Sqrt[e*Sin[c + d*x]])/(4*b^5*d) + (9*e^3*(7*a + 2*b*Cos[c + d*x])*(e*Sin[
c + d*x])^(5/2))/(20*b^3*d*(a + b*Cos[c + d*x])) + (e*(e*Sin[c + d*x])^(9/2
))/(2*b*d*(a + b*Cos[c + d*x])^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
```

```
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
```

```
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(9e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx}{4b} \\
&= \frac{9e^3(7a+2b\cos(c+dx))(e\sin(c+dx))^{5/2}}{20b^3d(a+b\cos(c+dx))} \\
&\quad + \frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} + \frac{(9e^4)\int \frac{\left(-b-\frac{7}{2}a\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx}{4b^3} \\
&= -\frac{3e^5(3(7a^2-2b^2)-7ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^5d} \\
&\quad + \frac{9e^3(7a+2b\cos(c+dx))(e\sin(c+dx))^{5/2}}{20b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} \\
&\quad + \frac{(3e^6)\int \frac{\frac{1}{2}b(7a^2-3b^2)+\frac{1}{4}a(21a^2-13b^2)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2b^5} \\
&= -\frac{3e^5(3(7a^2-2b^2)-7ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^5d} \\
&\quad + \frac{9e^3(7a+2b\cos(c+dx))(e\sin(c+dx))^{5/2}}{20b^3d(a+b\cos(c+dx))} \\
&\quad + \frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} + \frac{(3a(21a^2-13b^2)e^6)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{8b^6} \\
&\quad - \frac{(9(7a^4-9a^2b^2+2b^4)e^6)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{8b^6}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3e^5(3(7a^2-2b^2)-7ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^5d} \\
&\quad + \frac{9e^3(7a+2b\cos(c+dx))(e\sin(c+dx))^{5/2}}{20b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} \\
&\quad + \frac{(9a(7a^4-9a^2b^2+2b^4)e^6)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16b^6\sqrt{-a^2+b^2}} \\
&\quad + \frac{(9a(7a^4-9a^2b^2+2b^4)e^6)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16b^6\sqrt{-a^2+b^2}} \\
&\quad + \frac{(9(7a^4-9a^2b^2+2b^4)e^7)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx, x, e\sin(c+dx)\right)}{8b^5d} \\
&\quad + \frac{\left(3a(21a^2-13b^2)e^6\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{8b^6\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3a(21a^2-13b^2)e^6\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^6d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{3e^5(3(7a^2-2b^2)-7ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^5d} \\
&\quad + \frac{9e^3(7a+2b\cos(c+dx))(e\sin(c+dx))^{5/2}}{20b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} \\
&\quad + \frac{(9(7a^4-9a^2b^2+2b^4)e^7)\,\mathrm{Subst}\left(\int \frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{4b^5d} \\
&\quad + \frac{\left(9a(7a^4-9a^2b^2+2b^4)e^6\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16b^6\sqrt{-a^2+b^2}\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{\left(9a(7a^4-9a^2b^2+2b^4)e^6\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16b^6\sqrt{-a^2+b^2}\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$= \frac{3a(21a^2-13b^2)\,e^6\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^6d\sqrt{e\sin(c+dx)}}$$
$$-\frac{9a(7a^4-9a^2b^2+2b^4)\,e^6\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^6\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{9a(7a^4-9a^2b^2+2b^4)\,e^6\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^6\sqrt{-a^2+b^2}\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{3e^5(3(7a^2-2b^2)-7ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^5d}$$
$$+\frac{9e^3(7a+2b\cos(c+dx))(e\sin(c+dx))^{5/2}}{20b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}$$
$$-\frac{(9(7a^4-9a^2b^2+2b^4)\,e^6)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8b^5\sqrt{-a^2+b^2}d}$$
$$-\frac{(9(7a^4-9a^2b^2+2b^4)\,e^6)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8b^5\sqrt{-a^2+b^2}d}$$

$$= -\frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{11/2}\left(-a^2+b^2\right)^{3/4}d}$$
$$-\frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{11/2}\left(-a^2+b^2\right)^{3/4}d}$$
$$+\frac{3a(21a^2-13b^2)\,e^6\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^6d\sqrt{e\sin(c+dx)}}$$
$$-\frac{9a(7a^4-9a^2b^2+2b^4)\,e^6\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^6\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{9a(7a^4-9a^2b^2+2b^4)\,e^6\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^6\sqrt{-a^2+b^2}\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{3e^5(3(7a^2-2b^2)-7ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^5d}$$
$$+\frac{9e^3(7a+2b\cos(c+dx))(e\sin(c+dx))^{5/2}}{20b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.20 (sec) , antiderivative size = 2024, normalized size of antiderivative = 3.35

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (((2*a*Cos[c + d*x])/b^4 + (-a^2 + b^2)^2/(2*b^5*(a + b*Cos[c + d*x])^2) -
(17*a*(a^2 - b^2))/(4*b^5*(a + b*Cos[c + d*x])) - Cos[2*(c + d*x)]/(5*b^3))
*Csc[c + d*x]^5*(e*Sin[c + d*x])^(11/2))/d + (3*(e*Sin[c + d*x])^(11/2)*((2
*(25*a^3 - 37*a*b^2)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-
2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*Ar
cTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt
[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[
c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*
b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]
^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 -
b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c
+ d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 +
Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(30*a^
2*b - 16*b^3)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*
Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/
4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
 + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d
*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*
a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^
2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^
2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 +
b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*
x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^
2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin
[c + d*x]^2)))))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2]) + ((-40*a^
2*b + 14*b^3)*Cos[c + d*x]*Cos[2*(c + d*x)]*(a + b*Sqrt[1 - Sin[c + d*x]^2]
)*((((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x
]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^
2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)
])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2
 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c
```

```
 + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sq
rt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*
b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Sin[c + d*x]])/b -
(4*a*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/
4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin
[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1
[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a
^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(
-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*
Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x]^2])))/(40*b^5*d*
Sin[c + d*x]^(11/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2751 vs. $2(621) = 1242$.

Time = 100.88 (sec) , antiderivative size = 2752, normalized size of antiderivative = 4.56

method	result	size
default	Expression too large to display	2752

```
[In] int((e*sin(d*x+c))^(11/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2*e^3*b*(-1/5/b^6*(e*sin(d*x+c))^(1/2)*e^2*(b^2*cos(d*x+c)^2+30*a^2-11*b^2
)+e^4/b^6*(-1/8*(e*sin(d*x+c))^(1/2)*e^2*(-19*a^4*b^2*cos(d*x+c)^2+21*a^2*b
^4*cos(d*x+c)^2-2*b^6*cos(d*x+c)^2+15*a^6-13*a^4*b^2-2*a^2*b^4)/(-b^2*cos(d
*x+c)^2*e^2+a^2*e^2)^2+9/64*(7*a^4-9*a^2*b^2+2*b^4)*(e^2*(a^2-b^2)/b^2)^(1/
4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e
*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a
^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))
+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arcta
n(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))-(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)*e^6*a*(1/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(10*(1
-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin
(d*x+c))^(1/2),1/2*2^(1/2))*a^2-7*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/
2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-2*b^2*c
os(d*x+c)^2*sin(d*x+c))+(21*a^4-30*a^2*b^2+9*b^4)/b^6*(-1/2/(-a^2+b^2)^(1/2
)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c
)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1
/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x
+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2
+b^2)^(1/2)/b),1/2*2^(1/2)))+(-15*a^6+33*a^4*b^2-21*a^2*b^4+3*b^6)/b^6*(1/2
*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a
```

```
^2)+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/
2*2^(1/2))-5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^
2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^
(1/2))+1/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^
2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^
(1/2))+5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(
1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2
))-1/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(
1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2
)))+4*a^2*(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/b^6*(1/4*b^2/e/a^2/(a^2-b^2)*(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)^2+1/16*b^2*(13*a^2-6*b^
2)/a^4/(a^2-b^2)^2/e*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a
^2)+13/32/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2
),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*
x+c))^(1/2),1/2*2^(1/2))*b^2-45/64/(a^2-b^2)^2/(-a^2+b^2)^(1/2)/b*(1-sin(d*
x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x
+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^
2+b^2)^(1/2)/b),1/2*2^(1/2))+9/16/a^2/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*b*(1-sin
(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(
-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*b^3*(
1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1
/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+45/64/(a^2-b^2)^2/(-a^2+b^2)^(1/2)/b*(
1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1
/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-9/16/a^2/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*
b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2
),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+3/16/a^4/(a^2-b^2)^2/(-a^2+b^2)^(1/
2)*b^3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*
x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))
^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1
/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(11/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\,\sin(c+d\,x))^{11/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

```
[In] int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^3, x)
```

3.80 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 498

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = -\frac{7(5a^2-2b^2)\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}d} + \frac{7(5a^2-2b^2)\,e^{9/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}d} + \frac{7a(5a^2-2b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{7a(5a^2-2b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^5\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{35ae^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^4d\sqrt{\sin(c+dx)}} + \frac{7e^3(5a+2b\cos(c+dx))(e\sin(c+dx))^{3/2}}{12b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}$$

```
[Out] -7/8*(5*a^2-2*b^2)*e^(9/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(9/2)/(-a^2+b^2)^(1/4)/d+7/8*(5*a^2-2*b^2)*e^(9/2)*arctanh(
b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/(-a^2+b^2)^(
1/4)/d+7/12*e^3*(5*a+2*b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/b^3/d/(a+b*cos(d*
x+c))+1/2*e*(e*sin(d*x+c))^(7/2)/b/d/(a+b*cos(d*x+c))^2-7/8*a*(5*a^2-2*b^2)
*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
Pi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(
1/2)/b^5/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-7/8*a*(5*a^2-2*b^2)*e^
```

```
5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(
cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2
)/b^5/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+35/4*a*e^4*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+
1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^4/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2942, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx &= -\frac{7e^{9/2}(5a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}} \\ &+ \frac{7e^{9/2}(5a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}} \\ &+ \frac{7ae^5(5a^2-2b^2)\sqrt{\sin(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8b^5d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}} \\ &+ \frac{7ae^5(5a^2-2b^2)\sqrt{\sin(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8b^5d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}} \\ &- \frac{35ae^4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^4d\sqrt{\sin(c+dx)}} \\ &+ \frac{7e^3(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{12b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}\end{aligned}$$

```
[In] Int[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-7*(5*a^2 - 2*b^2)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*b^(9/2)*(-a^2 + b^2)^(1/4)*d) + (7*(5*a^2 - 2*b^2)
*e^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*b^(9/2)*(-a^2 + b^2)^(1/4)*d) + (7*a*(5*a^2 - 2*b^2)*e^5*EllipticPi[
(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8
*b^5*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (7*a*(5*a^2 - 2*b^2)*
e^5*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Si
n[c + d*x]])/(8*b^5*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (35*a*
e^4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(4*b^4*d*Sqrt[Si
n[c + d*x]]) + (7*e^3*(5*a + 2*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(12*
b^3*d*(a + b*Cos[c + d*x])) + (e*(e*Sin[c + d*x])^(7/2))/(2*b*d*(a + b*Cos[
c + d*x])^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = \frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(7e^2)\int\frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx}{4b}$$

$$= \frac{7e^3(5a+2b\cos(c+dx))(e\sin(c+dx))^{3/2}}{12b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} + \frac{(7e^4)\int \frac{\left(-b-\frac{5}{2}a\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{4b^3}$$

$$= \frac{7e^3(5a+2b\cos(c+dx))(e\sin(c+dx))^{3/2}}{12b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(35ae^4)\int \sqrt{e\sin(c+dx)}\,dx}{8b^4} + \frac{(7(5a^2-2b^2)e^4)\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{8b^4}$$

$$= \frac{7e^3(5a+2b\cos(c+dx))(e\sin(c+dx))^{3/2}}{12b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(7a(5a^2-2b^2)e^5)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16b^5} + \frac{(7a(5a^2-2b^2)e^5)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16b^5} - \frac{(7(5a^2-2b^2)e^5)\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx, x, e\sin(c+dx)\right)}{8b^3d} - \frac{\left(35ae^4\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{8b^4\sqrt{\sin(c+dx)}}$$

$$= -\frac{35ae^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^4d\sqrt{\sin(c+dx)}} + \frac{7e^3(5a+2b\cos(c+dx))(e\sin(c+dx))^{3/2}}{12b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(7(5a^2-2b^2)e^5)\,\mathrm{Subst}\left(\int \frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{4b^3d} - \frac{\left(7a(5a^2-2b^2)e^5\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16b^5\sqrt{e\sin(c+dx)}} + \frac{\left(7a(5a^2-2b^2)e^5\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16b^5\sqrt{e\sin(c+dx)}}$$

$$= \frac{7a(5a^2-2b^2)e^5 \operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{8b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{7a(5a^2-2b^2)e^5 \operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{8b^5\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{35ae^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^4d\sqrt{\sin(c+dx)}}$$
$$+ \frac{7e^3(5a+2b\cos(c+dx))(e\sin(c+dx))^{3/2}}{12b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}$$
$$+ \frac{(7(5a^2-2b^2)e^5)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{8b^4d}$$
$$- \frac{(7(5a^2-2b^2)e^5)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{8b^4d}$$

$$= -\frac{7(5a^2-2b^2)e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}d}$$
$$+ \frac{7(5a^2-2b^2)e^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}d}$$
$$+ \frac{7a(5a^2-2b^2)e^5 \operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{8b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{7a(5a^2-2b^2)e^5 \operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{8b^5\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{35ae^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^4d\sqrt{\sin(c+dx)}}$$
$$+ \frac{7e^3(5a+2b\cos(c+dx))(e\sin(c+dx))^{3/2}}{12b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.43 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.68

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{\csc^4(c+dx)(e\sin(c+dx))^{9/2}\left(\frac{2\sin(c+dx)}{3b^3}+\frac{11a\sin(c+dx)}{4b^3(a+b\cos(c+dx))}+\frac{-a^2\sin(c+dx)+b^2\sin(c}{2b^3(a+b\cos(c+dx))}\right.}{d}$$

$$-\frac{7(e\sin(c+dx))^{9/2}\left(\frac{5a\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}\right.\right.\right.}{}\right.}{}$$

```
[In] Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^4*(e*Sin[c + d*x])^(9/2)*((2*Sin[c + d*x])/(3*b^3) + (11*a*Si
n[c + d*x])/(4*b^3*(a + b*Cos[c + d*x])) + (-(a^2*Sin[c + d*x]) + b^2*Sin[c
 + d*x])/(2*b^3*(a + b*Cos[c + d*x])^2)))/d - (7*(e*Sin[c + d*x])^(9/2)*((5
*a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqr
t[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]
*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqr
t[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2
- b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d
*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c +
d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))
/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*b
*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x
]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])
/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^
(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 +
I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqr
t[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b
^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a +
b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2]
)))/(8*b^3*d*Sin[c + d*x]^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2735 vs. $2(522) = 1044$.

Time = 98.27 (sec) , antiderivative size = 2736, normalized size of antiderivative = 5.49

method	result	size
default	Expression too large to display	2736

```
[In] int((e*sin(d*x+c))^(9/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2*e^3*b*(1/3*(e*sin(d*x+c))^(3/2)/b^4-e^2/b^4*(-1/8*(e*sin(d*x+c))^(3/2)*e
^2*(-13*a^2*b^2*cos(d*x+c)^2+2*b^4*cos(d*x+c)^2+9*a^4+2*a^2*b^2)/(-b^2*cos(
d*x+c)^2*e^2+a^2*e^2)^2+1/8*(35/8*a^2-7/4*b^2)/b^2/(e^2*(a^2-b^2)/b^2)^(1/4
)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*
2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*
(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(
e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2
-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)*e^5*a*(-3/b^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(2*EllipticE((1-sin(d*x+c))^(1/2),1/2*
2^(1/2))-EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2)))-2*(5*a^2-3*b^2)/b^4*(
-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c
))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin(d*x+c))^(1/2)
*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/
(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2
)/b),1/2*2^(1/2)))+(11*a^4-14*a^2*b^2+3*b^4)/b^4*(1/2*b^2/e/a^2/(a^2-b^2)*s
in(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)-1/2/a^2
/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+
1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1
/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^
(1/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*Ellipti
cPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2
-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^
(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c)
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^
2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2
)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(
1/2)))-4*a^2*(a^4-2*a^2*b^2+b^4)/b^4*(1/4*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)^2+1/16*b^2*(11*a^2-
```

```
6*b^2)/a^4/(a^2-b^2)^2/e*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2
*cos(d*x+c)^2+a^2)-11/16/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-s
in(d*x+c))^(1/2),1/2*2^(1/2))+3/8/a^4/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Ellip
ticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+11/32/a^2/(a^2-b^2)^2*(1-sin(d*x
+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*
(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*
e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-21/64/(
a^2-b^2)^2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin
(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+7/16/a^2/(a^2-b^2)^2*(
1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1
/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^2*(1-sin(d*x+c)
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^
2)^(1/2)/b),1/2*2^(1/2))-21/64/(a^2-b^2)^2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2
+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2
*2^(1/2))+7/16/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*E
llipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a
^4/(a^2-b^2)^2*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1
-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*si
n(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx=\text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(9/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx=\text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx=\int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx=\int \frac{(e\,\sin(c+d\,x))^{9/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

```
[In] int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^3, x)
```

3.81 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 512

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx &= \frac{5(3a^2-2b^2)\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{7/2}\,(-a^2+b^2)^{3/4}\,d}\\
&+\frac{5(3a^2-2b^2)\,e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{7/2}\,(-a^2+b^2)^{3/4}\,d}\\
&-\frac{15ae^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^4d\sqrt{e\sin(c+dx)}}\\
&+\frac{5a(3a^2-2b^2)\,e^4\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{5a(3a^2-2b^2)\,e^4\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^4\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{5e^3(3a+2b\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}\end{aligned}$$

```
[Out] 5/8*(3*a^2-2*b^2)*e^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)^(3/4)/d+5/8*(3*a^2-2*b^2)*e^(7/2)*arctanh(b
^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)^(3
/4)/d+1/2*e*(e*sin(d*x+c))^(5/2)/b/d/(a+b*cos(d*x+c))^2+15/4*a*e^4*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1
/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/(e*sin(d*x+c))^(1/2)-5/8*a*(
3*a^2-2*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*
```

```
sin(d*x+c)^(1/2)/b^4/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-5/
8*a*(3*a^2-2*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+
1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1
/2))*sin(d*x+c)^(1/2)/b^4/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/
2)+5/4*e^3*(3*a+2*b*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^3/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2942, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{5e^{7/2}(3a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}} + \frac{5e^{7/2}(3a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}} + \frac{5ae^4(3a^2-2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8b^4d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}} + \frac{5ae^4(3a^2-2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8b^4d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}} - \frac{15ae^4\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{4b^4d\sqrt{e\sin(c+dx)}} + \frac{5e^3\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{4b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}$$

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (5*(3*a^2 - 2*b^2)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*b^(7/2)*(-a^2 + b^2)^(3/4)*d) + (5*(3*a^2 - 2*b^2)*
e^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/(8*b^(7/2)*(-a^2 + b^2)^(3/4)*d) - (15*a*e^4*EllipticF[(c - Pi/2 + d*x)/
2, 2]*Sqrt[Sin[c + d*x]])/(4*b^4*d*Sqrt[e*Sin[c + d*x]]) + (5*a*(3*a^2 - 2*
b^2)*e^4*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(8*b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c +
d*x]]) + (5*a*(3*a^2 - 2*b^2)*e^4*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*b^4*(a^2 - b*(b + Sqrt[-a^2 +
 b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (5*e^3*(3*a + 2*b*Cos[c + d*x])*Sqrt[e*Si
n[c + d*x]])/(4*b^3*d*(a + b*Cos[c + d*x])) + (e*(e*Sin[c + d*x])^(5/2))/(2
*b*d*(a + b*Cos[c + d*x])^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = \frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(5e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx}{4b}$$

$$= \frac{5e^3(3a+2b\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} + \frac{(5e^4)\int\frac{-b-\frac{3}{2}a\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{4b^3}$$

$$= \frac{5e^3(3a+2b\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(15ae^4)\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{8b^4} + \frac{(5(3a^2-2b^2)\,e^4)\int\frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{8b^4}$$

$$= \frac{5e^3(3a+2b\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(5a(3a^2-2b^2)\,e^4)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16b^4\sqrt{-a^2+b^2}} - \frac{(5a(3a^2-2b^2)\,e^4)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16b^4\sqrt{-a^2+b^2}} - \frac{(5(3a^2-2b^2)\,e^5)\,\text{Subst}\left(\int\frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx, x, e\sin(c+dx)\right)}{8b^3d} - \frac{\left(15ae^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{8b^4\sqrt{e\sin(c+dx)}}$$

$$= -\frac{15ae^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^4d\sqrt{e\sin(c+dx)}} + \frac{5e^3(3a+2b\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(5(3a^2-2b^2)\,e^5)\,\text{Subst}\left(\int\frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{4b^3d} - \frac{\left(5a(3a^2-2b^2)\,e^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16b^4\sqrt{-a^2+b^2}\sqrt{e\sin(c+dx)}} - \frac{\left(5a(3a^2-2b^2)\,e^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16b^4\sqrt{-a^2+b^2}\sqrt{e\sin(c+dx)}}$$

$$= -\frac{15ae^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^4 d\sqrt{e\sin(c+dx)}}$$
$$+\frac{5a(3a^2-2b^2)e^4 \operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{5a(3a^2-2b^2)e^4 \operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^4\sqrt{-a^2+b^2}\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{5e^3(3a+2b\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}$$
$$+\frac{(5(3a^2-2b^2)e^4)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8b^3\sqrt{-a^2+b^2}d}$$
$$+\frac{(5(3a^2-2b^2)e^4)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8b^3\sqrt{-a^2+b^2}d}$$

$$=\frac{5(3a^2-2b^2)e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d}$$
$$+\frac{5(3a^2-2b^2)e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d}$$
$$-\frac{15ae^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^4 d\sqrt{e\sin(c+dx)}}$$
$$+\frac{5a(3a^2-2b^2)e^4 \operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{5a(3a^2-2b^2)e^4 \operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^4\sqrt{-a^2+b^2}\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{5e^3(3a+2b\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.94 (sec) , antiderivative size = 946, normalized size of antiderivative = 1.85

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{\csc^3(c+dx)(e\sin(c+dx))^{7/2}\left(7a^2+2b^2+9ab\cos(c+dx)+\frac{}{\frac{1}{9}\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\,\mathrm{si}}\right.}{}$$

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(e*Sin[c + d*x])^(7/2)*(7*a^2 + 2*b^2 + 9*a*b*Cos[c + d*x]
+ ((a + b*Cos[c + d*x])*(-6*b - 7*a*Cos[c + d*x] + 4*b*Cos[2*(c + d*x)])*(8
*(a + b) - 5*(3*a + 2*b)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[(c + d*x)/2]^2, ((
-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sqrt[Sec[(c + d*x)/2]^2] + (3*a - 2*b)
*AppellF1[5/4, 1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]
^2)/(a + b)]*Sqrt[Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2))/((Sqrt[Sec[(c +
d*x)/2]^2]*Sin[c + d*x]*Tan[(c + d*x)/2]*(-45*(3*a + 2*b)*AppellF1[1/4, 1/2
, 1, 5/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)] + 18*
(3*a - 2*b)*AppellF1[5/4, 1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(
c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2 + (9*(3*a + 2*b)*(2*(a - b)*Appe
llF1[5/4, 1/2, 2, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(
a + b)] + (a + b)*AppellF1[5/4, 3/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)
*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2)/(a + b) + 9*(3*a - 2*b)*
AppellF1[5/4, 1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^
2)/(a + b)]*Tan[(c + d*x)/2]^2 - (5*(3*a - 2*b)*(2*(a - b)*AppellF1[9/4, 1/
2, 2, 13/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)] + (
a + b)*AppellF1[9/4, 3/2, 1, 13/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c +
d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)/(a + b)))/9 + C
os[c + d*x]*(8*(a + b) - 5*(3*a + 2*b)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[(c +
 d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sqrt[Sec[(c + d*x)/2]^2]
 + (3*a - 2*b)*AppellF1[5/4, 1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Ta
n[(c + d*x)/2]^2)/(a + b)]*Sqrt[Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2))))/
(4*b^3*d*(a + b*Cos[c + d*x])^2)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2588 vs. $2(536) = 1072$.

Time = 98.09 (sec) , antiderivative size = 2589, normalized size of antiderivative = 5.06

method	result	size
default	Expression too large to display	2589

```
[In] int((e*sin(d*x+c))^(7/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2*e^3*b*((e*sin(d*x+c))^(1/2)/b^4-e^2/b^4*(-1/8*(e*sin(d*x+c))^(1/2)*e^2*(
-11*a^2*b^2*cos(d*x+c)^2+2*b^4*cos(d*x+c)^2+7*a^4+2*a^2*b^2)/(-b^2*cos(d*x+
c)^2*e^2+a^2*e^2)^2+5/64*(3*a^2-2*b^2)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b
^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^
(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^
(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(
1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))-(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)*e^4*a*(-3/b^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1
/2*2^(1/2))+(-10*a^2+6*b^2)/b^4*(-1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2
)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1
/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2
)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(
1/2)))+1/b^4*(11*a^4-14*a^2*b^2+3*b^4)*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)
^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+1/4/a^2/(a^2-b^2)*(1-sin(d*x
+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/(-a^2+b
^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*
x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)/(-a^2+b
^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*
x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+5/8/(a^2-b^2)/(-a^2+b^2)^
(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c)
)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/4/a^2/(a^2-b^2)/(-a^2+b^2)^
(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c)
)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))-4*a^2*(a^4-2*a^2*b^2+b^4)/b^
4*(1/4*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+
c)^2+a^2)^2+1/16*b^2*(13*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+13/32/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(
```

```
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*(1-sin
(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-45/64/(a^2-b^
2)^2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*Elliptic
Pi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+9/16/a^2/(a^2
-b^2)^2/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*Ellip
ticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(
a^2-b^2)^2/(-a^2+b^2)^(1/2)*b^3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*
EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+45/64
/(a^2-b^2)^2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*
EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-9/16/
a^2/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/
b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+3/
16/a^4/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*b^3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(
1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2
))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a)^3, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\,\sin(c+d\,x))^{7/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

```
[In] int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^3, x)
```

3.82 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 520

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx &= -\frac{3(a^2-2b^2)\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d} \\
&+ \frac{3(a^2-2b^2)\,e^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d} \\
&- \frac{3a(a^2-2b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^3\left(a^2-b^2\right)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{3a(a^2-2b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^3\left(a^2-b^2\right)\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{3ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^2\left(a^2-b^2\right)d\sqrt{\sin(c+dx)}} \\
&+ \frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} - \frac{3ae(e\sin(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\cos(c+dx))}
\end{aligned}$$

```
[Out] -3/8*(a^2-2*b^2)*e^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/
4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(5/4)/d+3/8*(a^2-2*b^2)*e^(5/2)*arctanh(b^(1
/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(5/4)
/d+1/2*e*(e*sin(d*x+c))^(3/2)/b/d/(a+b*cos(d*x+c))^2-3/4*a*e*(e*sin(d*x+c))
^(3/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))+3/8*a*(a^2-2*b^2)*e^3*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+
1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^3/(a^2-b^2)/d
/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+3/8*a*(a^2-2*b^2)*e^3*(sin(1/2*c
```

```
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/
4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^3/(a^2-b
^2)/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-3/4*a*e^2*(sin(1/2*c+1/4*Pi
+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2
*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2943, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = & -\frac{3e^{5/2}(a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{5/2}d\left(b^2-a^2\right)^{5/4}} \\ & +\frac{3e^{5/2}(a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{5/2}d\left(b^2-a^2\right)^{5/4}} \\ & +\frac{3ae^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^2d\left(a^2-b^2\right)\sqrt{\sin(c+dx)}}-\frac{3ae(e\sin(c+dx))^{3/2}}{4bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)} \\ & -\frac{3ae^3(a^2-2b^2)\sqrt{\sin(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8b^3d\left(a^2-b^2\right)\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}} \\ & -\frac{3ae^3(a^2-2b^2)\sqrt{\sin(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8b^3d\left(a^2-b^2\right)\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}} \\ & +\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}\end{aligned}$$

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-3*(a^2 - 2*b^2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^
2)^(1/4)*Sqrt[e])])/(8*b^(5/2)*(-a^2 + b^2)^(5/4)*d) + (3*(a^2 - 2*b^2)*e^(
5/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/
(8*b^(5/2)*(-a^2 + b^2)^(5/4)*d) - (3*a*(a^2 - 2*b^2)*e^3*EllipticPi[(2*b)/
(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*b^3*(
a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (3*a*(a^2 - 2*b
^2)*e^3*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqr
t[Sin[c + d*x]])/(8*b^3*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c +
 d*x]]) + (3*a*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(
4*b^2*(a^2 - b^2)*d*Sqrt[Sin[c + d*x]]) + (e*(e*Sin[c + d*x])^(3/2))/(2*b*d
*(a + b*Cos[c + d*x])^2) - (3*a*e*(e*Sin[c + d*x])^(3/2))/(4*b*(a^2 - b^2)*
d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = \frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(3e^2)\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx}{4b}$$

$$= \frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} - \frac{3ae(e\sin(c+dx))^{3/2}}{4b(a^2-b^2)d(a+b\cos(c+dx))} + \frac{(3e^2)\int \frac{\left(b+\frac{1}{2}a\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{4b(a^2-b^2)}$$

$$= \frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} - \frac{3ae(e\sin(c+dx))^{3/2}}{4b(a^2-b^2)d(a+b\cos(c+dx))} + \frac{(3ae^2)\int\sqrt{e\sin(c+dx)}\,dx}{8b^2(a^2-b^2)} - \frac{(3(a^2-2b^2)e^2)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{8b^2(a^2-b^2)}$$

$$= \frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} - \frac{3ae(e\sin(c+dx))^{3/2}}{4b(a^2-b^2)d(a+b\cos(c+dx))} + \frac{(3a(a^2-2b^2)e^3)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16b^3(a^2-b^2)} - \frac{(3a(a^2-2b^2)e^3)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16b^3(a^2-b^2)} + \frac{(3(a^2-2b^2)e^3)\,\mathrm{Subst}\left(\int\frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx, x, e\sin(c+dx)\right)}{8b(a^2-b^2)d} + \frac{\left(3ae^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{8b^2(a^2-b^2)\sqrt{\sin(c+dx)}}$$

$$= \frac{3ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^2(a^2-b^2)d\sqrt{\sin(c+dx)}} + \frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} - \frac{3ae(e\sin(c+dx))^{3/2}}{4b(a^2-b^2)d(a+b\cos(c+dx))} + \frac{(3(a^2-2b^2)e^3)\,\mathrm{Subst}\left(\int\frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{4b(a^2-b^2)d} + \frac{\left(3a(a^2-2b^2)e^3\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16b^3(a^2-b^2)\sqrt{e\sin(c+dx)}} - \frac{\left(3a(a^2-2b^2)e^3\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16b^3(a^2-b^2)\sqrt{e\sin(c+dx)}}$$

$$= -\frac{3a(a^2-2b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^3\,(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{3a(a^2-2b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^3\,(a^2-b^2)\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{3ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{4b^2\,(a^2-b^2)\,d\sqrt{\sin(c+dx)}}$$
$$+\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3ae(e\sin(c+dx))^{3/2}}{4b\,(a^2-b^2)\,d(a+b\cos(c+dx))}$$
$$-\frac{(3(a^2-2b^2)\,e^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8b^2\,(a^2-b^2)\,d}$$
$$+\frac{(3(a^2-2b^2)\,e^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8b^2\,(a^2-b^2)\,d}$$

$$= -\frac{3(a^2-2b^2)\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{5/2}\,(-a^2+b^2)^{5/4}\,d}$$
$$+\frac{3(a^2-2b^2)\,e^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{5/2}\,(-a^2+b^2)^{5/4}\,d}$$
$$-\frac{3a(a^2-2b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^3\,(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{3a(a^2-2b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^3\,(a^2-b^2)\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{3ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{4b^2\,(a^2-b^2)\,d\sqrt{\sin(c+dx)}}$$
$$+\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3ae(e\sin(c+dx))^{3/2}}{4b\,(a^2-b^2)\,d(a+b\cos(c+dx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.24 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.60

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{\csc^2(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{\sin(c+dx)}{2b(a+b\cos(c+dx))^2}+\frac{3a\sin(c+dx)}{4b(-a^2+b^2)(a+b\cos(c+dx))}\right)}{d}$$

$$+\frac{3(e\sin(c+dx))^{5/2}\left(\frac{a\cos^2(c+dx)\left(3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{}\right.\right.\right.}{}\right.}{}$$

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*(Sin[c + d*x]/(2*b*(a + b*Cos[c + d*
x])^2) + (3*a*Sin[c + d*x])/(4*b*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d + (
3*(e*Sin[c + d*x])^(5/2)*((a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*
(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*A
rcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqr
t[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin
[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[S
in[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin
[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*
Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1
 - Sin[c + d*x]^2)) + (4*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I
)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*S
qrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1
+ I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Lo
g[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2,
1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/
2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x]
)*Sqrt[1 - Sin[c + d*x]^2])))/(8*(a - b)*b*(a + b)*d*Sin[c + d*x]^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. $2(544) = 1088$.

Time = 98.23 (sec) , antiderivative size = 2612, normalized size of antiderivative = 5.02

method	result	size
default	Expression too large to display	2612

```
[In] int((e*sin(d*x+c))^(5/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (e^3*b*(-1/4*(e*sin(d*x+c))^(3/2)*e^2*(-5*a^2*b^2*cos(d*x+c)^2+2*b^4*cos(d*
x+c)^2+a^4+2*a^2*b^2)/b^2/(a^2-b^2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2+3/32*
(a^2-2*b^2)/b^4/(a^2-b^2)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+
c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^
2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1
/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*
(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d
*x+c))^(1/2)-1)))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^3*a*(3/b^2*(-1/2/b^2*
(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*
e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),
1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+
b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*
2^(1/2)))-(7*a^2-3*b^2)/b^2*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)
^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)-1/2/a^2/(a^2-b^2)*(1-sin(d*x
+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-
sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/8/(a^2-b^2)/
b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c
)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1
/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)
^(1/2)/b),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^
(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/
2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi
((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+4*a^2*(a^2-b^2
)/b^2*(1/4*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)
/(-b^2*cos(d*x+c)^2+a^2)^2+1/16*b^2*(11*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*sin(d*
x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)-11/16/a^2/(a
^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+3
```

```
/8/a^4/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2
^(1/2))*b^2+11/32/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/
2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+
c))^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-21/64/(a^2-b^2)^2/b^2*(1-sin(d*x+c))^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^
(1/2)/b),1/2*2^(1/2))+7/16/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2
)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(
1/2))-3/16/a^4/(a^2-b^2)^2*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*E
llipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-21/64/
(a^2-b^2)^2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-si
n(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+7/16/a^2/(a^2-b^2)^2*
(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*
e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),
1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^2*(1-sin(d*x+c
))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b
^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\,\sin(c+d\,x))^{5/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

```
[In] int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3, x)
```

3.83 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 534

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx &= -\frac{(a^2+2b^2)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{3/2}\,(-a^2+b^2)^{7/4}\,d} \\ &- \frac{(a^2+2b^2)\,e^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{3/2}\,(-a^2+b^2)^{7/4}\,d} \\ &- \frac{ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^2\,(a^2-b^2)\,d\sqrt{e\sin(c+dx)}} \\ &+ \frac{a(a^2+2b^2)\,e^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^2\,(a^2-b^2)\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ &+ \frac{a(a^2+2b^2)\,e^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^2\,(a^2-b^2)\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ &+ \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{ae\sqrt{e\sin(c+dx)}}{4b\,(a^2-b^2)\,d(a+b\cos(c+dx))}\end{aligned}$$

```
[Out] -1/8*(a^2+2*b^2)*e^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/
4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(7/4)/d-1/8*(a^2+2*b^2)*e^(3/2)*arctanh(b^(1
/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(7/4)
/d+1/4*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/b^2/(a^2-b^2)
/d/(e*sin(d*x+c))^(1/2)-1/8*a*(a^2+2*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(
b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/(a^2-b^2)/d/(a^2-b*(b-(-a
```

```
^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-1/8*a*(a^2+2*b^2)*e^2*(sin(1/2*c+1/4*P
i+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1
/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/(a^2-b^2)/d/
(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+1/2*e*(e*sin(d*x+c))^(1/2
)/b/d/(a+b*cos(d*x+c))^2-1/4*a*e*(e*sin(d*x+c))^(1/2)/b/(a^2-b^2)/d/(a+b*co
s(d*x+c))
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2943, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = &-\frac{e^{3/2}(a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{3/2}d\left(b^2-a^2\right)^{7/4}} \\
&-\frac{e^{3/2}(a^2+2b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{3/2}d\left(b^2-a^2\right)^{7/4}} \\
&-\frac{ae^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{4b^2d\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}} \\
&+\frac{ae^2(a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8b^2d\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}} \\
&+\frac{ae^2(a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8b^2d\left(a^2-b^2\right)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}} \\
&-\frac{ae\sqrt{e\sin(c+dx)}}{4bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}
\end{aligned}$$

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -1/8*((a^2 + 2*b^2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*(-a^2 + b^2)^(7/4)*d) - ((a^2 + 2*b^2)*e^(3/
2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8
*b^(3/2)*(-a^2 + b^2)^(7/4)*d) - (a*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(4*b^2*(a^2 - b^2)*d*Sqrt[e*Sin[c + d*x]]) + (a*(a^2 + 2*
b^2)*e^2*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(8*b^2*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqr
t[e*Sin[c + d*x]]) + (a*(a^2 + 2*b^2)*e^2*EllipticPi[(2*b)/(b + Sqrt[-a^2 +
 b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*b^2*(a^2 - b^2)*(a^2
- b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (e*Sqrt[e*Sin[c + d*x
]])/(2*b*d*(a + b*Cos[c + d*x])^2) - (a*e*Sqrt[e*Sin[c + d*x]])/(4*b*(a^2 -
 b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{e^2\int\frac{\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx}{4b}$$

$$= \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{ae\sqrt{e\sin(c+dx)}}{4b\,(a^2-b^2)\,d(a+b\cos(c+dx))} - \frac{e^2\int\frac{-b+\frac{1}{2}a\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{4b\,(a^2-b^2)}$$

$$= \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{ae\sqrt{e\sin(c+dx)}}{4b(a^2-b^2)d(a+b\cos(c+dx))} - \frac{(ae^2)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{8b^2(a^2-b^2)} + \frac{((a^2+2b^2)e^2)\int\frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{8b^2(a^2-b^2)}$$

$$= \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{ae\sqrt{e\sin(c+dx)}}{4b(a^2-b^2)d(a+b\cos(c+dx))} + \frac{(a(a^2+2b^2)e^2)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}dx}{16b^2(-a^2+b^2)^{3/2}} + \frac{(a(a^2+2b^2)e^2)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}dx}{16b^2(-a^2+b^2)^{3/2}} - \frac{((a^2+2b^2)e^3)\operatorname{Subst}\left(\int\frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}dx, x, e\sin(c+dx)\right)}{8b(a^2-b^2)d} - \frac{\left(ae^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{8b^2(a^2-b^2)\sqrt{e\sin(c+dx)}}$$

$$= -\frac{ae^2\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^2(a^2-b^2)d\sqrt{e\sin(c+dx)}} + \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{ae\sqrt{e\sin(c+dx)}}{4b(a^2-b^2)d(a+b\cos(c+dx))} - \frac{((a^2+2b^2)e^3)\operatorname{Subst}\left(\int\frac{1}{(a^2-b^2)e^2+b^2x^4}dx, x, \sqrt{e\sin(c+dx)}\right)}{4b(a^2-b^2)d} + \frac{\left(a(a^2+2b^2)e^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}dx}{16b^2(-a^2+b^2)^{3/2}\sqrt{e\sin(c+dx)}} + \frac{\left(a(a^2+2b^2)e^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}dx}{16b^2(-a^2+b^2)^{3/2}\sqrt{e\sin(c+dx)}}$$

$$= -\frac{ae^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^2(a^2-b^2)\,d\sqrt{e\sin(c+dx)}} - \frac{a(a^2+2b^2)\,e^2 \operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^2(-a^2+b^2)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{a(a^2+2b^2)\,e^2 \operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^2(-a^2+b^2)^{3/2}\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{ae\sqrt{e\sin(c+dx)}}{4b(a^2-b^2)\,d(a+b\cos(c+dx))} - \frac{((a^2+2b^2)\,e^2)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8b(-a^2+b^2)^{3/2}\,d} - \frac{((a^2+2b^2)\,e^2)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8b(-a^2+b^2)^{3/2}\,d}$$

$$= -\frac{(a^2+2b^2)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{3/2}(-a^2+b^2)^{7/4}\,d} - \frac{(a^2+2b^2)\,e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{3/2}(-a^2+b^2)^{7/4}\,d} - \frac{ae^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^2(a^2-b^2)\,d\sqrt{e\sin(c+dx)}} - \frac{a(a^2+2b^2)\,e^2 \operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^2(-a^2+b^2)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{a(a^2+2b^2)\,e^2 \operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^2(-a^2+b^2)^{3/2}\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{ae\sqrt{e\sin(c+dx)}}{4b(a^2-b^2)\,d(a+b\cos(c+dx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 1211, normalized size of antiderivative = 2.27

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{\left(\frac{1}{2b(a+b\cos(c+dx))^2}+\frac{a}{4b(-a^2+b^2)(a+b\cos(c+dx))}\right)\csc(c+dx)(e\sin(c+dx))^{3/2}}{d}$$

$$-\frac{(e\sin(c+dx))^{3/2}\left(\frac{2a\cos^2(c+dx)\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b\right.\right.}{}\right.}{}\right.}{}$$

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((1/(2*b*(a + b*Cos[c + d*x])^2) + a/(4*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])
))*Csc[c + d*x]*(e*Sin[c + d*x])^(3/2))/d - ((e*Sin[c + d*x])^(3/2)*((2*a*C
os[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*
Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt
[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*
Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a
^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c
+ d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1
[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt
[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/
2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*Ap
pellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)
] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x
]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((
a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) - (4*b*Cos[c + d*x]*(a + b*Sqrt[1
 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*S
qrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt
[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b
]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2
 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c
 + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4,
 Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sq
rt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9
/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*Appel
lF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*S
```

```
in[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*S
qrt[1 - Sin[c + d*x]^2])))/(8*(a - b)*b*(a + b)*d*Sin[c + d*x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2492 vs. $2(558) = 1116$.

Time = 7.71 (sec) , antiderivative size = 2493, normalized size of antiderivative = 4.67

method	result	size
default	Expression too large to display	2493

```
[In] int((e*sin(d*x+c))^(3/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2*e^3*b*(1/8*(e*sin(d*x+c))^(1/2)*e^2*(3*a^2*b^2*cos(d*x+c)^2-2*b^4*cos(d*
x+c)^2+a^4-2*a^2*b^2)/b^2/(a^2-b^2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2-1/64*
(a^2+2*b^2)/b^2/(a^2-b^2)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/
2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)
+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(
d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^
2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b
^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^2*a
*(3/b^2*(-1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)
*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/
(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1
-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+(-7*a^2+3*b^2)/b^
2*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+
c)^2+a^2)+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1
/2),1/2*2^(1/2))-5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-
a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),
1/2*2^(1/2))+1/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-
a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),
1/2*2^(1/2))+5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+
b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*
2^(1/2))-1/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+
b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*
2^(1/2)))+4*a^2*(a^2-b^2)/b^2*(1/4*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)^2+1/16*b^2*(13*a^2-6*b^2)/a^4/(a^2-b^
2)^2/e*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+13/32/a^2/
```

```
(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))
-3/16/a^4/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/
2*2^(1/2))*b^2-45/64/(a^2-b^2)^2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-
(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b
),1/2*2^(1/2))+9/16/a^2/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)
*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/
(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2
)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*b^3*(1-sin(d*x+c))^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)
^(1/2)/b),1/2*2^(1/2))+45/64/(a^2-b^2)^2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)
^(1/2)/b),1/2*2^(1/2))-9/16/a^2/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c
))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b
^2)^(1/2)/b),1/2*2^(1/2))+3/16/a^4/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*b^3*(1-sin(
d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-
a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\,\sin(c+d\,x))^{3/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

```
[In] int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3, x)
```

3.84 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 529

$$\begin{aligned}
\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx
&= -\frac{(3a^2+2b^2)\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\,(-a^2+b^2)^{9/4}\,d}+\frac{(3a^2+2b^2)\sqrt{e}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\,(-a^2+b^2)^{9/4}\,d} \\
&+\frac{a(3a^2+2b^2)\,e\,\mathrm{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b\,(a^2-b^2)^2\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+\frac{a(3a^2+2b^2)\,e\,\mathrm{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b\,(a^2-b^2)^2\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+\frac{5aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4\,(a^2-b^2)^2\,d\sqrt{\sin(c+dx)}} \\
&-\frac{b(e\sin(c+dx))^{3/2}}{2\,(a^2-b^2)\,de(a+b\cos(c+dx))^2}-\frac{5ab(e\sin(c+dx))^{3/2}}{4\,(a^2-b^2)^2\,de(a+b\cos(c+dx))}
\end{aligned}$$

```
[Out] -1/2*b*(e*sin(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2-5/4*a*b*(e*sin
(d*x+c))^(3/2)/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))-1/8*(3*a^2+2*b^2)*arctan(b^
(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(9/
4)/d/b^(1/2)+1/8*(3*a^2+2*b^2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(9/4)/d/b^(1/2)-1/8*a*(3*a^2+2*b^2)*e
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(c
os(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)
/b/(a^2-b^2)^2/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-1/8*a*(3*a^2+2*b
^2)*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
```

```
cPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/b/(a^2-b^2)^2/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-5/4*a*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*
c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/sin(d*x+c)^(1
/2)
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2943, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\begin{aligned}
&\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx \\
&= -\frac{\sqrt{e}(3a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8\sqrt{b}d\,(b^2-a^2)^{9/4}}+\frac{\sqrt{e}(3a^2+2b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8\sqrt{b}d\,(b^2-a^2)^{9/4}} \\
&\quad -\frac{5ab(e\sin(c+dx))^{3/2}}{4de\,(a^2-b^2)^2\,(a+b\cos(c+dx))}-\frac{b(e\sin(c+dx))^{3/2}}{2de\,(a^2-b^2)\,(a+b\cos(c+dx))^2} \\
&\quad +\frac{5aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4d\,(a^2-b^2)^2\sqrt{\sin(c+dx)}} \\
&\quad +\frac{ae(3a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8bd\,(a^2-b^2)^2\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}} \\
&\quad +\frac{ae(3a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8bd\,(a^2-b^2)^2\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}
\end{aligned}$$

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -1/8*((3*a^2 + 2*b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(Sqrt[b]*(-a^2 + b^2)^(9/4)*d) + ((3*a^2 + 2*b^2)*S
qrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])]
)/(8*Sqrt[b]*(-a^2 + b^2)^(9/4)*d) + (a*(3*a^2 + 2*b^2)*e*EllipticPi[(2*b)/
(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*b*(a^
2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*(3*a^2 + 2*b
^2)*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/(8*b*(a^2 - b^2)^2*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d
*x]]) + (5*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(4*(a^2
 - b^2)^2*d*Sqrt[Sin[c + d*x]]) - (b*(e*Sin[c + d*x])^(3/2))/(2*(a^2 - b^2)
*d*e*(a + b*Cos[c + d*x])^2) - (5*a*b*(e*Sin[c + d*x])^(3/2))/(4*(a^2 - b^2
)^2*d*e*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b(e\sin(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2} - \frac{\int \frac{\left(-2a+\frac{1}{2}b\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b(e\sin(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2} - \frac{5ab(e\sin(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))} + \frac{\int \frac{\left(\frac{1}{2}\left(4a^2+b^2\right)+\frac{5}{4}ab\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= -\frac{b(e\sin(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2} - \frac{5ab(e\sin(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))} + \frac{(5a)\int \sqrt{e\sin(c+dx)}\,dx}{8\left(a^2-b^2\right)^2} + \frac{\left(3a^2+2b^2\right)\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{8\left(a^2-b^2\right)^2}$$

$$= -\frac{b(e\sin(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2} - \frac{5ab(e\sin(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))} - \frac{\left(a\left(3a^2+2b^2\right)e\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16b\left(a^2-b^2\right)^2} + \frac{\left(a\left(3a^2+2b^2\right)e\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16b\left(a^2-b^2\right)^2} - \frac{\left(b\left(3a^2+2b^2\right)e\right)\operatorname{Subst}\left(\int \frac{\sqrt{x}}{\left(a^2-b^2\right)e^2+b^2x^2}\,dx, x, e\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} + \frac{\left(5a\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{8\left(a^2-b^2\right)^2\sqrt{\sin(c+dx)}}$$

$$= \frac{5aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4\left(a^2-b^2\right)^2 d\sqrt{\sin(c+dx)}} - \frac{b(e\sin(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2} - \frac{5ab(e\sin(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))} - \frac{\left(b\left(3a^2+2b^2\right)e\right)\operatorname{Subst}\left(\int \frac{x^2}{\left(a^2-b^2\right)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{4\left(a^2-b^2\right)^2 d} - \frac{\left(a\left(3a^2+2b^2\right)e\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16b\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}} + \frac{\left(a\left(3a^2+2b^2\right)e\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16b\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}}$$

$$= \frac{a(3a^2+2b^2)\,e\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b\,(a^2-b^2)^2\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{a(3a^2+2b^2)\,e\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b\,(a^2-b^2)^2\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{5aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{4\,(a^2-b^2)^2\,d\sqrt{\sin(c+dx)}}$$
$$- \frac{b(e\sin(c+dx))^{3/2}}{2\,(a^2-b^2)\,de(a+b\cos(c+dx))^2} - \frac{5ab(e\sin(c+dx))^{3/2}}{4\,(a^2-b^2)^2\,de(a+b\cos(c+dx))}$$
$$+ \frac{((3a^2+2b^2)\,e)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8\,(a^2-b^2)^2\,d}$$
$$- \frac{((3a^2+2b^2)\,e)\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8\,(a^2-b^2)^2\,d}$$

$$= -\frac{(3a^2+2b^2)\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\,(-a^2+b^2)^{9/4}\,d}$$
$$+ \frac{(3a^2+2b^2)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\,(-a^2+b^2)^{9/4}\,d}$$
$$+ \frac{a(3a^2+2b^2)\,e\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b\,(a^2-b^2)^2\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{a(3a^2+2b^2)\,e\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b\,(a^2-b^2)^2\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{5aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{4\,(a^2-b^2)^2\,d\sqrt{\sin(c+dx)}}$$
$$- \frac{b(e\sin(c+dx))^{3/2}}{2\,(a^2-b^2)\,de(a+b\cos(c+dx))^2} - \frac{5ab(e\sin(c+dx))^{3/2}}{4\,(a^2-b^2)^2\,de(a+b\cos(c+dx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 12.24 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{\sqrt{e\sin(c+dx)}\left(-\frac{2b(7a^2-2b^2+5ab\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2}+\frac{\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)\left(\frac{5a\sec(c+dx)\left(3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(2\arctan\left(\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Sqrt[e*Sin[c + d*x]]*((-2*b*(7*a^2 - 2*b^2 + 5*a*b*Cos[c + d*x])*Sin[c + d
*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (Cos[c + d*x]*(a + b*Sqrt[Cos
[c + d*x]^2])*((5*a*Sec[c + d*x]*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1
 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (
Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2
] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]
+ Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]
] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2
, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)))/(Sqrt[b]*(-a^2 +
b^2)) + (48*(4*a^2 + b^2)*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt
[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Si
n[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(
-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 +
b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c +
d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c
+ d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 -
b^2))))/Sqrt[Cos[c + d*x]^2]))/(12*(a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])
*Sqrt[Sin[c + d*x]])))/(8*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2364 vs. 2(553) = 1106.

Time = 7.73 (sec) , antiderivative size = 2365, normalized size of antiderivative = 4.47

method	result	size
default	Expression too large to display	2365

```
[In] int((e*sin(d*x+c))^(1/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2*e^3*b*(-1/8/(a^4-2*a^2*b^2+b^4)*(e*sin(d*x+c))^(3/2)*(-3*a^2*b^2*cos(d*x
+c)^2-2*b^4*cos(d*x+c)^2+7*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2
-1/64*(3*a^2+2*b^2)/(a^4-2*a^2*b^2+b^4)/e^2/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2
^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(
1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*
sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2
*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e
*a*(3/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-
b^2*cos(d*x+c)^2+a^2)-3/2/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-si
n(d*x+c))^(1/2),1/2*2^(1/2))+3/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Elliptic
F((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-9/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(
1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)
/b),1/2*2^(1/2))+3/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/
b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-9/
8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-si
n(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+3/4/a^2/(a^2-b^2)*(1-
sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(
1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-4*a^2*(1/4*b^2/e/a^2/(a^2-b^2)*sin(d*x+c
)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)^2+1/16*b^2*(11*
a^2-6*b^2)/a^4/(a^2-b^2)^2/e*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(
-b^2*cos(d*x+c)^2+a^2)-11/16/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))+3/8/a^4/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*E
llipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+11/32/a^2/(a^2-b^2)^2*(1-sin
(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^2
)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c
```

```
)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-21/
64/(a^2-b^2)^2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1
-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+7/16/a^2/(a^2-b^2)
^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)
^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/
2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^2*(1-sin(d*
x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x
+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^
2+b^2)^(1/2)/b),1/2*2^(1/2))-21/64/(a^2-b^2)^2/b^2*(1-sin(d*x+c))^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(
-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b)
,1/2*2^(1/2))+7/16/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/
b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/
16/a^4/(a^2-b^2)^2*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticP
i((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(
e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{e\sin\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{e\sin\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
[Out] integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{e\,\sin\left(c+d\,x\right)}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

```
[In] int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3, x)
```

3.85 $\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 535

$$\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{3\sqrt{b}(5a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{11/4}d\sqrt{e}}+\frac{3\sqrt{b}(5a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{11/4}d\sqrt{e}} \\
&\quad-\frac{7a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4\left(a^2-b^2\right)^2d\sqrt{e\sin(c+dx)}} \\
&\quad+\frac{3a(5a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&\quad+\frac{3a(5a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^2\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&\quad-\frac{b\sqrt{e\sin(c+dx)}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2}-\frac{7ab\sqrt{e\sin(c+dx)}}{4\left(a^2-b^2\right)^2de(a+b\cos(c+dx))}
\end{aligned}$$

```
[Out] 3/8*(5*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1
/2))*b^(1/2)/(-a^2+b^2)^(11/4)/d/e^(1/2)+3/8*(5*a^2+2*b^2)*arctanh(b^(1/2)*
(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(11/4)/d/
e^(1/2)+7/4*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/
d/(e*sin(d*x+c))^(1/2)-3/8*a*(5*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(
-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a^2-b*(b-(-a^2+b^
2)^(1/2)))/(e*sin(d*x+c))^(1/2)-3/8*a*(5*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d
```

```
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x)
,2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a^2-b*(b
+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-1/2*b*(e*sin(d*x+c))^(1/2)/(a^2-b^
2)/d/e/(a+b*cos(d*x+c))^2-7/4*a*b*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/e/(a+b
*cos(d*x+c))
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2943, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{3\sqrt{b}(5a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8d\sqrt{e}\,(b^2-a^2)^{11/4}}+\frac{3\sqrt{b}(5a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8d\sqrt{e}\,(b^2-a^2)^{11/4}} \\
&\quad-\frac{7ab\sqrt{e\sin(c+dx)}}{4de\,(a^2-b^2)^2\,(a+b\cos(c+dx))}-\frac{b\sqrt{e\sin(c+dx)}}{2de\,(a^2-b^2)\,(a+b\cos(c+dx))^2} \\
&\quad-\frac{7a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{4d\,(a^2-b^2)^2\,\sqrt{e\sin(c+dx)}} \\
&\quad+\frac{3a(5a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8d\,(a^2-b^2)^2\,\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}} \\
&\quad+\frac{3a(5a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8d\,(a^2-b^2)^2\,\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}
\end{aligned}$$

```
[In] Int[1/((a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (3*Sqrt[b]*(5*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(11/4)*d*Sqrt[e]) + (3*Sqrt[b]*(5*a^2
+ 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*(-a^2 + b^2)^(11/4)*d*Sqrt[e]) - (7*a*EllipticF[(c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(4*(a^2 - b^2)^2*d*Sqrt[e*Sin[c + d*x]]) + (3*a*(5*a
^2 + 2*b^2)*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]
*Sqrt[Sin[c + d*x]])/(8*(a^2 - b^2)^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sq
rt[e*Sin[c + d*x]]) + (3*a*(5*a^2 + 2*b^2)*EllipticPi[(2*b)/(b + Sqrt[-a^2
+ b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*(a^2 - b^2)^2*(a^2 -
 b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (b*Sqrt[e*Sin[c + d*x]
])/(2*(a^2 - b^2)*d*e*(a + b*Cos[c + d*x])^2) - (7*a*b*Sqrt[e*Sin[c + d*x]]
)/(4*(a^2 - b^2)^2*d*e*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b\sqrt{e\sin(c+dx)}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2}-\frac{\int\frac{-2a+\frac{3}{2}b\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sqrt{e\sin(c+dx)}}{2(a^2-b^2)de(a+b\cos(c+dx))^2} - \frac{7ab\sqrt{e\sin(c+dx)}}{4(a^2-b^2)^2 de(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}(4a^2+3b^2)-\frac{7}{4}ab\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2(a^2-b^2)^2}$$

$$= -\frac{b\sqrt{e\sin(c+dx)}}{2(a^2-b^2)de(a+b\cos(c+dx))^2} - \frac{7ab\sqrt{e\sin(c+dx)}}{4(a^2-b^2)^2 de(a+b\cos(c+dx))} - \frac{(7a)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{8(a^2-b^2)^2} + \frac{(3(5a^2+2b^2))\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{8(a^2-b^2)^2}$$

$$= -\frac{b\sqrt{e\sin(c+dx)}}{2(a^2-b^2)de(a+b\cos(c+dx))^2} - \frac{7ab\sqrt{e\sin(c+dx)}}{4(a^2-b^2)^2 de(a+b\cos(c+dx))} - \frac{(3a(5a^2+2b^2))\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16(-a^2+b^2)^{5/2}} - \frac{(3a(5a^2+2b^2))\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16(-a^2+b^2)^{5/2}} - \frac{(3b(5a^2+2b^2)e)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx, x, e\sin(c+dx)\right)}{8(a^2-b^2)^2 d} - \frac{\left(7a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{8(a^2-b^2)^2\sqrt{e\sin(c+dx)}}$$

$$= -\frac{7a\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4(a^2-b^2)^2 d\sqrt{e\sin(c+dx)}} - \frac{b\sqrt{e\sin(c+dx)}}{2(a^2-b^2)de(a+b\cos(c+dx))^2} - \frac{7ab\sqrt{e\sin(c+dx)}}{4(a^2-b^2)^2 de(a+b\cos(c+dx))} - \frac{(3b(5a^2+2b^2)e)\,\mathrm{Subst}\left(\int \frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{4(a^2-b^2)^2 d} - \frac{\left(3a(5a^2+2b^2)\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16(-a^2+b^2)^{5/2}\sqrt{e\sin(c+dx)}} - \frac{\left(3a(5a^2+2b^2)\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16(-a^2+b^2)^{5/2}\sqrt{e\sin(c+dx)}}$$

$$= -\frac{7a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4\left(a^2-b^2\right)^2 d\sqrt{e\sin(c+dx)}} + \frac{3a(5a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(-a^2+b^2\right)^{5/2}\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{3a(5a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(-a^2+b^2\right)^{5/2}\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{b\sqrt{e\sin(c+dx)}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2} - \frac{7ab\sqrt{e\sin(c+dx)}}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))} + \frac{(3b(5a^2+2b^2))\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8\left(-a^2+b^2\right)^{5/2}d} + \frac{(3b(5a^2+2b^2))\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8\left(-a^2+b^2\right)^{5/2}d}$$

$$= \frac{3\sqrt{b}(5a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{11/4}d\sqrt{e}} + \frac{3\sqrt{b}(5a^2+2b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{11/4}d\sqrt{e}} - \frac{7a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4\left(a^2-b^2\right)^2 d\sqrt{e\sin(c+dx)}} + \frac{3a(5a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(-a^2+b^2\right)^{5/2}\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{3a(5a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(-a^2+b^2\right)^{5/2}\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{b\sqrt{e\sin(c+dx)}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2} - \frac{7ab\sqrt{e\sin(c+dx)}}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.45 (sec) , antiderivative size = 1226, normalized size of antiderivative = 2.29

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx$$

$$= \frac{\left(-\frac{b}{2(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{7ab}{4(a^2-b^2)^2(a+b\cos(c+dx))}\right)\sin(c+dx)}{d\sqrt{e\sin(c+dx)}}$$

$$+\frac{\sqrt{\sin(c+dx)}\left(-\frac{14ab\cos^2(c+dx)\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] ((-1/2*b/((a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (7*a*b)/(4*(a^2 - b^2)^2*(a
 + b*Cos[c + d*x])))*Sin[c + d*x])/(d*Sqrt[e*Sin[c + d*x]]) + (Sqrt[Sin[c +
 d*x]]*((-14*a*b*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*Ar
cTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan
[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2
 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c +
d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c
+ d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a
^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/
(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)
*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b
^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*
x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2
, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[
c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(8*a^2 + 6
*b^2)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*
(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*
ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[S
qrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I
*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 -
 b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*Appel
```

```
lF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] -
2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-
a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*
Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x
]^2)))))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(8*(a - b)^2*(a
+ b)^2*d*Sqrt[e*Sin[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2219 vs. 2(559) = 1118.

Time = 7.95 (sec) , antiderivative size = 2220, normalized size of antiderivative = 4.15

method	result	size
default	Expression too large to display	2220

```
[In] int(1/(a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*b*e^3*(-1/8/(a^4-2*a^2*b^2+b^4)*(e*sin(d*x+c))^(1/2)*(-5*a^2*b^2*cos(d*x
+c)^2-2*b^4*cos(d*x+c)^2+9*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2
-3/64*(5*a^2+2*b^2)/(a^4-2*a^2*b^2+b^4)/e^2*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*
e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x
+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/
b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arcta
n(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2
)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)*a*(3/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(
-b^2*cos(d*x+c)^2+a^2)+3/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-s
in(d*x+c))^(1/2),1/2*2^(1/2))-15/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+
c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+
b^2)^(1/2)/b),1/2*2^(1/2))+3/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+
c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+
b^2)^(1/2)/b),1/2*2^(1/2))+15/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^
(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2
)^(1/2)/b),1/2*2^(1/2))-3/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^
(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2
)^(1/2)/b),1/2*2^(1/2))-4*a^2*(1/4*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)^2+1/16*b^2*(13*a^2-6*b^2)/a^4/(a^2-b^
2)^2/e*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+13/32/a^2/
(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))
```

```
-3/16/a^4/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/
2*2^(1/2))*b^2-45/64/(a^2-b^2)^2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-
(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b
),1/2*2^(1/2))+9/16/a^2/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)
*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/
(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2
)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*b^3*(1-sin(d*x+c))^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)
^(1/2)/b),1/2*2^(1/2))+45/64/(a^2-b^2)^2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)
^(1/2)/b),1/2*2^(1/2))-9/16/a^2/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c
))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b
^2)^(1/2)/b),1/2*2^(1/2))+3/16/a^4/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*b^3*(1-sin(
d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-
a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3\sqrt{e\sin(dx+c)}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sqrt(e*sin(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\,\sin(c+d\,x)}\,(a+b\,\cos(c+d\,x))^3}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3),x)
[Out] int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3), x)
```

3.86 $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 611

$$\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \\
&-\frac{5b^{3/2}(7a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}} \\
&+\frac{5b^{3/2}(7a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}} \\
&-\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} \\
&-\frac{9ab}{4\left(a^2-b^2\right)^2de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} \\
&+\frac{5b(7a^2+2b^2)-a(8a^2+37b^2)\cos(c+dx)}{4\left(a^2-b^2\right)^3de\sqrt{e\sin(c+dx)}} \\
&-\frac{5ab(7a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^3\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\
&-\frac{5ab(7a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^3\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\
&-\frac{a(8a^2+37b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4\left(a^2-b^2\right)^3de^2\sqrt{\sin(c+dx)}}
\end{aligned}$$

```
[Out] -5/8*b^(3/2)*(7*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/e^(3/2)+5/8*b^(3/2)*(7*a^2+2*b^2)*arctanh
```

```
(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d
/e^(3/2)-1/2*b/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2)-9/4*a*
b/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2)+1/4*(5*b*(7*a^2+2*b
^2)-a*(8*a^2+37*b^2)*cos(d*x+c))/(a^2-b^2)^3/d/e/(e*sin(d*x+c))^(1/2)+5/8*a
*b*(7*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*
sin(d*x+c)^(1/2)/(a^2-b^2)^3/d/e/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+
5/8*a*b*(7*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+
1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1
/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3/d/e/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(
1/2)+1/4*a*(8*a^2+37*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1
/2)/(a^2-b^2)^3/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2773, 2943, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx =$$
$$-\frac{5b^{3/2}(7a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{3/2}\left(b^2-a^2\right)^{13/4}}$$
$$+\frac{5b^{3/2}(7a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{3/2}\left(b^2-a^2\right)^{13/4}}$$
$$-\frac{a(8a^2+37b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4de^2\left(a^2-b^2\right)^3\sqrt{\sin(c+dx)}}$$
$$-\frac{9ab}{4de\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$
$$-\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$
$$+\frac{5b(7a^2+2b^2)-a(8a^2+37b^2)\cos(c+dx)}{4de\left(a^2-b^2\right)^3\sqrt{e\sin(c+dx)}}$$
$$-\frac{5ab(7a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8de\left(a^2-b^2\right)^3\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}$$
$$-\frac{5ab(7a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8de\left(a^2-b^2\right)^3\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}$$

```
[In] Int[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-5*b^(3/2)*(7*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(13/4)*d*e^(3/2)) + (5*b^(3/2)*(7*a^2
 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(8*(-a^2 + b^2)^(13/4)*d*e^(3/2)) - b/(2*(a^2 - b^2)*d*e*(a + b*Cos[c
+ d*x])^2*Sqrt[e*Sin[c + d*x]]) - (9*a*b)/(4*(a^2 - b^2)^2*d*e*(a + b*Cos[c
 + d*x])*Sqrt[e*Sin[c + d*x]]) + (5*b*(7*a^2 + 2*b^2) - a*(8*a^2 + 37*b^2)*
Cos[c + d*x])/(4*(a^2 - b^2)^3*d*e*Sqrt[e*Sin[c + d*x]]) - (5*a*b*(7*a^2 +
2*b^2)*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/(8*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Sin[c +
d*x]]) - (5*a*b*(7*a^2 + 2*b^2)*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c
 - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*(a^2 - b^2)^3*(b + Sqrt[-a^2 +
b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (a*(8*a^2 + 37*b^2)*EllipticE[(c - Pi/2 +
 d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(4*(a^2 - b^2)^3*d*e^2*Sqrt[Sin[c + d*x]]
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
```

```
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

integral

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} - \frac{\int \frac{-2a+\frac{5}{2}b\cos(c+dx)}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} - \frac{9ab}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{\int \frac{\frac{1}{2}\left(4a^2+5b^2\right)-\frac{27}{4}ab\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} - \frac{9ab}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)}{4\left(a^2-b^2\right)^3 de\sqrt{e\sin(c+dx)}} - \frac{\int \frac{\left(\frac{1}{4}\left(4a^4+36a^2b^2+5b^4\right)+\frac{1}{8}ab\left(8a^2+37b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{\left(a^2-b^2\right)^3 e^2}$$

$$
\begin{aligned}
&= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{9ab}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{5b(7a^2+2b^2)-a(8a^2+37b^2)\cos(c+dx)}{4\left(a^2-b^2\right)^3 de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{(5b^2(7a^2+2b^2))\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{8\left(a^2-b^2\right)^3 e^2} - \frac{(a(8a^2+37b^2))\int\sqrt{e\sin(c+dx)}\,dx}{8\left(a^2-b^2\right)^3 e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{9ab}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{5b(7a^2+2b^2)-a(8a^2+37b^2)\cos(c+dx)}{4\left(a^2-b^2\right)^3 de\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{(5ab(7a^2+2b^2))\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16\left(a^2-b^2\right)^3 e} \\
&\quad - \frac{(5ab(7a^2+2b^2))\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16\left(a^2-b^2\right)^3 e} \\
&\quad + \frac{(5b^3(7a^2+2b^2))\,\mathrm{Subst}\left(\int\frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx,x,e\sin(c+dx)\right)}{8\left(a^2-b^2\right)^3 de} \\
&\quad - \frac{\left(a(8a^2+37b^2)\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{8\left(a^2-b^2\right)^3 e^2\sqrt{\sin(c+dx)}}
\end{aligned}
$$

$$= -\frac{b}{2(a^2-b^2)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} - \frac{9ab}{4(a^2-b^2)^2de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5b(7a^2+2b^2)-a(8a^2+37b^2)\cos(c+dx)}{4(a^2-b^2)^3de\sqrt{e\sin(c+dx)}} - \frac{a(8a^2+37b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4(a^2-b^2)^3de^2\sqrt{\sin(c+dx)}} + \frac{(5b^3(7a^2+2b^2))\,\text{Subst}\left(\int\frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{4(a^2-b^2)^3de} + \frac{\left(5ab(7a^2+2b^2)\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16(a^2-b^2)^3e\sqrt{e\sin(c+dx)}} - \frac{\left(5ab(7a^2+2b^2)\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16(a^2-b^2)^3e\sqrt{e\sin(c+dx)}}$$

$$= -\frac{b}{2(a^2-b^2)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} - \frac{9ab}{4(a^2-b^2)^2de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5b(7a^2+2b^2)-a(8a^2+37b^2)\cos(c+dx)}{4(a^2-b^2)^3de\sqrt{e\sin(c+dx)}} - \frac{5ab(7a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8(a^2-b^2)^3\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} - \frac{5ab(7a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8(a^2-b^2)^3\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} - \frac{a(8a^2+37b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4(a^2-b^2)^3de^2\sqrt{\sin(c+dx)}} - \frac{(5b^2(7a^2+2b^2))\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8(a^2-b^2)^3de} + \frac{(5b^2(7a^2+2b^2))\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8(a^2-b^2)^3de}$$

$$= -\frac{5b^{3/2}(7a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8(-a^2+b^2)^{13/4}de^{3/2}} + \frac{5b^{3/2}(7a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8(-a^2+b^2)^{13/4}de^{3/2}}$$
$$- \frac{b}{2(a^2-b^2)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}$$
$$- \frac{9ab}{4(a^2-b^2)^2de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}$$
$$+ \frac{5b(7a^2+2b^2)-a(8a^2+37b^2)\cos(c+dx)}{4(a^2-b^2)^3de\sqrt{e\sin(c+dx)}}$$
$$- \frac{5ab(7a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8(a^2-b^2)^3\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$- \frac{5ab(7a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8(a^2-b^2)^3\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$- \frac{a(8a^2+37b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4(a^2-b^2)^3de^2\sqrt{\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.89 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \frac{\sin^2(c+dx)\left(-\frac{2(-3a^2b-b^3+a^3\cos(c+dx)+3ab^2\cos(c+dx))\csc(c+dx)}{(a^2-b^2)^3} + \frac{\quad}{2(a}\right.}{d(e\sin(c+dx))^{3/2}}$$
$$- \frac{\sin^{\frac{3}{2}}(c+dx)\left(\frac{(8a^3b+37ab^3)\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b}\right.\right.\right.}{\quad}\right.}{\quad}$$

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sin[c + d*x]^2*((-2*(-3*a^2*b - b^3 + a^3*Cos[c + d*x] + 3*a*b^2*Cos[c + d
*x])*Csc[c + d*x])/(a^2 - b^2)^3 + (b^3*Sin[c + d*x])/(2*(a^2 - b^2)^2*(a +
 b*Cos[c + d*x])^2) + (13*a*b^3*Sin[c + d*x])/(4*(a^2 - b^2)^3*(a + b*Cos[c
 + d*x]))))/(d*(e*Sin[c + d*x])^(3/2)) - (Sin[c + d*x]^(3/2)*(((8*a^3*b + 3
7*a*b^3)*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[
2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*S
```

```
qrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqr
t[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin
[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Si
n[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x
]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2))
+ (2*(8*a^4 + 72*a^2*b^2 + 10*b^4)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 -
 ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((
1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^
2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*
x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c
+ d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/
4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c +
d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[
c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(8*(a - b)^3*(a + b)^3*d*(e*Sin[c + d
*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2836 vs. 2(631) = 1262.

Time = 10.14 (sec) , antiderivative size = 2837, normalized size of antiderivative = 4.64

method	result	size
default	Expression too large to display	2837

```
[In] int(1/(a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*e^3*b*(b^2/e^4/(a-b)^3/(a+b)^3*(1/8*(e*sin(d*x+c))^(3/2)*e^2*(-11*a^2*b^
2*cos(d*x+c)^2-2*b^4*cos(d*x+c)^2+15*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e^2+
a^2*e^2)^2+1/8*(35/8*a^2+5/4*b^2)/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln
((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*
(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c)
)^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1
/4)*(e*sin(d*x+c))^(1/2)-1)))-(-3*a^2-b^2)/e^4/(a^2-b^2)^3/(e*sin(d*x+c))^(
1/2))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e*a*((-a^2-3*b^2)/(a^2-b^2)^3*(2*(1
-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin
(d*x+c))^(1/2),1/2*2^(1/2))-(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2)/(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)+4*a^2*b^2/(a-b)/(a+b)*(1/4*b^2/e/a^2/(a^2-
b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)^2
+1/16*b^2*(11*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)-11/16/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+3/8/a^4/(a^2-b^2)^2*(1-sin(
```

d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+11/32/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-21/64/(a^2-b^2)^2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+7/16/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-21/64/(a^2-b^2)^2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+7/16/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+b^2*(a^2+3*b^2)/(a-b)^3/(a+b)^3*(-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+b^2*(a^2+3*b^2)/(a-b)^2/(a+b)^2*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)-1/2/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1

```
/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)
)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(e\,\sin\left(c+d\,x\right)\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3),x)
[Out] int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3), x)
```

3.87 $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 629

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \frac{7b^{5/2}(9a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}}$$
$$+\frac{7b^{5/2}(9a^2+2b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}}$$
$$-\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}$$
$$-\frac{11ab}{4\left(a^2-b^2\right)^2de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}$$
$$+\frac{7b(9a^2+2b^2)-a(8a^2+69b^2)\cos(c+dx)}{12\left(a^2-b^2\right)^3de(e\sin(c+dx))^{3/2}}$$
$$+\frac{a(8a^2+69b^2)\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{12\left(a^2-b^2\right)^3de^2\sqrt{e\sin(c+dx)}}$$
$$-\frac{7ab^2(9a^2+2b^2)\,\mathrm{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^3\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}}$$
$$-\frac{7ab^2(9a^2+2b^2)\,\mathrm{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^3\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}}$$

```
[Out] 7/8*b^(5/2)*(9*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/(-a^2+b^2)^(15/4)/d/e^(5/2)+7/8*b^(5/2)*(9*a^2+2*b^2)*arctanh(
b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(15/4)/d/
e^(5/2)-1/2*b/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2)-11/4*a*
```

```
b/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2)+1/12*(7*b*(9*a^2+2*
b^2)-a*(8*a^2+69*b^2)*cos(d*x+c))/(a^2-b^2)^3/d/e/(e*sin(d*x+c))^(3/2)-1/12
*a*(8*a^2+69*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2
)^3/d/e^2/(e*sin(d*x+c))^(1/2)+7/8*a*b^2*(9*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/
2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d
*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3/d/e^2/(a
^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+7/8*a*b^2*(9*a^2+2*b^2)*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1
/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^
2-b^2)^3/d/e^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 2.14 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2773, 2943, 2945, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \frac{7b^{5/2}(9a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{5/2}\left(b^2-a^2\right)^{15/4}}$$
$$+\frac{7b^{5/2}(9a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{5/2}\left(b^2-a^2\right)^{15/4}}$$
$$+\frac{a(8a^2+69b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{12de^2\left(a^2-b^2\right)^3\sqrt{e\sin(c+dx)}}$$
$$-\frac{7ab^2(9a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8de^2\left(a^2-b^2\right)^3\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}$$
$$-\frac{7ab^2(9a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8de^2\left(a^2-b^2\right)^3\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}$$
$$-\frac{11ab}{4de\left(a^2-b^2\right)^2(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$
$$-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$
$$+\frac{7b(9a^2+2b^2)-a(8a^2+69b^2)\cos(c+dx)}{12de\left(a^2-b^2\right)^3(e\sin(c+dx))^{3/2}}$$

```
[In] Int[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (7*b^(5/2)*(9*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(15/4)*d*e^(5/2)) + (7*b^(5/2)*(9*a^2
```

```
+ 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*(-a^2 + b^2)^(15/4)*d*e^(5/2)) - b/(2*(a^2 - b^2)*d*e*(a + b*Cos[c +
 d*x])^2*(e*Sin[c + d*x])^(3/2)) - (11*a*b)/(4*(a^2 - b^2)^2*d*e*(a + b*Cos
[c + d*x])*(e*Sin[c + d*x])^(3/2)) + (7*b*(9*a^2 + 2*b^2) - a*(8*a^2 + 69*b
^2)*Cos[c + d*x])/(12*(a^2 - b^2)^3*d*e*(e*Sin[c + d*x])^(3/2)) + (a*(8*a^2
 + 69*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(12*(a^2 -
b^2)^3*d*e^2*Sqrt[e*Sin[c + d*x]]) - (7*a*b^2*(9*a^2 + 2*b^2)*EllipticPi[(2
*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*(
a^2 - b^2)^3*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Sin[c + d*x]]) -
 (7*a*b^2*(9*a^2 + 2*b^2)*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*(a^2 - b^2)^3*(a^2 - b*(b + Sqrt[-a^2
 + b^2]))*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} - \frac{\int \frac{-2a+\frac{7}{2}b\cos(c+dx)}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} - \frac{11ab}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}\left(4a^2+7b^2\right)-\frac{55}{4}ab\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} - \frac{11ab}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{7b(9a^2+2b^2)-a(8a^2+69b^2)\cos(c+dx)}{12\left(a^2-b^2\right)^3 de(e\sin(c+dx))^{3/2}} - \frac{\int \frac{\frac{1}{4}\left(-4a^4+60a^2b^2+21b^4\right)-\frac{1}{8}ab\left(8a^2+69b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3\left(a^2-b^2\right)^3 e^2}$$

$$
\begin{aligned}
&= -\frac{b}{2\left(a^2-b^2\right) de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} \\
&\quad - \frac{11ab}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{7b(9a^2+2b^2)-a(8a^2+69b^2)\cos(c+dx)}{12\left(a^2-b^2\right)^3 de(e\sin(c+dx))^{3/2}} \\
&\quad - \frac{(7b^2(9a^2+2b^2))\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{8\left(a^2-b^2\right)^3 e^2} + \frac{(a(8a^2+69b^2))\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{24\left(a^2-b^2\right)^3 e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b}{2\left(a^2-b^2\right) de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} \\
&\quad - \frac{11ab}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{7b(9a^2+2b^2)-a(8a^2+69b^2)\cos(c+dx)}{12\left(a^2-b^2\right)^3 de(e\sin(c+dx))^{3/2}} \\
&\quad - \frac{(7ab^2(9a^2+2b^2))\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16\left(-a^2+b^2\right)^{7/2} e^2} \\
&\quad - \frac{(7ab^2(9a^2+2b^2))\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16\left(-a^2+b^2\right)^{7/2} e^2} \\
&\quad + \frac{(7b^3(9a^2+2b^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{x}((a^2-b^2)e^2+b^2x^2)}\,dx, x, e\sin(c+dx)\right)}{8\left(a^2-b^2\right)^3 de} \\
&\quad + \frac{\left(a(8a^2+69b^2)\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{24\left(a^2-b^2\right)^3 e^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} \\
&\quad - \frac{11ab}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{7b(9a^2+2b^2)-a(8a^2+69b^2)\cos(c+dx)}{12\left(a^2-b^2\right)^3 de(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{a(8a^2+69b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{12\left(a^2-b^2\right)^3 de^2\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{(7b^3(9a^2+2b^2))\,\text{Subst}\left(\int\frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{4\left(a^2-b^2\right)^3 de} \\
&\quad - \frac{\left(7ab^2(9a^2+2b^2)\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16\left(-a^2+b^2\right)^{7/2}e^2\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{\left(7ab^2(9a^2+2b^2)\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16\left(-a^2+b^2\right)^{7/2}e^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} \\
&\quad - \frac{11ab}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{7b(9a^2+2b^2)-a(8a^2+69b^2)\cos(c+dx)}{12\left(a^2-b^2\right)^3 de(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{a(8a^2+69b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{12\left(a^2-b^2\right)^3 de^2\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{7ab^2(9a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(-a^2+b^2\right)^{7/2}\left(b-\sqrt{-a^2+b^2}\right)de^2\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{7ab^2(9a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(-a^2+b^2\right)^{7/2}\left(b+\sqrt{-a^2+b^2}\right)de^2\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{(7b^3(9a^2+2b^2))\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8\left(-a^2+b^2\right)^{7/2}de^2} \\
&\quad + \frac{(7b^3(9a^2+2b^2))\,\text{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8\left(-a^2+b^2\right)^{7/2}de^2}
\end{aligned}
$$

$$= \frac{7b^{5/2}(9a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}} + \frac{7b^{5/2}(9a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}} - \frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} - \frac{11ab}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{7b(9a^2+2b^2)-a(8a^2+69b^2)\cos(c+dx)}{12\left(a^2-b^2\right)^3 de(e\sin(c+dx))^{3/2}} + \frac{a(8a^2+69b^2)\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{12\left(a^2-b^2\right)^3 de^2\sqrt{e\sin(c+dx)}} + \frac{7ab^2(9a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(-a^2+b^2\right)^{7/2}\left(b-\sqrt{-a^2+b^2}\right)de^2\sqrt{e\sin(c+dx)}} - \frac{7ab^2(9a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(-a^2+b^2\right)^{7/2}\left(b+\sqrt{-a^2+b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.26 (sec) , antiderivative size = 1308, normalized size of antiderivative = 2.08

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \frac{\left(\frac{b^3}{2(a^2-b^2)^2(a+b\cos(c+dx))^2} + \frac{15ab^3}{4(a^2-b^2)^3(a+b\cos(c+dx))} - \frac{2(-3a^2b-b^3+a^3\cos\ldots}{\ldots}\right.}{d(e\sin(c+dx))^{5/2}} + \frac{\sin^{\frac{5}{2}}(c+dx)\left(\frac{2\left(8a^3b+69ab^3\right)\cos^2(c+dx)\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{\ldots}\right.\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] ((b^3/(2*(a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (15*a*b^3)/(4*(a^2 - b^2)^
3*(a + b*Cos[c + d*x])) - (2*(-3*a^2*b - b^3 + a^3*Cos[c + d*x] + 3*a*b^2*C
os[c + d*x])*Csc[c + d*x]^2)/(3*(a^2 - b^2)^3))*Sin[c + d*x]^3)/(d*(e*Sin[c
```

```
 + d*x])^(5/2)) + (Sin[c + d*x]^(5/2)*((2*(8*a^3*b + 69*a*b^3)*Cos[c + d*x]
^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqr
t[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Si
n[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^
2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] +
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(
4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2,
 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*
x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4,
Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4,
 -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 -
b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c
 + d*x])*(1 - Sin[c + d*x]^2)) + (2*(8*a^4 - 120*a^2*b^2 - 42*b^4)*Cos[c +
d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 -
((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1
 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2
] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x
]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c +
 d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1
[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[
Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2,
1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*Appel
lF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] +
(-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2
)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a +
 b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(24*(a - b)^3*(a + b)^3*d*(e*S
in[c + d*x])^(5/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2680 vs. $2(649) = 1298$.

Time = 11.22 (sec) , antiderivative size = 2681, normalized size of antiderivative = 4.26

method	result	size
default	Expression too large to display	2681

```
[In] int(1/(a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*e^3*b*(b^2/e^4/(a-b)^3/(a+b)^3*(1/8*(e*sin(d*x+c))^(1/2)*e^2*(-13*a^2*b^
2*cos(d*x+c)^2-2*b^4*cos(d*x+c)^2+17*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e^2+
a^2*e^2)^2+7/64*(9*a^2+2*b^2)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2
^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(
1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*
sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2
```

```
*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-1/3*(-3*a^2-b^2)/e^4/(a^2-b^2)^3/(e
*sin(d*x+c))^(3/2))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e^2*a*(1/3*(-a^2-3*b^
2)/(a^2-b^2)^3/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*((1-sin(d
*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c
))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+c))+4*a^2*b^2/(a+b)/(a-b)*(1/4
*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a
^2)^2+1/16*b^2*(13*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+13/32/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(
2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*El
lipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*(1-sin(d*x+c
))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-45/64/(a^2-b^2)^2/(
-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1
/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-
sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+9/16/a^2/(a^2-b^2)^
2/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi(
(1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^
2)^2/(-a^2+b^2)^(1/2)*b^3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*Ellipt
icPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+45/64/(a^2-
b^2)^2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*Ellipt
icPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-9/16/a^2/(a
^2-b^2)^2/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*si
n(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*Ell
ipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+3/16/a^4
/(a^2-b^2)^2/(-a^2+b^2)^(1/2)*b^3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/
2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b
)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+b^
2*(a^2+3*b^2)/(a-b)^2/(a+b)^2*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)
*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)
/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)
^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/
2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)
*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)
^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/
2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(
1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1
/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b*(
1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
```

```
*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1
/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+b^2*(a^2+3*b^2)/(a-b)^3/(a+b)^3*(-1/2
/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((
1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1
/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^
(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/
2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(e\,\sin(c+d\,x))^{5/2}\,(a+b\,\cos(c+d\,x))^3}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3),x)
[Out] int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3), x)
```

3.88 $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 700

$$\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \\
&-\frac{9b^{7/2}(11a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{17/4}de^{7/2}} \\
&+\frac{9b^{7/2}(11a^2+2b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{17/4}de^{7/2}} \\
&-\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} \\
&-\frac{13ab}{4\left(a^2-b^2\right)^2de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} \\
&+\frac{9b(11a^2+2b^2)-a(8a^2+109b^2)\cos(c+dx)}{20\left(a^2-b^2\right)^3de(e\sin(c+dx))^{5/2}} \\
&-\frac{3(15b^3(11a^2+2b^2)+a(8a^4-64a^2b^2-139b^4)\cos(c+dx))}{20\left(a^2-b^2\right)^4de^3\sqrt{e\sin(c+dx)}} \\
&+\frac{9ab^3(11a^2+2b^2)\,\mathrm{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^4\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
&+\frac{9ab^3(11a^2+2b^2)\,\mathrm{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^4\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
&-\frac{3a(8a^4-64a^2b^2-139b^4)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20\left(a^2-b^2\right)^4de^4\sqrt{\sin(c+dx)}}
\end{aligned}$$

```
[Out] -9/8*b^(7/2)*(11*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^
(1/4)/e^(1/2))/(-a^2+b^2)^(17/4)/d/e^(7/2)+9/8*b^(7/2)*(11*a^2+2*b^2)*arcta
nh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(17/4)
/d/e^(7/2)-1/2*b/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2)-13/4
*a*b/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2)+1/20*(9*b*(11*a^
2+2*b^2)-a*(8*a^2+109*b^2)*cos(d*x+c))/(a^2-b^2)^3/d/e/(e*sin(d*x+c))^(5/2)
-3/20*(15*b^3*(11*a^2+2*b^2)+a*(8*a^4-64*a^2*b^2-139*b^4)*cos(d*x+c))/(a^2-
b^2)^4/d/e^3/(e*sin(d*x+c))^(1/2)-9/8*a*b^3*(11*a^2+2*b^2)*(sin(1/2*c+1/4*P
i+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1
/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^4/d/e^
3/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-9/8*a*b^3*(11*a^2+2*b^2)*(sin(1
/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*
c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b
^2)^4/d/e^3/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+3/20*a*(8*a^4-64*a^2*
b^2-139*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)
^4/d/e^4/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 2.86 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules

used = {2773, 2943, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = -\frac{9b^{7/2}(11a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{7/2}\left(b^2-a^2\right)^{17/4}} + \frac{9b^{7/2}(11a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{7/2}\left(b^2-a^2\right)^{17/4}} - \frac{13ab}{4de\left(a^2-b^2\right)^2(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))} - \frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2} + \frac{9b(11a^2+2b^2)-a(8a^2+109b^2)\cos(c+dx)}{20de\left(a^2-b^2\right)^3(e\sin(c+dx))^{5/2}} + \frac{9ab^3(11a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8de^3\left(a^2-b^2\right)^4\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}} + \frac{9ab^3(11a^2+2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{8de^3\left(a^2-b^2\right)^4\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}} - \frac{3a(8a^4-64a^2b^2-139b^4)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20de^4\left(a^2-b^2\right)^4\sqrt{\sin(c+dx)}} - \frac{3(15b^3(11a^2+2b^2)+a(8a^4-64a^2b^2-139b^4)\cos(c+dx))}{20de^3\left(a^2-b^2\right)^4\sqrt{e\sin(c+dx)}}$$

```
[In] Int[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] (-9*b^(7/2)*(11*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 +
 b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(17/4)*d*e^(7/2)) + (9*b^(7/2)*(11*a
^2 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(8*(-a^2 + b^2)^(17/4)*d*e^(7/2)) - b/(2*(a^2 - b^2)*d*e*(a + b*Cos[
c + d*x])^2*(e*Sin[c + d*x])^(5/2)) - (13*a*b)/(4*(a^2 - b^2)^2*d*e*(a + b*
Cos[c + d*x])*(e*Sin[c + d*x])^(5/2)) + (9*b*(11*a^2 + 2*b^2) - a*(8*a^2 +
109*b^2)*Cos[c + d*x])/(20*(a^2 - b^2)^3*d*e*(e*Sin[c + d*x])^(5/2)) - (3*(
15*b^3*(11*a^2 + 2*b^2) + a*(8*a^4 - 64*a^2*b^2 - 139*b^4)*Cos[c + d*x]))/(
20*(a^2 - b^2)^4*d*e^3*Sqrt[e*Sin[c + d*x]]) + (9*a*b^3*(11*a^2 + 2*b^2)*El
lipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
d*x]])/(8*(a^2 - b^2)^4*(b - Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Sin[c + d*x]])
+ (9*a*b^3*(11*a^2 + 2*b^2)*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - P
i/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*(a^2 - b^2)^4*(b + Sqrt[-a^2 + b^2]
)*d*e^3*Sqrt[e*Sin[c + d*x]]) - (3*a*(8*a^4 - 64*a^2*b^2 - 139*b^4)*Ellipti
```

```
cE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(20*(a^2 - b^2)^4*d*e^4*Sqr
t[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b}{2(a^2-b^2)\,de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{\int \frac{-2a+\frac{9}{2}b\cos(c+dx)}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx}{2(a^2-b^2)}$$

$$= -\frac{b}{2(a^2-b^2)\,de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{13ab}{4(a^2-b^2)^2\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}(4a^2+9b^2)-\frac{91}{4}ab\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx}{2(a^2-b^2)^2}$$

$$= -\frac{b}{2(a^2-b^2)\,de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{13ab}{4(a^2-b^2)^2\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9b(11a^2+2b^2)-a(8a^2+109b^2)\cos(c+dx)}{20(a^2-b^2)^3\,de(e\sin(c+dx))^{5/2}} - \frac{\int \frac{-\frac{3}{4}(4a^4-28a^2b^2-15b^4)-\frac{3}{8}ab(8a^2+109b^2)\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx}{5(a^2-b^2)^3\,e^2}$$

$$= -\frac{b}{2(a^2-b^2)\,de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{13ab}{4(a^2-b^2)^2\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9b(11a^2+2b^2)-a(8a^2+109b^2)\cos(c+dx)}{20(a^2-b^2)^3\,de(e\sin(c+dx))^{5/2}} - \frac{3(15b^3(11a^2+2b^2)+a(8a^4-64a^2b^2-139b^4)\cos(c+dx))}{20(a^2-b^2)^4\,de^3\sqrt{e\sin(c+dx)}} + \frac{2\int \frac{\left(-\frac{3}{8}(4a^6-32a^4b^2-152a^2b^4-15b^6)-\frac{3}{16}ab(8a^4-64a^2b^2-139b^4)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{5(a^2-b^2)^4\,e^4}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{13ab}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9b(11a^2+2b^2)-a(8a^2+109b^2)\cos(c+dx)}{20\left(a^2-b^2\right)^3 de(e\sin(c+dx))^{5/2}} - \frac{3(15b^3(11a^2+2b^2)+a(8a^4-64a^2b^2-139b^4)\cos(c+dx))}{20\left(a^2-b^2\right)^4 de^3\sqrt{e\sin(c+dx)}} + \frac{(9b^4(11a^2+2b^2))\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{8\left(a^2-b^2\right)^4 e^4} - \frac{(3a(8a^4-64a^2b^2-139b^4))\int\sqrt{e\sin(c+dx)}\,dx}{40\left(a^2-b^2\right)^4 e^4}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{13ab}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9b(11a^2+2b^2)-a(8a^2+109b^2)\cos(c+dx)}{20\left(a^2-b^2\right)^3 de(e\sin(c+dx))^{5/2}} - \frac{3(15b^3(11a^2+2b^2)+a(8a^4-64a^2b^2-139b^4)\cos(c+dx))}{20\left(a^2-b^2\right)^4 de^3\sqrt{e\sin(c+dx)}} - \frac{(9ab^3(11a^2+2b^2))\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16\left(a^2-b^2\right)^4 e^3} + \frac{(9ab^3(11a^2+2b^2))\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16\left(a^2-b^2\right)^4 e^3} - \frac{(9b^5(11a^2+2b^2))\,\mathrm{Subst}\left(\int\frac{\sqrt{x}}{(a^2-b^2)e^2+b^2x^2}\,dx,x,e\sin(c+dx)\right)}{8\left(a^2-b^2\right)^4 de^3} - \frac{\left(3a(8a^4-64a^2b^2-139b^4)\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{40\left(a^2-b^2\right)^4 e^4\sqrt{\sin(c+dx)}}$$

$$= -\frac{b}{2(a^2-b^2)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{13ab}{4(a^2-b^2)^2de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9b(11a^2+2b^2)-a(8a^2+109b^2)\cos(c+dx)}{20(a^2-b^2)^3de(e\sin(c+dx))^{5/2}} - \frac{3(15b^3(11a^2+2b^2)+a(8a^4-64a^2b^2-139b^4)\cos(c+dx))}{20(a^2-b^2)^4de^3\sqrt{e\sin(c+dx)}} - \frac{3a(8a^4-64a^2b^2-139b^4)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20(a^2-b^2)^4de^4\sqrt{\sin(c+dx)}} - \frac{(9b^5(11a^2+2b^2))\,\mathrm{Subst}\left(\int\frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{4(a^2-b^2)^4de^3} - \frac{\left(9ab^3(11a^2+2b^2)\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{16(a^2-b^2)^4e^3\sqrt{e\sin(c+dx)}} + \frac{\left(9ab^3(11a^2+2b^2)\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{-a^2+b^2}+b\sin(c+dx)\right)}\,dx}{16(a^2-b^2)^4e^3\sqrt{e\sin(c+dx)}}$$

$$= -\frac{b}{2(a^2-b^2)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{13ab}{4(a^2-b^2)^2de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9b(11a^2+2b^2)-a(8a^2+109b^2)\cos(c+dx)}{20(a^2-b^2)^3de(e\sin(c+dx))^{5/2}} - \frac{3(15b^3(11a^2+2b^2)+a(8a^4-64a^2b^2-139b^4)\cos(c+dx))}{20(a^2-b^2)^4de^3\sqrt{e\sin(c+dx)}} + \frac{9ab^3(11a^2+2b^2)\,\mathrm{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8(a^2-b^2)^4\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} + \frac{9ab^3(11a^2+2b^2)\,\mathrm{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8(a^2-b^2)^4\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} - \frac{3a(8a^4-64a^2b^2-139b^4)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20(a^2-b^2)^4de^4\sqrt{\sin(c+dx)}} + \frac{(9b^4(11a^2+2b^2))\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e-bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8(a^2-b^2)^4de^3} - \frac{(9b^4(11a^2+2b^2))\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{-a^2+b^2}e+bx^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{8(a^2-b^2)^4de^3}$$

$$= -\frac{9b^{7/2}(11a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8(-a^2+b^2)^{17/4}de^{7/2}} + \frac{9b^{7/2}(11a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8(-a^2+b^2)^{17/4}de^{7/2}} - \frac{b}{2(a^2-b^2)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{13ab}{4(a^2-b^2)^2de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9b(11a^2+2b^2)-a(8a^2+109b^2)\cos(c+dx)}{20(a^2-b^2)^3de(e\sin(c+dx))^{5/2}} - \frac{3(15b^3(11a^2+2b^2)+a(8a^4-64a^2b^2-139b^4)\cos(c+dx))}{20(a^2-b^2)^4de^3\sqrt{e\sin(c+dx)}} + \frac{9ab^3(11a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8(a^2-b^2)^4\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} + \frac{9ab^3(11a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8(a^2-b^2)^4\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} - \frac{3a(8a^4-64a^2b^2-139b^4)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20(a^2-b^2)^4de^4\sqrt{\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.02 (sec) , antiderivative size = 1014, normalized size of antiderivative = 1.45

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \frac{\sin^4(c+dx)\left(-\frac{2(50a^2b^3+10b^5+3a^5\cos(c+dx)-24a^3b^2\cos(c+dx)-39ab^4\cos}{5(a^2-b^2)^4}\right.}{}$$

$$-\frac{3\sin^{\frac{7}{2}}(c+dx)\left(\frac{(8a^5b-64a^3b^3-139ab^5)\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)\right.\right.}{}\right.}{}$$

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] (Sin[c + d*x]^4*((-2*(50*a^2*b^3 + 10*b^5 + 3*a^5*Cos[c + d*x] - 24*a^3*b^2
*Cos[c + d*x] - 39*a*b^4*Cos[c + d*x])*Csc[c + d*x])/(5*(a^2 - b^2)^4) - (2
*(-3*a^2*b - b^3 + a^3*Cos[c + d*x] + 3*a*b^2*Cos[c + d*x])*Csc[c + d*x]^3)
/(5*(a^2 - b^2)^3) - (b^5*Sin[c + d*x])/(2*(a^2 - b^2)^3*(a + b*Cos[c + d*x
])^2) - (21*a*b^5*Sin[c + d*x])/(4*(a^2 - b^2)^4*(a + b*Cos[c + d*x]))))/(d
*(e*Sin[c + d*x])^(7/2)) - (3*Sin[c + d*x]^(7/2)*(((8*a^5*b - 64*a^3*b^3 -
139*a*b^5)*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqr
t[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]
*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[S
qrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*S
in[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*
Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d
*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)
) + (2*(8*a^6 - 64*a^4*b^2 - 304*a^2*b^4 - 30*b^6)*Cos[c + d*x]*(((1/8 + I/
8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] -
 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Lo
g[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(
1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))
+ (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2
]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(40*(a - b)^4*(a + b)
^4*d*(e*Sin[c + d*x])^(7/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3078 vs. 2(716) = 1432.

Time = 12.10 (sec) , antiderivative size = 3079, normalized size of antiderivative = 4.40

method	result	size
default	Expression too large to display	3079

```
[In] int(1/(a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*e^3*b*(-b^4/e^6/(a+b)^4/(a-b)^4*(1/8*(e*sin(d*x+c))^(3/2)*e^2*(-19*a^2*b
^2*cos(d*x+c)^2-2*b^4*cos(d*x+c)^2+23*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e^2
+a^2*e^2)^2+1/8*(99/8*a^2+9/4*b^2)/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(l
n((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2
*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c
))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2
)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(
1/4)*(e*sin(d*x+c))^(1/2)-1)))-1/5*(-3*a^2-b^2)/e^4/(a-b)^3/(a+b)^3/(e*sin(
d*x+c))^(5/2)-2*b^2*(5*a^2+b^2)/e^6/(a+b)^4/(a-b)^4/(e*sin(d*x+c))^(1/2))-(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e^3*a*(-1/5*(-a^2-3*b^2)/(a^2-b^2)^3/(cos(
```

d*x+c)^2*e*sin(d*x+c))^(1/2)/sin(d*x+c)/(cos(d*x+c)^2-1)*(6*(1-sin(d*x+c))^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2
),1/2*2^(1/2))-3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/
2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+6*sin(d*x+c)*cos(d*x+c)^4-8*
cos(d*x+c)^2*sin(d*x+c))+6*b^2*(a^2+b^2)/(a^2-b^2)^4*(2*(1-sin(d*x+c))^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/
2*2^(1/2))-(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ell
ipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2)/(cos(d*x+c)^2*e*si
n(d*x+c))^(1/2)-4*a^2*b^4/(a-b)^2/(a+b)^2*(1/4*b^2/e/a^2/(a^2-b^2)*sin(d*x+
c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)^2+1/16*b^2*(11
*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/
(-b^2*cos(d*x+c)^2+a^2)-11/16/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE
((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+3/8/a^4/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)
*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*
EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+11/32/a^2/(a^2-b^2)^2*(1-si
n(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin
(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^
2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-21
/64/(a^2-b^2)^2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((
1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+7/16/a^2/(a^2-b^2
)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c
)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1
/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^2*(1-sin(d
x+c))^(1/2)(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*
x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a
^2+b^2)^(1/2)/b),1/2*2^(1/2))-21/64/(a^2-b^2)^2/b^2*(1-sin(d*x+c))^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+
(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b
),1/2*2^(1/2))+7/16/a^2/(a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)
/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3
/16/a^4/(a^2-b^2)^2*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*Elliptic
Pi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))-6*b^4*(a^2+b
^2)/(a+b)^4/(a-b)^4*(-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*El
lipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2
*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2)
,1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))-b^4*(5*a^2+3*b^2)/(a-b)^3/(a+b)^3*(
1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*
cos(d*x+c)^2+a^2)-1/2/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(

```
1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*
x+c))^(1/2),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1
-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-
a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),
1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*E
llipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/8/(a
^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*
x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(
d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-
a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(e\,\sin(c+d\,x))^{7/2}\,(a+b\,\cos(c+d\,x))^3}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3),x)
[Out] int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```